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Estimating the Coefficients of Polynomial Regression Model when the Error Distributed Generalized Logistic with Three Parameters

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Abstract--*In this research, estimates of polynomial regression model coefficients are derived when the random error is logistically distributed with three parameters representing scale, shape and location. By employing simulation method, the model coefficients are compared using methods of General Least Squares and the modified maximum likelihood. Based on statistical criteria, the Mean Square Error (MSE) indicated that the modified maximum likelihood (MML1) method is the best. This method is applied to data of Gross Domestic Product (GDP) at current prices in Iraq.*

Key words--*Polynomial regression, Generalized Logistic distribution, Modified maximum likelihood method, General least squares, Mean square error.*

I. INTRODUCTION

In polynomial linear regression model (Akkaya & Tiku, 2008), the ordinary least squares method for estimating the regression model parameters is sensitive towards outliers. To address this problem, Akkaya and Tiku (Akkaya & Tiku, 2008) reparametrized the regression model:

$$y_i = \theta_0 + \sum_{j=1}^{q} \theta_j \Lambda_{ij} + e_i \quad (1 \le i \le n, 1 \le j \le q) \quad \ldots (1)$$

Where (Λ_{ij}) is the standard score of explanatory variables $(x_{ij}),(y_i)$ refers to the dependent variable, (θ_j) represents the polynomial regression coefficients, and (q) is the number of variables.

When the relationship between the dependent variable and the explanatory variables is non-linear, the polynomial regression method can be used [5, 10, 6]. In this study, second order multifactor polynomial regression model (Akkaya & Tiku, 2010) is used:

$$y_i = \theta_0 + \theta_1\Lambda_{i1} + \theta_2\Lambda_{i2} + \theta_3\Lambda_{i3} + \theta_{11}\Lambda_{i1}^2 + \theta_{22}\Lambda_{i2}^2 + \theta_{33}\Lambda_{i3}^2 + \theta_{12}\Lambda_{i1}\Lambda_{i2} + \theta_{13}\Lambda_{i1}\Lambda_{i3} + \theta_{23}\Lambda_{i2}\Lambda_{i3} + e_i \quad \ldots(2)$$

Assuming that random error (e_i) which follows Generalized Logistic Distribution (GLD) with three parameters (Akkaya & Tiku, 2010) with probability density function (p.d.f) is as follows:

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$$f(e)=\left[b\exp\left(\frac{-(e-\zeta)}{\sigma}\right)\right]\Bigg/\left[\sigma\left[1+\exp\left(\frac{-(e-\zeta)}{\sigma}\right)\right]^{b+1}\right], \quad -\infty < e < \infty \quad \text{... (3)}$$

Where (b), (σ) and (ξ) are the parameters of shape and scale and location; while cumulative distribution function (c.d.f) is as follows:

$$F(e)=\left[1+\exp\left(\frac{-(e-\xi)}{\sigma}\right)\right]^{-b} \quad \text{… (4)}$$

Among researchers who have employed the polynomial regression model and modified maximum likelihood (MMLE) are Braune and Shacham (Braune & Shacham, 1998) in 1998, Akkaya and Tiku (Akkaya & Tiku, 2010) in 2010, Puthenpura and Sinha, (1989); (Islam & Tiku, 2004) in 1986, Tiku and Suresh (Tiku & Suresh, 1992) in 1992 and Ahmed and Kaeth (Ahmed & Kadhim, 2017) in 2016. In 2018 Ebtisam et al. (Ebtisam, Ahmed & Baydaa, 2019) introduced another procedure for multiple linear regression model where they used three methods in estimating the model's parameters when the error distributed General Logistic. This study aimed at estimating the coefficients of second order multifactor polynomial regression model. The formula of estimating the coefficients using (GLS) method is as follows:

$$\hat{\theta}=(\Lambda'\Lambda)^{-1}\Lambda' y \quad \text{... (5)}$$

Modified Maximum Likelihood Method (MMLM)

