The Concircular Curvature Tensor of Viasman-Grey Manifold

[1]Ebtihal Q. Rashad, [2]Ali A. Shihab, [3]Qasim N. Husain

Abstract: *This paper deals with calculation of components concircular curvature tensor in some aspects of Hermitian manifold especially in Viasman-Grey Manifold. The study also shows that this tensor possesses the classical symmetry properties of the Riemannian curvature. Furthermore, relationships between the components of the tensor in this manifold of Viasman-Grey manifold have been established.*

Keywords: *concircular curvature tensor, Viasman-Grey Manifold, Riemannian curvature*

I. Introduction

Concircular curvature tensors invariant under concircular transformations , i.e. with conformal transformations1of space keeping a harmony of functions. Conformal transformations of Riemannian structures are the important object of differential geometry, in this paper, we investigated the "concirculac curvature tensor of Viasman-Grey Manifold, i.e. the geometrical properties of one of the Almost Hermitian manifold structures is denoted by $\mathbf{W_1} \oplus \mathbf{W_4}$, where $\mathbf{W_1}$ and $\mathbf{W_4}$ respectively denoted to the nearly kahler manifold and locally conformal kahler manifold have been studied .

One of the representative work of differential geometry is an almost Hermititian structure .

In 1975 a great change was made on these studies by The Russian researcher Kirichenko found an interesting method to study the different classes of almost Hermitian manifold ,this method is depending on the orinciple fiber bundle of allcomplex frames of manifold Mwith structure group is the unitary group U(n).This space is called an adjoined G-structure space. The Russian researcher Kirichenko studied the almost Hermitian manifold by adjoined G-structure space in particular, he defined two tensors which were the structure and virtual tensors [6]. These tensors helped him to find the structure group of almost Hermitian manifold. In 1993, Banaru [4] succed in re-classifying the sixteen classes of almost Hermitian manifold by using the structure and Virtual tensors, which were named Kirichenko's tensors [3].

[1]*Tikrit University, College1of Education1for1Pure1Science, Mathematics Department, Tikrit, Iraq*
[2]*Tikrit University, College1of Education1for1Women, Mathematics Department, Tikrit, Iraq*
[3]*Tikrit University, College1of Education1for1Pure1Science, Mathematics Department, Tikrit, Iraq*

In 2015 [2]Ali A.Shihab and Rawah Abdul Mohsin Zaben were studied concircular curvature tensor of nearly Kahler manifold. In this paper we have studied concircular curvature tensor of Vaisman-Grey manifold .

II. Preliminaries

Let M –"smooth manifold of dimension 2n" , C^{∞}(M) is algebra of smooth function on M; X(M) is the module of smooth vector fields on manifold of M; g= <. , .> is Riemannian metrics, ∇ is Riemannian connection of the metrics g on M; d is the operator of exterior differentiation. In the further all manifold, Tensor field, etc. objects are assumed smooth a class C^{∞}(M). The concircular curvature tensor was introduced will be Reminded Yano as a tensor of type (4, 0) on n-dimensional Riemannian manifold.

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We concentrate our attention on the concircular curvature tensor of Viasman-Gray manifold, where Viasman Gray manifold is considered as one of the most important classes of almost Hermitian manifold which is denoted by $W_1 \oplus W_4$ and represents a generalization of the W_1 and W_4 classes ,the space W_1 is called nearly Kähler manifold (NK -manifold) andW_4 is called a locally conformal Kähler manifold (LCK-manifold).

