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Effect of Teachers' Mathematical Knowledge for Teaching (Mkt) on Mathematical Content Knowledge (Mck) and Mathematics Achievement of Students at Secondary Level

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***ABSTRACT**--The purpose of this study was to find out the effect of teachers' Mathematical Knowledge for Teaching (MKT) on the students' Mathematical Content Knowledge (MCK) and the students' achievement in mathematics at secondary level. The research was causal comparative. The participants of the study were 100 mathematics teachers and 1500 grade 10th and 11th students randomly selected from 33 secondary and all the 17 higher secondary public schools of district Lahore. Two instruments were developed out of the Study of Instructional Improvement (SII) survey (Hill, Schilling, & Ball, 2008) to measure teachers' MKT and students' MCK. A mathematics achievement test was developed by the researchers according to the national curriculum of secondary level. All the three instrument were validated from experts and pilot tested. Descriptive statistics, correlation and ANOVA were applied on the data. The result showed that there was a significantly positive relationship between teachers' MKT and students' MCK and between students' MCK and students' achievement in mathematics. However, no significant relationship was found between teachers' MKT and students' achievement in mathematics. Further analysis of demographic variables revealed that another variable "help in study at home" is affecting the MCK and achievement of students as an intervening variable.*

***Keywords**-- Mathematical knowledge for teaching (MKT), mathematical content knowledge (MCK), mathematics achievement, help in study at home, teachers, students*

I. INTRODUCTION

Nothing is more foundational to teacher competency than knowing content (Ball, Thames, & Phelps, 2008). Teachers' orientation to subject and disciplinary background shape the way in which they represent and approach content with the student (Hill, Rowan, & Ball, 2005). Importance of subject matter for teaching is recognized by majority of people. However, what it comprises is loosely understood (Ball et al., 2008). There are two milestones related to the related to the research on mathematical content knowledge (Thanheiser et al., 2014). First was the introduction of *Pedagogical Content Knowledge* by Shulman and his colleagues (Shulman, 1986, 1987; Wilson, Shulman, & Richert, 1987) and the second was the introduction of Mathematical Knowledge for Teaching (Ball, 1990; Ball & Bass, 2000, 2003; Ball, Hill, & Bass, 2005; Hill et al., 2005).

Pedagogical Content Knowledge is the special type of content knowledge unique for teaching. In contrast to the general subject matter and general pedagogical knowledge, it is the combination of knowledge of content and

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pedagogy and is important for the effectiveness of teachers. Content knowledge is the facts, concepts, and the amount of knowledge and its organization in the mind of teacher.

The new domain of teacher knowledge *pedagogical content knowledge* goes beyond the subject matter per se to the content knowledge required *for* teaching. It includes the variety of useful examples, explanations, illustrations, demonstrations, analogies, and forms of representation and formulations of subjects for the comprehension of others. It also includes commonly taught ideas, preconceptions and misconceptions of learners of different background and age regarding the topics and how the learning of particular topics becomes difficult or easy. Mere content knowledge is useless just like skills without content (Shulman, 1986). The notion of *pedagogical content knowledge* provoked immediate and widespread interest among researchers and they used it in thousands of researches across a wide variety of subjects like science, chemistry, mathematics, engineering, social studies, physical education, special education, English, English language learning, communication, religion, music, higher education, and many others (Ball et al., 2008).

In the field of mathematics, the fundamental work was the introduction of Mathematical Knowledge for Teaching (MKT) (Ball & Bass, 2000, 2003; Ball et al., 2005; Hill et al., 2005). *Mathematical knowledge for teaching* means the mathematical knowledge used to carry out the "work of teaching" mathematics. In addition to basic mathematical skills, it includes how to represent the concepts and procedures of mathematics accurately to students by using diagrams, pictures, examples, explanations for common mathematical terms, concepts, rules and procedures; analyzing students' statements, explanations, and solutions; and judging and correcting textbook treatments of particular topics (Hill et al., 2005). Teachers require multidisciplinary knowledge for teaching mathematics. They require knowledge about various topics as well as about different domains (Hill, Schilling, & Ball, 2004).

Research on mathematical content knowledge is growing every year. Meta-analysis study on content knowledge of elementary school prospective mathematics teachers revealed that most of the studies on mathematical content knowledge were conducted in United States (65 %), followed by Austria (8 %), Canada (6.2 %), Turkey (4.5 %), and Taiwan (4.5 %) (Thanheiser et al., 2014). It is an unpacked research area in Pakistan. The current study focuses on identifying the effect Mathematical Knowledge for Teaching (MKT) of teachers on the Mathematical Content Knowledge (MCK) and achievement of students in mathematics at secondary level in Pakistan.