Tiku [4, 8] has developed this method and explained that it is used when the estimates of maximum likelihood contain intractable function that cannot be solved, requiring its replacement by linear function. Given the natural logarithm of likelihood function after substituting formula (3), the result is as follows:

$$Ln\,L = n\ln b - n\ln\sigma + \sum_{i=1}^{n}\left(\frac{-(e_i-\xi)}{\sigma}\right) - (b+1)\sum_{i=1}^{n}\ln\left(1+\exp\left(\frac{-(e_i-\xi)}{\sigma}\right)\right) \quad \text{... (6)}$$

By deriving (6) w.r.t. (θ) and (σ) respectively and equalizing it to zero, the result is as follows:

$$\frac{\partial Ln\,L}{\partial\theta_0}=\frac{n}{\sigma}-\frac{(b+1)}{\sigma}\sum_{i=1}^{n}\frac{\exp\left(\frac{-(e_i-\xi)}{\sigma}\right)}{\left[1+\exp\left(\frac{-(e_i-\xi)}{\sigma}\right)\right]}=0 \quad \text{... (7)}$$

$$\frac{\partial Ln\,L}{\partial\theta_j}=\sum_{i=1}^{n}\sum_{j=1}^{q}\frac{\Lambda_{ij}}{\sigma}-\frac{(b+1)}{\sigma}\sum_{i=1}^{n}\sum_{j=1}^{q}\frac{\Lambda_{ij}\exp\left(\frac{-(e_i-\xi)}{\sigma}\right)}{\left[1+\exp\left(\frac{-(e_i-\xi)}{\sigma}\right)\right]}=0 \quad \text{... (8)}$$

$$\frac{\partial Ln\,L}{\partial\theta_{jj}}=\sum_{i=1}^{n}\sum_{j=1}^{q}\frac{\Lambda_{ij}{}^2}{\sigma}-\frac{(b+1)}{\sigma}\sum_{i=1}^{n}\sum_{j=1}^{q}\frac{\Lambda_{ij}{}^2\exp\left(\frac{-(e_i-\xi)}{\sigma}\right)}{\left[1+\exp\left(\frac{-(e_i-\xi)}{\sigma}\right)\right]}=0 \quad \text{... (9)}$$

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$$\frac{\partial Ln\,L}{\partial \theta_{jk}} = \sum_{i=1}^{n}\sum_{j=1}^{q-1}\sum_{k=j+1}^{q}\frac{\Lambda_{ij}\Lambda_{ik}}{\sigma} - \frac{(b+1)}{\sigma}\sum_{i=1}^{n}\sum_{j=1}^{q-1}\sum_{k=j+1}^{q}\frac{\Lambda_{ij}\Lambda_{ik}\exp(\frac{-(e_i-\xi)}{\sigma})}{\left[1+\exp(\frac{-e_i+\xi}{\sigma})\right]} = 0 \quad \ldots (10)$$

$$\frac{\partial Ln\,L}{\partial \sigma} = \frac{-n}{\sigma} + \sum_{i=1}^{n}\frac{(e_i-\xi)}{\sigma^2} - (b+1)\sum_{i=1}^{n}\frac{\frac{(e_i-\xi)}{\sigma^2}\exp(\frac{-(e_i-\xi)}{\sigma})}{\left[1+\exp(\frac{-(e_i-\xi)}{\sigma})\right]} = 0 \quad \ldots (11)$$

$$\frac{\partial Ln\,L}{\partial \sigma} = \frac{-n}{\sigma} + \frac{1}{\sigma}\sum_{i=1}^{n}S_i - \frac{(b+1)}{\sigma}\sum_{i=1}^{n}\frac{S_i\exp(-S_i)}{[1+\exp(-S_i)]} = 0 \quad \ldots (12)$$