Definition 2.1[5]

A Viasman -Gray structure is an G-structure{J, g = < . , . >} such that:

$$"\nabla_X(F)(X,Y) = \frac{-1}{2(n-1)}\{<X,X>\delta F(Y) - <X,Y>\delta F(X) - <JX,Y>\delta F(JX)\}" \qquad (1)$$

where ∇ is the Riemannian connection of g , $F(X,Y) = <JX,Y>$ is the Kähler form, δ is a coderivative and $X,Y,Z \in X(M)$. An AH-structure (J, g = < . , .>) is called a structure of class W_1 or nearly Kähler(NK -structure) if it's Kähler form is a killing form or equivalently,

$$\nabla_X(J) = 0\,; \qquad X \in X(M). \qquad (2)$$

An AH –structure (J, g = < .,. >) is called a structure of classW_4 or locally

conformal Kähler structure (LCK -structure) if,

$$\nabla_X(F)(Y,Z) = \frac{-1}{2(n-1)}\{<X,Y>\delta F(Z) - <X,Z>\delta F(Y) - <X,JY>\delta F(JZ) + X,JZ>\delta F(JY)\}$$

A manifold M with Vaisman-Gray structure is called a Viasman-Gray manifold (VG -manifold) .

Example 2.2

The main example of Viasman-Gray manifold is the diagonal Hopf manifold ([OV3]) .

Theorem 2.3 [1]

The collection of the structure equations of VG -manifold in the adjointG-structure space has the following forms:

i) $d\omega^a = \omega^a_b \wedge \omega^b + B^{ab}_c \omega^c \wedge \omega_b + B^{abc} \omega_b \wedge \omega_c$;

ii) $d\omega_a = -\omega^b_a \wedge \omega_b + B^c_{ab} \omega_c \wedge \omega^b + \omega_{abc} \omega^b \wedge \omega_c$;

iii) $d\omega^a_b \omega^a_c \wedge \omega^c_b + \left(2B^{adh} B_{hbc} + A^{ad}_{bc}\right)\omega^c \wedge \omega_d + \left(B^{ah}{}_{[C} B_{d]bh} + A^a_{bcd}\right)\omega^c \wedge \omega^d + (B_{bh}{}^{[c} B^{d]ah} + A^{acd}_b)\, \omega_c \wedge \omega_d$

Theorem 2.4 [3]

In the adjoined G-structure space, the components of Riemannian curvature tensor of VG-manifold aregiven by the following forms:

i) $R_{abcd} = 2(B_{ab[cd]} + \alpha_{[a} B_{b]cd})$;

ii)$R_{\hat{a}bcd} = 2A^a_{bcd}$;

iii)$R_{\hat{a}\hat{b}cd} = 2(-B^{abh} B_{hcd} + \alpha^{[a}_{[c} \delta^{b]}_{d]})$;

iv) $R_{\hat{a}bc\hat{d}} = A^{ad}_{bc} + B^{adh} B_{hbc} - B^{ah}{}_c B_{hb}{}^d$,

where $\{\alpha^a{}_b , \alpha_a{}^b , \alpha_{ab} , \alpha^{ab}\}$ are some functions on adjoined G-structure space such that:

$d\alpha_a + \alpha_b \omega^b_a = \alpha^b_a \omega_b + \alpha_{ab} \omega^b$ and $d\alpha^a - \alpha^b \omega^a_b = \alpha^a_b \omega^b + \alpha^{ab} \omega_b$.

Definition 2.5 [2]

A tensor of type (2,0) which is defined as is $r_{ij} = R^k_{ijk} = g^{kl} R_{kijl}$ is called a Ricci tensor.

Definition 2.6[3]

In the adjoint G-structure space, the components of Ricci tensor of Viasman- Grey manifold are given as the following forms:

1) $r_{ab} = \frac{1-n}{2}(\alpha_{ab} + \alpha_{ba} + \alpha_a + \alpha_b)$

2) $r_{\hat{a}b} = r^a_b = 3B^{cah} B_{cbh} + A^{ca}_{cb} + \frac{n-1}{2}\left(\alpha^a \alpha_b - \alpha^h \alpha_h\right) - \frac{1}{2}\alpha^h{}_h \delta^a_b + (n-2)\alpha^a{}_b$

Remark 2.7 [4]

From the Banaru's classification of AH-manifold, the class VG-manifoldsatisfies the following conditions:

$B^{abc} = -B^{bac}$, $B^{ab}_c = \alpha^{[a} \delta^{b]}_c$.

