Proof without words: The Sum $\sum_{k=1}^{+\infty} \frac{a}{n^k}$

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*Abstract—**Objective**In this note, we prove geometrically that* $\sum_{k=1}^{+\infty} \frac{a}{n^k} = \frac{a}{n-1}$.

***Methodology**: In mathematics, a proof without words is a proof of an identity or mathematical statement, which can be demonstrated as self-evident by a diagram without any accompanying explanatory text. Using this method, we give the value of* $\sum_{k=1}^{+\infty} \frac{a}{n^k}$.

Keywords—fraction, proof without words, sum, sum of fractions.

I. INTRODUCTION

The method of proof without words, or the geometric proof, dates back to about 1300 years ago, where Al-Khwārizmi used geometry to determine the solution of a quadratic equation. Then, The relationship between geometria and analysis has been established.

From the mid-1970s onwards in almost every issue of the undergraduate mathematics journals Mathematics Magazine and College Mathematics Journal there is at least one ‘proof without words' [3].

A proof without words does not contain any words other than literal or numerical symbols and geometrical drawings, It can be thought of as a ‘proof’ that makes use of visual representations, that is, pictures or other visual means to show a mathematical idea, equation or theorem [1].

Writing explanations for and discussing a suitable proof without words can present opportunities to develop insights about and connections between different mathematical ideas. These are also ways to popularise proof in general in the secondary mathematics curriculum.

Mathematicians admire proofs that are ingenious. But mathematicians especially admire proofs that are ingenious and economical—lean, spare arguments that cut directly to the heart of the matter and achieve their objectives with a striking immediacy. Such proofs are said to be elegant.

Mathematical elegance is not unlike that of other creative enterprises. It has much in common with the artistic elegance of a Monet canvas that depicts a French landscape with a few deft brushstrokes or a haiku poem that says more than its words. Elegance is ultimately an aesthetic, not a mathematical property [3].

In [2], We give a generalization of a Roger B. Nelsen result, by giving a closed form expression for $x = [a_0, a_1, \dots, a_k, \overline{b_1, \dots, b_m}]$.

In mathematics, we call geometric series any series of the form $\sum_{k=1}^{+\infty} a\, x^k$.

Of course Alsina and Nelsen [5] were able to give a nice geometric proof of the convergence of a geometric serie example and give the value of its limit. In fact, they rewrite a previews result of Bivens and Klein [4]. The result of consider two similar triangles: large white triangle and gray triangle; they use an iterative procedure consisting in partitioning the large white triangle into similar trapezoids, hence the ratio of horizontal to vertical sides in each is the same, which yields the result.

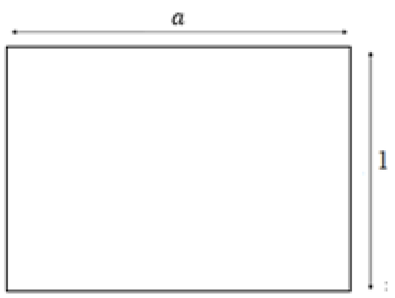
In this note, we give a new geometric proof of the same problem. We consider a rectangle of sides 1 and a, And we use an iterative procedure consisting in partitioning a rectangle in n rectangles. , which yields$\sum_{k=1}^{+\infty} \frac{a}{n^k} = \frac{a}{n-1}$.

II. Main Result

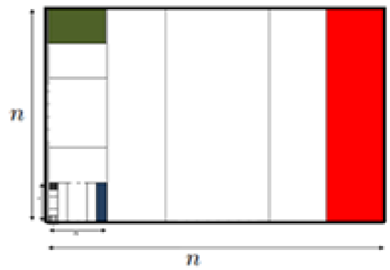
Theorem: Let $n \geq 2$, be an integer, then$\sum_{k=1}^{+\infty} \frac{a}{n^k} = \frac{a}{n-1}$.

Proof

Consider a rectangle of sides 1 and a.



We apply the iterative procedure consisting in partitioning a rectangle in n similar rectangles, which yields



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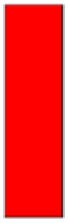
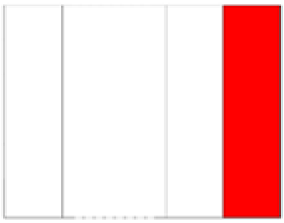
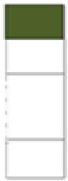
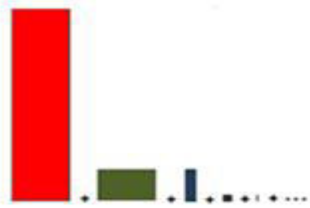
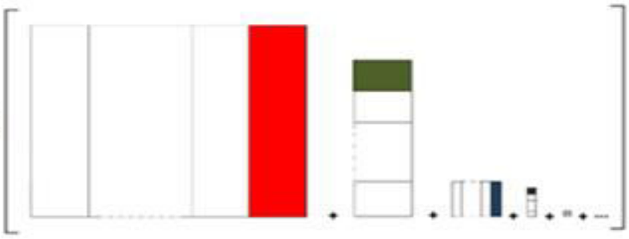
 $= \frac{a}{n} = \frac{1}{n-1}$  ,

 $= \frac{a}{n^2} = \frac{1}{n-1}$  , ...

Then, we get

$\sum_{k=1}^{+\infty} \frac{a}{n^k} =$ 

$= \frac{1}{n-1}$ 

$= \frac{1}{n-1}$ 

$= \frac{a}{n-1}$

Which gives, $\sum_{k=1}^{+\infty} \frac{a}{n^k} = \frac{a}{n-1}$, for all $n \geq 2$.

III. Acknowledgement

The author would like to thank the editor and the anonymous referee who kindly reviewed the earlier version of this manuscript and provided valuable suggestions and comments.

IV. References

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