II. LITERATURE REVIEW

Researchers try to identify the factors that affect the achievement of students. The Process-Product Literature on Teaching tries to attempt to identify the relationship between teacher behaviors and student achievement and to predict the student achievement from teacher characteristics. This tradition goes beyond the affective factors and focuses what teachers actually do in classrooms and its effect on student achievement. Certain behaviors affect student achievement gains like focusing class time on active academic instruction rather than nonacademic classroom management tasks, presenting materials in a structured format via advance organizers, making salient linkages explicit, and calling attention to main ideas. However, subject matter knowledge of teachers and subject specific teachers' behaviors were missed in process product research (Hill et al., 2005).

Educational production function research tradition focuses on the relationship between the student achievement on standardized tests and educational resources possessed by school, teachers, students, and others. Most of the educational production function studies focused on proxy variable such as teacher preparation (degree attained by teachers, courses studied), experience, and verbal ability of teachers to predict the achievement of students. Only limited number of studies in education production function research focused on measuring teachers' knowledge by looking at their performance in the certification exams or other test of subject-matter competence. The reason to focus on the subject matter knowledge of teachers is that the student learning is significantly affected by intellectual resources of teachers. These studies were conducted in few subjects and only three were conducted for identifying the effect of mathematical knowledge of teachers for students' achievement (Harbison & Hanushek, 1992; Mullens, Murnane, & Willett, 1996; Rowan, Chiang, & Miller, 1997) but unfortunately with flaws in design and methods. Measuring teachers' scores on subject tests is the recognition of importance of subject matter knowledge of teachers for students' achievement. However, educational production function research was not successful in adequately measuring the teachers' knowledge, hence couldn't identify the magnitude of the effect of teachers' knowledge on student learning and the kinds of teacher knowledge that matter most in producing student learning. The knowledge a teacher has is not sufficient for effective teaching but the thing that matters the most is how this knowledge is used in classrooms. In addition to the proficiency in subject, teachers must be able to use their own knowledge to perform the tasks related to the teaching (Hill et al., 2005).

Another school of thought tried to conceptualize teachers' knowledge differently and argued that it is the teachers' ability to understand and use subject-matter knowledge to carry out the tasks of teaching that really affects students' achievement. They tried to identify the kinds of teacher knowledge and skill that matter the most in helping students learn academic content. Professional development and teacher preparation programs should be content focused and measures of teachers' knowledge should also be content specific particularly content used in classroom teaching (Hill et al., 2005).

Content knowledge predicts student achievement more than any other factor. However, all content demands of teaching are not met by just knowing more advanced mathematics. Mathematics studied in general and teacher education programs doesn't prepare the graduates for mathematics instruction. It is more academic, scholarly and irrelevant and organized towards disciplines and not according to the requirement of teaching (Ball et al., 2008). What teachers actually need to know to teach mathematics is not included in teacher license exams (Hill et al., 2004). Hence, teachers' attainment in mathematics courses doesn't predict achievement gains of students (Panel, 2008). It is necessary to measure teachers' mathematical content knowledge used for teaching mathematics.

Domains of Mathematical Knowledge for Teaching (MKT) (Ball et al., 2008) comprises mainly two types of knowledge: Subject matter knowledge and pedagogical content knowledge that were the (Shulman, 1986)'s two initial categories. Simply calculating a mathematical problem is Common content knowledge (CCK), a subcategory of subject matter knowledge. It includes knowing the material, recognizing a wrong answer of students, or identifying inaccurate material in the textbook. The second subcategory of subject matter Specialized Content Knowledge (SCK) includes mathematical knowledge and skill unique to teaching and usually not required by adults other than teachers. It involves nimble thinking about numbers, flexible thinking about meaning in ways and attention to patterns to size up the nature of an error, particularly an unfamiliar error.

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Knowing both about students and mathematics is the Knowledge of Content and Students (KCS), a subcategory of pedagogical content knowledge. It includes anticipating the thinking and confusions of the students, knowledge of common errors and recognizing the common errors of students. Knowing about teaching and mathematics Knowledge of Content and Teaching (KCT), is another subcategory of pedagogical content knowledge. It involves decisions about the selection and sequencing of content for instruction for the deeper understanding of the students, evaluating the instructional pros and cons of representations for teaching a concept and identifying the instructional feasibility of various methods and procedures. Hence, teachers move from CCK (ordering a list of items related to a topic) to SCK (generating and ordering a list of items that would reveal key mathematical issues), KCS (recognizing the item that would cause difficulty to most students), and KCT (deciding what to do about their difficulties) (Ball et al., 2008).