Theorem 2.8 [6]

Let (M,J,g) is AH -manifold $T \in T_r^1(M)$,then the tensor$T_{(k)}, k = 1,2,\dots,2^r - 1$ as nonzero. The component, can have only components of the form $\{T^a_{(k)}\,\alpha_1 \dots \alpha_r, T^{\hat{a}}_{(k)}\,\hat{\alpha}_1 \dots \hat{\alpha}_r\}$ where $\alpha_j = a_i$ or$\alpha_j = \hat{a}_i$ depending on ,whether that there is on a *j*-th a place in binary representation of number k a zero or unit respectively

j=1,2,…,*r* ; $\hat{a} = a + n$. Thus $T^a_{(k)}\,\alpha_1 \dots \alpha_r = T^a \alpha_1 \dots \alpha_r$, $T^{\hat{a}}_{(k)}\,\hat{\alpha}_1 \dots \hat{\alpha}_r$.

Definition 2.9 [8]

"Aconcircular curvature tensor on VG-manifold M is a tensor of type (4,0) and satisfied the relation

$e^{-2f}\bar{C}(X,Y,Z,W) = C(X,Y,Z,W)$, which is defined as the form: $C(X,Y,Z,W) = R(X,Y,Z,W)) - \frac{k}{n(n-1)}\{r(X,W)g(Y,Z) - r(X,Z)g(Y,W)\}$ (3)

Where R is the Riemannian curvature tensor, r is Ricci, g is the Riemannian metric and k-is the scalar curvature $X,Y,Z \in X(M)$. Where$X(M)$ is the Lie algebra of C^∞vector fields on M.

Let's consider properties tensor concircular curvature".

Remark 2.10[2]

Thus,concircular curvature tensor satisfies all the properties of algebraic curvature tensor:

1) $(X,Y,Z,W) = -C(Y,X,Z,W)$;

2) $(X,Y,Z,W) = -C(X,Y,Z,W)$;

3)$(X,Y,Z,W) + C(Y,Z,X,W) + (Z,X,Y,W) = 0$;

4) $(X,Y,Z,W) = C(Z,W,X,Y)$;$X,Y,Z,W \in X(M)$. (4)

Covarient –tensor concircular curvature C type (3,1) have form

$C(X,Y)Z = R(X,Y)Z - \frac{k}{n(n-1)}\{< X,Z > Y - < Y,Z > X\}$ (5)

Where R-is the Riemannian curvature tensor and k-is the scalar curvature, $X,Y,Z \in X(M)$.

By definition of a spectrum tensor.

$C(X,Y)Z = C_0(X,Y)Z + C_1(X,Y)Z + C_2(X,Y)Z + C_3(X,Y)Z + C_4(X,Y)Z + C_5(X,Y)Z + C_6(X,Y)Z +$

$C_7(X,Y)Z$; $X,Y,Z \in X(M)$ (6)

It agreestheorem (7)

tensor$C_0(X,Y)Z$as nonzero. The component can have only components of the form

$\{C^a_{0\ bcd}, C^{\hat{a}}_{0\ \hat{b}\hat{c}\hat{d}}\} = \{C^a{}_{bcd}, C^{\hat{a}}_{\hat{b}\hat{c}\hat{d}}\}$;

tensor $C_1(X,Y)Z$- components of the form $\{C^a_{1\ bc\hat{d}}, C^{\hat{a}}_{1\ \hat{b}\hat{c}d}\} = \{C^a{}_{bc\hat{d}}, C^{\hat{a}}_{\hat{b}\hat{c}d}\}$;

tensor$C_2(X,Y)$Z- components of the form $\{C_{2\ b\hat{c}d}^{a}, C_{2\ \hat{b}c\hat{d}}^{\hat{a}}\} = \{C^{a}{}_{b\hat{c}d}, C_{\hat{b}c\hat{d}}^{\hat{a}}\}$;

tensor$C_3(X,Y)$Z- components of the form $\{C_{3\ b\hat{c}\hat{d}}^{a}, C_{3\ \hat{b}cd}^{\hat{a}}\} = \{C^{a}{}_{b\hat{c}\hat{d}}, C_{\hat{b}cd}^{\hat{a}}\}$;