The above equations include intractable function and do not have explicit solutions (Akkaya & Tiku, 2010) and that:

$$g(S_i) = \exp(-S_i)/[1+\exp(-S_i)] \quad \ldots (13)$$

$(S_i = (e_i - \xi)/\sigma)$ the equations (7), (8), (9), (10) and (12) are termed as order variates (Draper & smith, 1981) for $S_{(i)}$, i.e., $S_{(1)} \le S_{(2)} \le \ldots \le S_{(n)}$. By replacing (S_i) in the above equations with $(S_{(i)})$, the following equations are obtained:

$$\frac{\partial Ln\,L^*}{\partial \theta_0} = \frac{n}{\sigma} - \frac{(b+1)}{\sigma}\sum_{i=1}^{n}g(S_{(i)}) = 0 \quad \ldots (14)$$

$$\frac{\partial Ln\,L^*}{\partial \theta_j} = \sum_{i=1}^{n}\sum_{j=1}^{q}\frac{\Lambda_{ij}}{\sigma} - \frac{(b+1)}{\sigma}\sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}\,g(S_{(i)}) = 0 \quad \ldots (15)$$

$$\frac{\partial Ln\,L^*}{\partial \theta_{jj}} = \sum_{i=1}^{n}\sum_{j=1}^{q}\frac{\Lambda_{ij}^{\ 2}}{\sigma} - \frac{(b+1)}{\sigma}\sum_{i=1}^{n}\sum_{j=1}^{g}\Lambda_{ij}^{\ 2}\,g(S_{(i)}) = 0 \quad \ldots (16)$$

$$\frac{\partial Ln\,L^*}{\partial \theta_{jk}} = \sum_{i=1}^{n}\sum_{j=1}^{q-1}\sum_{k=j+1}^{q}\frac{\Lambda_{ij}\Lambda_{ik}}{\sigma} - \frac{(b+1)}{\sigma}\sum_{i=1}^{n}\sum_{j=1}^{q-1}\sum_{k=j+1}^{q}\Lambda_{ij}\Lambda_{ik}\,g(S_{(i)}) = 0 \quad \ldots (17)$$

$$\frac{\partial Ln\,L^*}{\partial \sigma} = \frac{-n}{\sigma} + \frac{1}{\sigma}\sum_{i=1}^{n}S_{(i)} - \frac{(b+1)}{\sigma}\sum_{i=1}^{n}S_{(i)}\,g(S_{(i)}) = 0 \quad \ldots (18)$$

And by replacing $g(S_{(i)})$ with linear function $g(S_{(i)}) \cong \alpha_i - \beta_i S_{(i)}$, and to find estimates of (β_i) and (α_i) parameters by using the first two terms in Taylor series for $g(S_{(i)})$ about $(t_{(i)})$ population quintiles, i.e., $g(S_{(i)}) \cong g(t_{(i)}) + (S_{(i)} - t_{(i)})g'(t_{(i)})$, and $g'(t_{(i)})$ is the first derivative of $g(t_{(i)})$ and the formulas of estimates (α_i) and (β_i) are (Ahmed & Kadhim, 2017):

$$\beta_i = \exp(t_{(i)})/(1+\exp(t_{(i)}))^2 \quad , \quad \alpha_i = [1+\exp(t_{(i)}) + t_{(i)}\exp(t_{(i)})]/(1+\exp(t_{(i)}))^2$$

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Assuming that (q_i) represents the estimation of cumulative function $F(t_{(i)})$ and takes two methods. The first one is $\left(q_i = \frac{i}{n+1}\right)$ referring to MML1. The second method is $\left(q_i = \frac{i-0.5}{n}\right)$ referring to MML2. $\left(t_{(i)} = -Ln(q_i^{\frac{1}{b}} - 1)\right)$ is the inverse of cumulative function, i.e.:

$$F(t_{(i)}) = 1 \Big/ \left[1 + \exp(-t_{(i)})\right]^b$$

By substituting $g(S_{(i)})$ in (14), (15), (16), (17) and (18) respectively is as follows:

$$\frac{\partial Ln\, L^*}{\partial \theta_0} = \frac{n}{\sigma} - \frac{(b+1)}{\sigma}\sum(\alpha_i - \beta_i S_{(i)}) = 0 \quad \ldots (19)$$

$$\frac{\partial Ln\, L^*}{\partial \theta j} = \sum_{i=1}^{n}\sum_{j=1}^{q}\frac{\Lambda_{ij}}{\sigma} - \frac{(b+1)}{\sigma}\sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}(\alpha_i - \beta_i S_{(i)}) = 0 \quad \ldots (20)$$