"Horizon knowledge" is to know how mathematical topics included in the curriculum are related and visualizing useful connections to later mathematical ideas (Ball, 1993). Curriculum knowledge is the (Shulman, 1986)'s third category that includes the arrangements of topics in curriculum over the school year, ability to

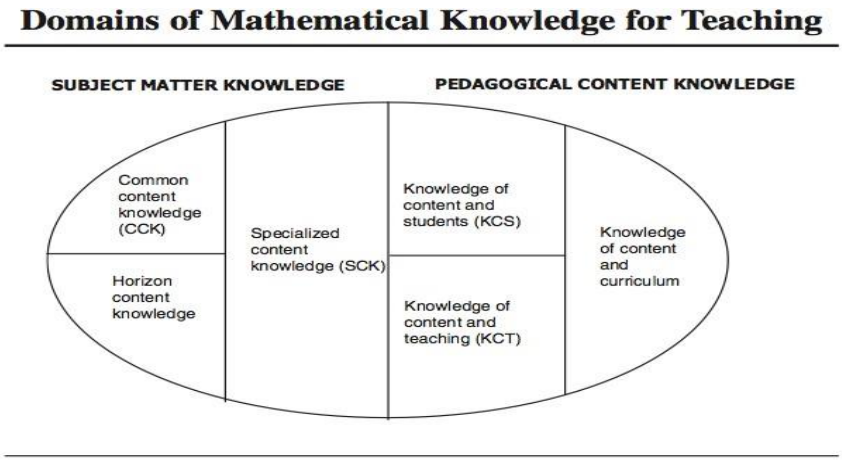

Figure 1: Domains of MKT (Ball et al., 2008)

relate topics being discussed simultaneously in different classes, and using curriculum resources for organizing a program of study for students. The horizon knowledge was provisionally placed within subject matter knowledge and curricular knowledge within pedagogical content knowledge. It was not sure whether Horizon knowledge fit within subject matter knowledge and Knowledge of Content and Curriculum (KCC) within pedagogical content knowledge or whether they may run across the other categories provisionally placed (Ball et al., 2008).

On the basis of education production function research, it is known that teacher knowledge as defined in teacher knowledge research is important for student achievement but what this knowledge is and how it affects student learning was not empirically established. Study of Instructional Improvement (SII) (Ball & Bass, 2000, 2003) started in 1999 to address this issue measures both teachers mathematical content knowledge as well as specialized content knowledge. An instrument for mathematical knowledge for teaching was developed in that study that validly and reliably measures mathematical knowledge and teaching-specific mathematical skills of teachers used in teaching elementary school mathematics (Hill et al., 2004). Scores on this measure were used to predict the gains of mathematics achievement of 1st and 3rd grade students. The independent contribution of mathematical

knowledge for teaching of teachers to achievement of students in mathematics was identified controlling the possible effect of other measures of teacher quality like certification, courses studied, and experience and it was found that teachers' mathematical knowledge for teaching predicted the achievement gains of 1st and 3rd grade students in mathematics (Hill et al., 2005).

The present study identifies the effect Mathematical Knowledge for Teaching (MKT) of teachers on the Mathematical Content Knowledge (MCK) and achievement of students in mathematics at secondary level in Pakistan keeping in view the demographic variables of teachers and students.

Objective of the study

1) To identify the relationship between Mathematical Knowledge for Teaching (MKT) of teachers and Mathematical Content Knowledge (MCK) and mathematics achievement of students studying at secondary and higher secondary schools.

2) To identify the effect of Mathematical Knowledge for Teaching (MKT) of teachers on Mathematical Content Knowledge (MCK) and mathematics achievement of students studying at secondary and higher secondary schools.

III. METHODOLOGY/MATERIALS

The study was causal comparative. The students of 10th and 11th grade studying at secondary and higher secondary schools of district Lahore and their mathematics teachers were the population of the study. All of the 17 higher secondary and about 33 out of the 323 public sector secondary schools (10 %) were randomly selected. About 2 mathematics teachers from each school and 15 students of each teacher were randomly selected. Hence, a total of 100 mathematics teachers and 1500 students of the 50 schools were the participants of the study. About 42% teachers were male and 58 % were females. Qualification of teachers ranged from BA, BSc (16%) to MSc/ BS (Hons) (46%), and to MS/ PhD (38%). Experience of teachers was ranging from 0-5 years (40%) to 5-10 years (22%), 10-15 years (16%), 15-20 years (14%), and to 20-30 years (8%).