tensor$C_4(X,Y)$Z- components of the form$\{C_{4\ \hat{b}cd}^{a}, C_{4\ b\hat{c}\hat{d}}^{\hat{a}}\} = \{C_{\hat{b}cd}^{a}, C^{\hat{a}}{}_{b\hat{c}\hat{d}}\}$;

tensor$C_5(X,Y)$Z- components of the form $\{C_{5\ \hat{b}c\hat{d}}^{a}, C_{5\ b\hat{c}d}^{a}\} = \{C_{\hat{b}c\hat{d}}^{a}, C^{\hat{a}}{}_{b\hat{c}d}\}$;

tensor $C_6(X,Y)$Z- components of the form $\{C_{6\ \hat{b}\hat{c}d}^{a}, C_{6\ bc\hat{d}}^{\hat{a}}\} = \{C_{\hat{b}\hat{c}d}^{a}, C^{\hat{a}}{}_{bc\hat{d}}\}$;

tensor$C_7(X,Y)$Z- components of the form$\{C_{7\ \hat{b}\hat{c}\hat{d}}^{a}, C_{7\ bcd}^{\hat{a}}\} = \{C_{\hat{b}\hat{c}\hat{d}}^{a}, C^{\hat{a}}{}_{bcd}\}$.

Tensors$C_0 = C_0(X,Y)Z$, $C_1 = C_1(X,Y)Z$, …,$C_7 = C_7(X,Y)Z$.

The basic invariants concircularVG -manifold will be named.

Proposition 2.11

The concircular curvature of *VG*- manifold satisfies all the properties of the algebraic curvature tensor:-

1) $C(VG)(X_a,X_b,X_c,X_d) = -C(VG)(X_b,X_a,X_c,X_d)$

2) $C(VG)(X_a,X_b,X_c,X_d) = -C(VG)(X_a,X_b,X_d,X_c)$

3) $C(VG)(X_a,X_b,X_c,X_d) + C(VG)(X_b,X_a,X_c,X_d) + C(VG)(X_c,X_a,X_b,X_d) = 0$

4) $C(VG)(X_a,X_b,X_c,X_d) = -C(VG)(X_b,X_c,X_a,X_d)$ Where $X_i \in X(M)$, $i = 1,2,3,4$

Proof:

We shall prove just (1)

1) $C(VG)(X_a,X_b,X_c,X_d) = R(X_a,X_b,X_c,X_d) - \frac{k}{n(n-1)}\{g(X_a,X_c)r(X_b,X_d) - g(X_b,X_c)r(X_a,X_d)\}$

$$= -R(X_a,X_b,X_c,X_d) + \frac{k}{n(n-1)}\{g(X_a,X_c)r(X_b,X_d) - g(X_b,X_c)r(X_a,X_d)\} = -C(X_b,X_a,X_c,X_d)$$

Properties are similarly proved:

2) $C(VG)(X_a,X_b,X_c,X_d) = -C(VG)(X_a,X_b,X_d,X_c)$

3) $C(VG)(X_a,X_b,X_c,X_d) + C(VG)(X_b,X_a,X_c,X_d) + C(VG)(X_c,X_a,X_b,X_d) = 0$

4) $C(VG)(X_a,X_b,X_c,X_d) = -C(VG)(X_c,X_d,X_a,X_b)$

$X_i \in X(M)$, $i = 1,2,3,4$

(1),(2),(3) and (4) is called an algebra curvature tensor of VG-manifolds.

Definition 2.12:

VG - manifold for whichC_i=0 is*VG*- manifold of classC_i, i= 0, 1, …, 7.