$$\frac{\partial Ln\, L^*}{\partial \theta_{jj}} = \sum_{i=1}^{n}\sum_{j=1}^{q}\frac{\Lambda_{ij}{}^2}{\sigma} - \frac{(b+1)}{\sigma}\sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}{}^2(\alpha_i - \beta_i S_{(i)}) = 0 \quad \ldots (21)$$

$$\frac{\partial Ln\, L^*}{\partial \theta_{jk}} = \sum_{i=1}^{n}\sum_{j=1}^{q-1}\sum_{k=j+1}^{q}\frac{\Lambda_{ij}\Lambda_{ik}}{\sigma} - \frac{(b+1)}{\sigma}\sum_{i=1}^{n}\sum_{j=1}^{q-1}\sum_{k=1+j}^{q}\Lambda_{ij}\Lambda_{ik}(\alpha_i - \beta_i S_{(i)}) = 0 \quad \ldots (22)$$

$$\frac{\partial Ln\, L^*}{\partial \sigma} = \frac{-n}{\sigma} + \frac{1}{\sigma}\sum_{i=1}^{n}S_{(i)} - \frac{(b+1)}{\sigma}\sum_{i=1}^{n}S_{(i)}(\alpha_i - \beta_i S_{(i)}) = 0 \quad \ldots (23)$$

When solving the equations above (appendix A), the estimates of MML are obtained as follows [4, 8, 1]:

$$\tilde{\theta} = G - H\tilde{\sigma} \quad \ldots (24)$$

$$G = (\lambda' \beta \lambda)^{-1}(\lambda' \beta Y) = G_\ell \quad \ldots (25)$$

$$H = (\lambda' \beta \lambda)^{-1}(\lambda' \Delta 1) = H_\ell \quad \ldots (26)$$

$$\beta = diag(\beta_i) \;\; \Delta = diag(\Delta_i) \; , \; \Delta_i = \alpha_i - (b+1)^{-1}$$

$$1 = [1, \;\; 1, \;\; \ldots \;\; 1] \, , \; \tilde{\sigma} = \left(-B + \sqrt{B^2 + 4nC}\right) \Big/ \left(2\sqrt{n(n-p)}\right)$$

Where (λ) represents the matrix (Λ) and $p = 1 + 2q + \frac{1}{2}q(q-1)$.

Simulation

The default values that will be used in the simulation of distribution parameters are $(b = 1, 2, 5)$, $(\sigma = 1, 2)$ and $(\zeta = 1)$. While the default values for the regression model coefficients are (Ahmed & Kadhim, 2017):

$$\begin{pmatrix} \theta_0 = -0.1414, \theta_1 = -0.0037, \theta_2 = -0.0896, \theta_3 = -0.0593, \theta_{11} = 0.1490, \theta_{22} = 0.0313, \\ \theta_{33} = 0.0667, \theta_{12} = -0.1739, \theta_{13} = -0.1059, \theta_{23} = 0.2013 \end{pmatrix}$$

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Generating data for the random variable (e_i) which follows (GLD) is done according to the formula $\left(e = \left(-Ln\left(u^{-\frac{1}{b}} - 1 \right)\sigma \right) + \xi \right)$. Three sizes of samples were selected (n = 25, 60, 100) and experiment is repeated (10000) times. Comparison between the methods of estimation to determine the best of them is performed using the statistical criteria which is the MSE of parameters in accordance with the formula:

$$MSE\left(\hat{\theta}\right) = \sum_{i=1}^{g} \left(E\left(\hat{\theta}\right) - \theta \right)^2 \Big/ RP$$

Where (RP) is the number of repeating experiment and (g) the number of groups. The program is written using (MATLAB). The following Tables illustrate the results of simulation:

Table 1: values of MSE for parameters when $(b = 1,\ \sigma = 1, \zeta = 1)$

n	**25**			**60**			**100**		
Cof.	**GLS**	**MML1**	**MML2**	**GLS**	**MML1**	**MML2**	**GLS**	**MML1**	**MML2**
θ_0	0.1469	0.0894	1.8954	0.0315	0.0280	1.3070	0.0175	0.0165	1.1835
θ_1	0.0391	0.0208	0.2196	0.0074	0.0065	0.0653	0.0040	0.0038	0.0358
θ_2	0.0397	0.0214	0.2214	0.0078	0.0068	0.0651	0.0041	0.0039	0.0363
θ_3	0.1109	0.0526	0.2187	0.0198	0.0146	0.0660	0.0109	0.0090	0.0365
θ_{11}	0.0528	0.0291	0.2900	0.0099	0.0086	0.0824	0.0053	0.0049	0.0450
θ_{22}	0.0479	0.0267	0.2926	0.0091	0.0080	0.0816	0.0050	0.0048	0.0449
θ_{33}	0.0566	0.0302	0.2871	0.0107	0.0093	0.0804	0.0056	0.0052	0.0454
θ_{12}	0.0620	0.0302	0.2916	0.0097	0.0081	0.0717	0.0052	0.0048	0.0381
θ_{13}	0.0690	0.0337	0.2893	0.0104	0.0088	0.0710	0.0058	0.0053	0.0379
θ_{23}	0.0837	0.0396	0.2903	0.0132	0.0108	0.0718	0.0070	0.0062	0.0371

Table 2: values of MSE for parameters when $(b = 2,\ \sigma = 1, \zeta = 1)$

n	**25**			**60**			**100**		
Cof.	**GLS**	**MML1**	**MML2**	**GLS**	**MML1**	**MML2**	**GLS**	**MML1**	**MML2**
θ_0	1.8546	1.6911	4.6729	1.4400	1.4078	4.2050	1.3717	1.3531	4.1223
θ_1	0.0372	0.0243	0.1530	0.0080	0.0073	0.0447	0.0045	0.0043	0.0252
θ_2	0.0343	0.0224	0.1560	0.0073	0.0068	0.0456	0.0043	0.0041	0.0249
θ_3	0.1136	0.0651	0.1532	0.0247	0.0191	0.0457	0.0150	0.0125	0.0256

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θ_{11}	0.0595	0.0390	0.2053	0.0122	0.0108	0.0582	0.0069	0.0064	0.0321
θ_{22}	0.0442	0.0296	0.2033	0.0098	0.0088	0.0587	0.0053	0.0050	0.0317
θ_{33}	0.0989	0.0666	0.2018	0.0199	0.0183	0.0580	0.0118	0.0113	0.0312
θ_{12}	0.0565	0.0340	0.2024	0.0100	0.0085	0.0490	0.0055	0.0050	0.0255
θ_{13}	0.0710	0.0418	0.2036	0.0111	0.0096	0.0481	0.0061	0.0056	0.0261
θ_{23}	0.0696	0.0400	0.1966	0.0119	0.0099	0.0495	0.0065	0.0057	0.0265

Table 3: values of MSE for parameters when $(b = 5,\ \sigma = 1, \zeta = 1)$

n	**25**			**60**			**100**		
Cof.	**GLS**	**MML1**	**MML2**	**GLS**	**MML1**	**MML2**	**GLS**	**MML1**	**MML2**
θ_0	10.7955	10.2388	9.9812	7.6717	7.5718	9.6501	7.1533	7.0920	9.6351
θ_1	0.0833	0.0575	0.1250	0.0168	0.0159	0.0369	0.0099	0.0096	0.0207
θ_2	0.0760	0.0525	0.1283	0.0156	0.0148	0.0363	0.0099	0.0096	0.0208
θ_3	0.3007	0.2032	0.1250	0.0526	0.0467	0.0363	0.0338	0.0313	0.0207
θ_{11}	0.1166	0.0831	0.1652	0.0196	0.0185	0.0476	0.0108	0.0104	0.0254
θ_{22}	0.0860	0.0619	0.1635	0.0149	0.0142	0.0469	0.0087	0.0085	0.0258
θ_{33}	0.2925	0.2034	0.1642	0.0593	0.0547	0.0475	0.0385	0.0368	0.0263
θ_{12}	0.1265	0.0824	0.1593	0.0151	0.0141	0.0396	0.0081	0.0078	0.0218
θ_{13}	0.1554	0.0986	0.1599	0.0187	0.0174	0.0399	0.0102	0.0098	0.0215
θ_{23}	0.1421	0.0902	0.1627	0.0174	0.0161	0.0391	0.0094	0.0089	0.0217