Instrument of** the **study

Mathematics released items (Hill et al., 2005) of Study of Instructional Improvement (SII) instrument were administered both for teachers and students with the permission of developer Deborah Ball. After pilot testing, elementary content knowledge items 1, 2, 4, 5, 7, 8, 9; knowledge of content and students items 11, 17; elementary and middle school knowledge of content and teaching items 20, 21, 22, 23, 24, 25, 26, 27; and middle school content knowledge items 30, 32, 33, and 35 of the instrument were retained for the instrument for measuring teachers' MKT and rest of the items were excluded. For measuring students' MCK, items of elementary content knowledge 1, 2, 4, 5; and middle school content knowledge items 29, 31, 32, 33, and 35 of the instrument were retained and other items were excluded. Hence, the final instrument for measuring teachers' MKT comprised 21 items and final instrument for measuring students' MCK comprised 9 items. About 7 items i. e. item number 1, 2, 4, 5, 32, 33, and 35 were common in the both the instruments and rest were different. Maximum scores on teachers' instrument were 36 and on students' instrument were 22.

An achievement test was developed by the researchers. Achievement test comprised 34 items; two items from each of the 17 units of national curriculum of grade 10. About 15 questions were related to knowledge level, 13 questions related to comprehension level, and 6 questions related to application level. After pilot testing, 3 items were excluded and final achievement test comprised 31 items. Maximum score on achievement test were 31. The Cronbach's Alpha value of the achievement test was 0.669. All of the three instruments were validated by 3 experts and pilot tested. About 10% of the sample, i. e., about 5 schools were selected for pilot testing.

IV. RESULTS

It was found that gender ($t(98) = .390$, $p = .697$) and qualification ($F(2, 97) = .183$, $p = .833$) has no significant effect on MKT of teachers. However, teachers with 11-15 years of experience have significantly better MKT ($M = 22.06$, $SD = 4.61$) than the teachers with 0-5 years of experience ($M = 18.18$, $SD = 4.55$, $F(4, 95) = 2.712$, $p < .05$).

The Pearson r correlation was administered to identify the relationship between Mathematical Knowledge for Teaching (MKT) of teachers and Mathematical Content Knowledge (MCK) and achievement of students in mathematics. The results are shown in Table 1.

Table 1: Relationship between MKT of Teachers and MCK and achievement of students in mathematics

Variable	**M**	**SD**	**Variable**	**M**	**SD**	**Pearson r**
Teachers' MKT	19.17	4.73	Students' MCK	12.11	4.28	.283***
Teachers' MKT	19.17	4.73	Mathematics Achievement	15.57	4.53	-.019
Students' MCK	12.11	4.28	Mathematics Achievement	15.57	4.53	.350***

Table 1 shows that there was a significantly positive relationship between Mathematical Knowledge for Teaching (MKT) of teachers and Mathematical Content Knowledge of students (MCK), $r(1498) = .283$, $p < 0.001$. There was a significantly positive relationship between MCK of students and achievement of students in mathematics ($r(1498) = .350$, $p < 0.001$). However, no significant relationship was found between Mathematical Knowledge for Teaching (MKT) of teachers and achievement of students in mathematics ($r (1498) = -.019$, $p = .474$).

Table 2 presents the effect of Mathematical Knowledge for Teaching (MKT) of teachers on students' Mathematical Content Knowledge (MCK) and achievement of students in mathematics.

Table 2: Effect of MKT of teachers on MCK and achievement of students in mathematics

Variable	Teachers' MKT	Students' MCK	Students achievement

	Range	M	SD	Range	M	SD	Range	M	SD
Teachers with high MKT	21-28	23.88	2.59	1-22	13.89	3.38	3-26	15.58	4.26
Teachers with average MKT	18-21	19.47	.88	1-22	11.95	4.20	3-27	15.10	4.18
Teachers with low MKT	3-17	14.15	3.48	1-20	10.48	4.47	0-31	16.04	5.08
F				88.621***			5.372**		

Table 2 showed that the Mathematical Knowledge for Teaching (MKT) of teachers has a significant effect on Mathematical Content Knowledge (MCK) of students ($F(2, 1497) = 88.621$, $p < 0.001$). Tukey HSD post hoc test revealed that MCK of students ($M = 13.89$, $SD = 3.38$) whose teachers have high MKT was significantly higher than the MCK of students ($M = 11.95$, $SD = 4.20$) whose teachers have average MKT and the MCK of students ($M = 10.48$, $SD = 4.47$) whose teachers have low MKT. Similarly, MCK of students ($M = 11.95$, $SD = 4.20$) whose teachers have average MKT was significantly higher than the MCK of students ($M = 10.48$, $SD = 4.47$) whose teachers have low MKT ($F(2, 1497) = 88.621$, $p < 0.001$).