Theorem2.13:

1) VG- manifold of class $C_0(VG)$characterized by identity

$$C(VG)(X,Y)Z - C(VG)(X,JY)JZ - C(VG)(JX,Y)JZ - C(VG)(JX,JY)Z - JC(VG)(X,Y)JZ$$

$$-JC(VG)(X,JY)Z - JC(VG)(JX,Y)Z + JC(VG)(JX,JY)JZ = 0 \quad , \; X,Y,Z \in (XM). \quad (7)$$

2) VG- manifold of class$C_1(VG)$characterized by identity

$$C(VG)(X,Y)Z + C(VG)(X,JY)JZ - C(VG)(JX,Y)JZ + C(VG)(JX,JY)Z + JC(VG)(X,Y)JZ$$

$$-JC(VG)(X,JY)Z - JC(VG)(JX,Y)Z - JC(VG)(JX,JY)JZ = 0 \quad , X,Y,Z \in (XM) \quad (8)$$

3) VG- manifold of class$C_2(VG)$characterized by identity

$$C(VG)(X,Y)Z - C(VG)(X,JY)JZ + C(VG)(JX,Y)JZ + C(VG)(JX,JY)Z - JC(VG)(X,Y)JZ$$

$$-JC(VG)(X,JY)Z + JC(VG)(JX,Y)Z - JC(VG)(JX,JY)JZ = 0, \; X,Y,Z \in (XM)(9)$$

4) VG- manifold of class$C_3(VG)$characterized by identity

$$C(VG)(X,Y)Z + C(VG)(X,JY)JZ + C(VG)(JX,Y)JZ - C(VG)(JX,JY)Z - JC(VG)(X,Y)JZ$$

$$+JC(VG)(X,JY)Z + JC(VG)(JX,Y)Z + JC(VG)(JX,JY)JZ = 0 \; , \; X,Y,Z \in (XM) \quad (10)$$

5) VG- manifold of class$C_4(VG)$characterized by identity

$$C(VG)(X,Y)Z + C(VG)(X,JY)JZ + C(VG)(JX,Y)JZ - C(VG)(JX,JY)Z + JC(VG)(X,Y)JZ$$

$$-JC(VG)(X,JY)Z - JC(VG)(JX,Y)Z - JC(VG)(JX,JY)JZ = 0 \quad , X,Y,Z \in (XM) \quad (11)$$

6) VG- manifold of class$C_5(VG)$characterized by identity

$$C(VG)(X,Y)Z - C(VG)(X,JY)JZ + C(VG)(JX,Y)JZ + C(VG)(JX,JY)Z + JC(VG)(X,Y)JZ$$

$$+JC(VG)(X,JY)Z - JC(VG)(JX,Y)Z + JC(VG)(JX,JY)JZ = 0 \; , \; X,Y,Z \in (XM) \; (12)$$

7) VG- manifold of class$C_6(VG)$characterized by identity

$$C(VG)(X,Y)Z + C(VG)(X,JY)JZ - C(VG)(JX,Y)JZ + C(VG)(JX,JY)Z + JC(VG)(X,Y)JZ$$

$$-JC(VG)(X,JY)Z + JC(VG)(JX,Y)Z + JC(VG)(JX,JY)JZ = 0 \; , \; X,Y,Z \in (XM) \quad (13)$$

8) VG - manifold of class$C_7(VG)$characterized by identity

$$C(VG)(X,Y)Z - C(VG)(X,JY)JZ - C(VG)(JX,Y)JZ - C(VG)(JX,JY)Z + JC(VG)(X,Y)JZ$$

$$+JC(VG)(X,JY)Z + JC(VG)(JX,Y)Z - JC(VG)(JX,JY)JZ = 0 \quad , X,Y,Z \in (XM) \, (14)$$

Proof:

1) LetVG- manifold of class$C_0(VG)$,the manifold of class$C_0(VG)$characterized by a condition

$C_0(VG)^a_{bcd} = 0$, *or* $C(VG)^a_{bcd} = 0$

i.e. $\left[C(VG)\left(\varepsilon_{c,\varepsilon_d}\right)\varepsilon_b\right]^a \varepsilon_a$.