Table 4: values of MSE for parameters when $(b = 1,\ \sigma = 2, \zeta = 1)$

n	**25**			**60**			**100**		
Cof.	**GLS**	**MML1**	**MML2**	**GLS**	**MML1**	**MML2**	**GLS**	**MML1**	**MML2**
θ_0	0.3846	0.2695	4.6796	0.0983	0.0901	2.1800	0.0607	0.0580	1.6484
θ_1	0.0896	0.0558	0.8830	0.0215	0.0197	0.2550	0.0131	0.0126	0.1455
θ_2	0.0930	0.0584	0.8838	0.0220	0.0201	0.2699	0.0134	0.0129	0.1464
θ_3	0.1324	0.0732	0.8755	0.0262	0.0224	0.2557	0.0150	0.0138	0.1460
θ_{11}	0.1098	0.0716	1.1751	0.0272	0.0249	0.3385	0.0161	0.0154	0.1776
θ_{22}	0.1141	0.0754	1.1570	0.0287	0.0263	0.3339	0.0167	0.0160	0.1806

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θ_{33}	0.1131	0.0737	1.2033	0.0281	0.0257	0.3315	0.0172	0.0165	0.1827
θ_{12}	0.1094	0.0657	1.1697	0.0228	0.0205	0.2779	0.0131	0.0124	0.1501
θ_{13}	0.1171	0.0685	1.1596	0.0238	0.0215	0.2783	0.0135	0.0128	0.1481
θ_{23}	0.1189	0.0690	1.1088	0.0260	0.0232	0.2839	0.0148	0.0140	0.1475

Table 5: values of MSE for parameters when $(b = 2,\ \sigma = 2, \zeta = 1)$

n	**25**			**60**			**100**		
Cof.	**GLS**	**MML1**	**MML2**	**GLS**	**MML1**	**MML2**	**GLS**	**MML1**	**MML2**
θ_0	5.5891	5.3219	11.4810	4.7658	4.7158	10.0039	4.5587	4.5322	9.4970
θ_1	0.0688	0.0534	0.6277	0.0230	0.0216	0.1826	0.0133	0.0128	0.0999
θ_2	0.0671	0.0519	0.6141	0.0228	0.0214	0.1844	0.0128	0.0124	0.1002
θ_3	0.1008	0.0695	0.6190	0.0270	0.0237	0.1873	0.0150	0.0139	0.1015
θ_{11}	0.1071	0.0836	0.8104	0.0326	0.0303	0.2321	0.0180	0.0173	0.1279
θ_{22}	0.1004	0.0792	0.8046	0.0295	0.0277	0.2279	0.0169	0.0163	0.1237
θ_{33}	0.1264	0.0989	0.8266	0.0340	0.0319	0.2236	0.0198	0.0191	0.1262
θ_{12}	0.0894	0.0666	0.7992	0.0245	0.0227	0.1964	0.0136	0.0130	0.1038
θ_{13}	0.0944	0.0704	0.7955	0.0253	0.0235	0.1977	0.0145	0.0140	0.1038
θ_{23}	0.0920	0.0681	0.8091	0.0246	0.0226	0.1952	0.0149	0.0142	0.1068

Table 6: values of MSE for parameters when $(b = 5,\ \sigma = 2, \zeta = 1)$

n		**25**	**60**				**100**		
Cof.	**GLS**	**MML1**	**MML2**	**GLS**	**MML1**	**MML2**	**GLS**	**MML1**	**MML2**
θ_0	31.1266	30.3082	28.8291	22.9749	22.8351	27.2339	21.6406	21.5537	27.1425
θ_1	0.1092	0.0957	0.5042	0.0362	0.0356	0.1449	0.0220	0.0218	0.0830
θ_2	0.1009	0.0871	0.5124	0.0336	0.0331	0.1467	0.0207	0.0205	0.0843
θ_3	0.1967	0.1542	0.4939	0.0430	0.0413	0.1489	0.0251	0.0245	0.0801
θ_{11}	0.1696	0.1485	0.6865	0.0488	0.0478	0.1883	0.0274	0.0270	0.1038
θ_{22}	0.1573	0.1376	0.6662	0.0456	0.0449	0.1892	0.0266	0.0263	0.1001
θ_{33}	0.2391	0.2011	0.6479	0.0640	0.0622	0.1881	0.0369	0.0364	0.1036
θ_{12}	0.1514	0.1235	0.6660	0.0351	0.0344	0.1591	0.0209	0.0207	0.0866
θ_{13}	0.1745	0.1418	0.6629	0.0390	0.0380	0.1580	0.0231	0.0228	0.0856