Table 2 shows that the difference in achievement of students in Mathematics with high, average, and low MKT of teachers is less than one point ($M = .934$). Achievement of students in Mathematics ($M= 16.04$, $SD = 5.08$) with low MKT of teachers was significantly higher than the achievement of students in Mathematics ($M = 15.10$, $SD = 4.18$) with average MKT of teachers ($F(2, 1497) = 5.372$, $p < 0.01$) but otherwise it was not following any pattern as evident from the Table 1 ($r (1498) = -.019$, $p = .474$).

Further analysis of demographic variables revealed that help in study at home significantly affected the MCK and achievement of student as evident from Table 3 and it acted as an intervening variable.

Table 3: Effect of help in study at home on MCK and achievement of student in mathematics

Variable	**Students' MCK**				**Achievement**			
	Range	**M**	**SD**	**F**	**Range**	**M**	**SD**	**F**
Father/ Mother (144)	3-20	13.78	3.60	15.364***	6-25	16.80	3.97	6.433***
Brother/Sister (160)	1-22	12.89	4.54		3-24	16.21	4.45	
Self-study (500)	1-21	12.29	4.03		0-26	15.56	4.46	
Home tutor/ Academy (696)	1-22	11.45	4.38		3-31	15.17	4.65	

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Tukey HSD test revealed that Students who receive help in their study from their fathers/ mothers have significantly better MCK (M = 13.78, SD = 3.60) than the students who engage in self-study (M = 12.29, SD = 4.03) or receive help in study from home tutor/ academy (M = 11.45, SD = 4.38), F(3, 1496) = 15.364, p < .000. MCK of students who receive help in their study from their brothers/ sisters (M = 12.89, SD = 4.54) or engaged in self-study (M = 12.29, SD = 4.03) was significantly better than the MCK of students who receive help in study from home tutor/ academy (M = 11.45, SD = 4.38), F(3, 1496) = 15.364, p < .000.

Similarly achievement of students who receive help in study from their fathers/ mothers was significantly better (M= 16.80, SD= 3.97) than the students who engage in self-study (M = 15.56, SD = 4.46) or receive help in study from home tutor/ academy (M = 15.17, SD = 4.65) and the achievement of students who receive help in their study from their brothers/ sisters (M= 16.21, SD = 4.45) was significantly better than the achievement of the students who receive help in study from home tutor/ academy achievement (M = 15.17, SD = 4.65), F(3, 1496) = 6.433, p < .000.

V. CONCLUSION

The result of the study showed that Mathematical Knowledge for Teaching (MKT) for teachers and Mathematical Content Knowledge (MCK) of students were significantly and positively related with each other. Mathematical Knowledge for Teaching (MKT) of teachers has a significant effect on Mathematical Content Knowledge (MCK) of students. The Tukey HSD post-hoc test revealed that the Mathematical Content Knowledge (MCK) of students whose teachers have high MKT was significantly better than the Mathematical Content Knowledge (MCK) of students whose teachers have average and low MKT. Similarly, the Mathematical Content Knowledge (MCK) of students whose teachers have average MKT was significantly better than the mathematical content knowledge (MCK) of students whose teachers have low MKT.

The present study showed that Mathematical Content Knowledge (MCK) and the achievement of students in mathematics were also significantly and positively related with each other. However, no significant relationship was found between MKT of teachers and achievement of students in mathematics. Further analysis of demographic variables revealed that another factor “help in study at home” is affecting the MCK and achievement of students as an intervening variable. Students who receive help in their study from their fathers/ mothers have significantly better MCK and achievement than the students who engage in self-study or receive help in study from home tutor/ academy. Students engaged in self-study also have better MCK than the students who receive help in study from home tutor/ academy. Similarly, students who receive help in their study from their brothers/ sisters have significantly better MCK and achievement than the students who receive help in study from home tutor/ academy.

Mathematical Knowledge for Teaching (MKT) of teachers predicted the mathematical content knowledge of students but not the achievement of students in mathematics. The study identifies another intervening variable “help in study at home” that affects both MCK and achievement of students in mathematics at secondary level.

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