Asσ- a projector on, that$D_J^{\sqrt{-1}}\sigma \circ \{C(VG)(\sigma X_1, \sigma Y_1)\sigma Z_1\} = 0$;

i.e(id-$\sqrt{-1}\,J$*)*$\{C(VG)(X - \sqrt{-1}\,JX, Y - \sqrt{-1}\,JY)(Z - \sqrt{-1}\,JZ)\}$ *= 0.*

Removing the brackets can be received: i.e.

$$C(VG)(X,Y)Z - C(VG)(X,JY)JZ - C(VG)(JX,Y)JZ - C(VG)(JX,JY)Z – JC(VG)(X,Y)JZ - JC(VG)(X,JY)Z$$

$$-JC(VG)(JX,Y)Z + JC(VG)(JX,JY)JZ - \sqrt{-1}\{\,C(VG)(X,Y)JZ + C(VG)(X,JY)Z + C(VG)(JX,Y)Z -$$

$$C(VG)(JX,JY)JZ\} - \{JC(VG)(X,Y)Z - JC(VG)(X,JY)JZ - JC(VG)(JX,Y)JZJC - (VG)(JX,JY)Z\} = 0$$

i.e

1) $C(VG)(X,Y)Z - C(VG)(X,JY)JZ - C(VG)(JX,Y)JZ - C(VG)(JX,JY)Z - JC(VG)(X,Y)JZ$

$-JC(VG)(X,JY)Z - JC(VG)(JX,Y)ZJ + JC(VG)(JX,JY)JZ = 0$(15)

2) $C(VG)(X,Y)JZ + C(VG)(X,JY)Z + C(VG)(JX,Y)Z - C(VG)(JX,JY)JZ + JC(VG)(X,Y)Z$

$-JC(VG)(X,JY)JZ - JC(VG)(JX,Y)JZ - JC(VG)(JX,JY)JZ = 0$(16)

These equalities (15) and (16) are equivalent. The second equality turns out from the first replacement Zon J Z. Thus *VG* - manifold of class$C_0(VG)$characterized by identity .

$$C(VG)(X,Y)Z - C(VG)(X,JY)JZ - C(VG)(JX,Y)JZ - C(VG)(JX,JY)Z + JC(VG)(X,Y)JZ$$

$-JC(VG)(X,JY)Z - JC(VG)(JX,Y)Z + JC(VG)(JX,JY)JZ = 0$, $X,Y,Z \in$ *(XM)*(17)

Similarly considering *VG*- manifold of classes $C_1(VG)$-$C_7(VG)$ can be received the 2,3,4,5,6,7 and 8.

Theorem 2.14:

We have the following inclusion relations

i) $C_1(VG) = -C_2(VG)$

ii)$C_0(VG) = C_3(VG) = C_5(VG) = C_6(VG)$

proof:

A coordinated to theorem (7) and proposition (10) we shall prove

1) prove inclusion $C_1(VG) = -C_2(VG)$

Let (M, J, g) - *VG*-AHmanifold of a class $C_2(VG)$,i.e. take place equality $C^a_{bc\hat{a}} = C^a_{b\hat{a}c} = 0$.

According to proposition (10) we have:

$C^{a}_{\hat{b}cd} + C^{a}_{cd\hat{b}} + C^{a}_{d\hat{b}c} = 0$ i.e $C^{a}_{\hat{b}cd} = 0$. This the *VG*-manifold of a class is $C_1(VG) = -C_2(VG)$ VG-manifold .

Putting (Folding) equality (8) and (9) gives identity describing *VG*- manifold of class

$C_1(VG) = -C_2(VG)$

2)we shall prove $C_5(VG) = C_6(VG)$ and similary prove the other.

For example we prove equality Let (*M, J, g*) be *VG*-manifold of class $C_5(VG)$, i.e $C^{a}_{\hat{b}c\hat{d}}$.

Then according to proposition (10) we have $C^{a}_{\hat{b}\hat{c}d}$=0, i.e. The *VG*- manifold is manifold of class $C_6(VG)$,

let M - *VG*- manifold of class $C_6(VG)$, then $C^{a}_{\hat{b}\hat{c}d}$, so, according to proposition (10) and $C^{a}_{\hat{b}c\hat{d}}$ =0.

Thus, classes $C_5(VG)$ and $C_6(VG)$ of $VG -$ □□□□□□□□□□□□□□□□□.

C(X,Y)Z+C(JX,JY)Z+JC(X,Y)JZ-JC(JX,JY)JZ=0; X,Y,Z∈□*(*□*)*(18)

The equality (7), (10), (12), (13) gives the identity describing *VG*- manifold of classes

$C_0(VG) = C_3(VG) = C_5(VG) = C_6(VG)$

C(X,Y)Z+JC(JX,JY)JZ = 0 ; X,Y.Z∈ X(M). (19)

Theorem 2.15:

The components of the concircular tensor of the *VG* -manifold in the adjoined *G*-structure space are given as the following forms:

1) $C_{abcd} = 2(B_{ab[cd]} + \alpha_{[a}B_{b]cd})$.

2) $C_{\hat{a}bcd} = 2A^{a}_{bcd}$.

3) $C_{\hat{a}\hat{b}cd} = 2(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}) - \frac{k}{n(n-1)}\left\{r^{[a}_{[d}\delta^{b]}_{c]} - r^{[a}_{[c}\delta^{b]}_{d]}\right\}$.

4) $C_{\hat{a}bc\hat{d}} = A^{ad}_{bc} + B^{adh}B_{hbc} - B^{ah}{}_{c}B_{h\hat{b}}^{d} + \frac{k}{n(n-1)}\{r^{[a}_{[c}\delta^{d]}_{b]}\}$.

And the other components its conjugate to the above or equal zero.

And the others conjugate to the above components or equal to zero.

Proof:

By using definition (8) we compute the components of concircular tensor as follows :

1) Put $i = a$, $j = b$, $k = c$, $l = d$

$$C_{abcd} = R_{abcd} - \frac{k}{n(n-1)}\{r_{ad}g_{bc} - r_{ac}g_{bd}\}$$

$$C_{abcd} = R_{abcd}$$

$C_{abcd} = 2(B_{ab[cd]} + \alpha_{[a}B_{b]cd})$

2) Put $i = \hat{a}, j = b, k = c, l = d$

$$C_{\hat{a}bcd} = R_{\hat{a}bcd} - \frac{k}{n(n-1)}\{r_{\hat{a}d}g_{bc} - r_{\hat{a}c}g_{bd}\}$$

$$C_{\hat{a}bcd} = R_{\hat{a}bcd}$$

$$C_{\hat{a}bcd} = 2A^{a}_{bcd}$$

3) Put $i = a, j = \hat{b}, k = c$ and $l = d$

$$C_{a\hat{b}cd} = R_{a\hat{b}cd} - \frac{k}{n(n-1)}\{r_{ad}g_{\hat{b}c} - r_{ac}g_{\hat{b}d}\}$$

$$C_{a\hat{b}cd} = 0 - \frac{k}{n(n-1)}\{r_{ad}g_{\hat{b}c} - r_{ac}g_{\hat{b}d}\}$$

If $c \leftrightarrow d$ then

$$C_{a\hat{b}cd} = 0 - \frac{k}{n(n-1)}\{r_{ac}g_{\hat{b}c} - r_{ac}g_{\hat{b}c}\}$$

$$C_{a\hat{b}cd} = 0$$

4) Put $i = a, j = b, k = \hat{c}$, and $l = d$

$$C_{ab\hat{c}d} = R_{ab\hat{c}d} - \frac{k}{(n-1)}\{r_{ad}g_{b\hat{c}} - r_{a\hat{c}}g_{bd}\}$$

If $c\hat{c} \leftrightarrow d$ then

$$C_{ab\hat{c}d} = 0 - \frac{k}{n(n-1)}\{(0)(0) - (0)(0)\}$$

$$C_{ab\hat{c}d} = 0$$

5) Put $i = a, j = b, k = c$ and $l = \hat{d}$

$$C_{abc\hat{d}} = R_{abc\hat{d}} - \frac{k}{n(n-1)}\{r_{a\hat{d}}g_{bc} - r_{ac}g_{b\hat{d}}\}$$

If $c \leftrightarrow \hat{d}$ then

$$C_{abc\hat{d}} = 0 - \frac{k}{n(n-1)}\{(0)(0) - (0)(0)\}$$

$$C_{abc\hat{d}} = 0$$

6) Put $i = \hat{a}, j = \hat{b}, k = c, l = d$

$$C_{\hat{a}\hat{b}cd} = R_{\hat{a}\hat{b}cd} - \frac{k}{n(n-1)}\{r_{\hat{a}d}g_{\hat{b}c} - r_{\hat{a}c}g_{\hat{b}d}\}$$

$$C_{\hat{a}\hat{b}cd} = R_{\hat{a}\hat{b}cd} - \frac{K}{n(n-1)}\{r_{\hat{a}d}g_{\hat{b}c} - r_{\hat{a}c}g_{\hat{b}d}\}$$

$$C_{\hat{a}\hat{b}cd} = 2(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}) - \frac{k}{n(n-1)}\{r^{[a}_{[d}\delta^{b]}_{c]} - r^{[a}_{[c}\delta^{b]}_{d]}\}$$

7) Put $i = \hat{a}$, $j = b$, $k = \hat{c}$ and $l = d$

$$C_{\hat{a}b\hat{c}d} = R_{\hat{a}b\hat{c}d} - \frac{k}{n(n-1)}\{r_{\hat{a}d}g_{b\hat{c}} - r_{\hat{a}\hat{c}}g_{bd}\}$$

If d$\hat{c} \leftrightarrow d$ then

$$C_{\hat{a}b\hat{c}d} = 0 - \frac{k}{n(n-1)}\{(0)(0) - (0)(0)\}$$

$$C_{\hat{a}b\hat{c}d} = 0$$

8) Put $i = \hat{a}, j = b$, $k = c$ and $l = \hat{d}$

$$C_{\hat{a}bc\hat{d}} = R_{\hat{a}bc\hat{d}} - \frac{k}{n(n-1)}\{r_{\hat{a}\hat{d}}g_{bc} - r_{\hat{a}c}g_{b\hat{d}}\}$$

$$C_{\hat{a}bc\hat{d}} = R_{\hat{a}bc\hat{d}} - \frac{k}{n(n-1)}\{(0)(0) - r_{\hat{a}c}g_{b\hat{d}}\}$$

$$C_{\hat{a}bc\hat{d}} = R_{\hat{a}bc\hat{d}} + \frac{k}{n(n-1)}\{r_{\hat{a}c}g_{b\hat{d}}\}$$

$$C_{\hat{a}bc\hat{d}} = A^{ad}_{bc} + B^{adh}B_{hbc} - B^{ah}{}_{c}B^{d}_{hb} + \frac{k}{n(n-1)}\{r^{[a}_{[c}\delta^{d]}_{b]}\}$$

By using the properties of concircular tensor we obtained :

$C_{\hat{a}bc\hat{d}} = C_{\hat{a}b\hat{c}d}$ as follows

$C_{\hat{a}b\hat{d}c} = -C_{\hat{a}bc\hat{d}}$

$C_{\hat{a}b\hat{d}c} = -A^{ad}_{bc} - \frac{K}{2n(n+1)}\delta^{ad}_{bc}$

Therefore,

$C_{\hat{a}b\hat{c}d} = -\mathcal{A}^{ac}_{bd} - \frac{K}{2n(n+1)}\delta^{ac}_{bd}$.

In above theorem I calculated components concircular tensor curvature on space of the adjoint G-structure for VG -manifold.

for VG -manifold only four concircular curvature tensor donts equal zero C_0 , C_1 , C_4 *and* C_7.

III. The main resuits of this paper are stated below:

1)Computing the components of Concircular curvature tensor of Viasman-Graymanifold (*VG*-manifold) , only four components concircular curvature tensor donts equal zero and others four components equal zero .

2) Roved that cocircular curvature tensor possesses the classical symmrtry properties of the Rimmanian curvature .

3) Find equations for these components class C_i where i =0,1,2,3,4,5,6,7 .

4) Find a relation between classes C_0, C_1, C_2, C_3, C_5 *and* C_6 with each other .

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