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θ_{23}	0.1660	0.1354	0.6557	0.0371	0.0362	0.1588	0.0219	0.0216	0.0850

Based on Tables (1), (2), (3), (4), (5) and (6), mean square error (MSE) of the estimated parameters decreases with increasing sample size for all methods. In addition, MML1 has less MSE in comparison with other methods and for all sample sizes. As for the values of shape and scale parameters, the values $(b=1, \sigma=1, \zeta=1)$ give less MSE for the estimated parameters in MML1 for all sample sizes. The efficiency of MML2 method decreases with increasing sample size and the increased value of *b*.

Data were obtained from Ministry of Planning, Central Statistical Organization, Iraq for (GDP) at current prices during the period (2006 – 2017) measured by million Iraqi dinars (Ministry of planning, central statistical organization, Iraq) representing the depenent variable (y_i). Table (7) illustrates the values of dependent variable and explanatory variables measured in million dinars. Hence, (x_{i1}) represents imports of goods and services, (x_{i2}) refers to exports of goods and services and (x_{i3}) denotes the government consumption expenditure.

Table 7: the values of dependent variable (y_i) and the explanatory variables (x_{i1}, x_{i2}, x_{i3})

y_i	x_{i1}	x_{i2}	x_{i3}
95587954.8	36914707.8	48780390.6	14984454.1
111455813.4	31422753.0	51158039.1	20871484.0
157026061.6	48249768.6	79028558.7	26139166.0
130643200.4	51326145.0	51473565.0	27517759.7
162064565.5	55232658.0	63880713.0	30660743.7
217327107.4	60316542.0	96531318.0	3699956.9
254225490.7	73980251.4	113151788.2	42158634.3
273587529.2	75910914.2	108514489.6	47755742.7
266420384.5	69948806.4	102738475.4	47946900.1
199715699.9	68289455.7	67192475.7	36339342.1
203869832.2	49267178.4	56312489.4	45872859.1
225995179.1	57333501.0	75180282.6	45918514.4

By applying Kolmogorov-Smirnov test to data according to hypothesis (H_0: data follow Generalized Logistic Distribution) and then processing data by taking the standard score of dependent variable (Y), the calculated value of test (0.3916) is less than the tabulated value (0.3926) at significance level (α = 0.05). This means that hypothesis (H_0) is accepted.

After dividing data by (100000000) to simplify the calculations using (MML1), which is the best method in simulation, and as this method needs initial estimates therefore, (GLS) method is used. The results are:

$$\theta_0 = 1.8065, \theta_1 = 0.1845, \theta_2 = 0.2205, \theta_3 = 0.3045, \theta_{11} = 0.1830, \theta_{22} = -0.1071,$$
$$\theta_{33} = 0.2103, \theta_{12} = -0.0179, \theta_{13} = -0.3973, \theta_{23} = 0.1717$$

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II. RECOMMENDATIONS

Based on analysis of results, it is found that the relationship between the explanatory variables (imports of goods and services, exports of goods and services, and government consumption expenditure) and the dependent variable (GDP at current prices) was positive. In other words, the increase in the ratio of these variables leads to an increase in the ratio of GDP. Second order multifactor polynomial regression model is:

$$\hat{y}_i = 3.2863 + 2.2882\Lambda_{i1} + 0.9362\Lambda_{i2} + 0.6017\Lambda_{i3} - 0.0193\Lambda_{i1}^2 - 3.3103\Lambda_{i2}^2 - 0.1682\Lambda_{i3}^2 + 3.8552\Lambda_{i1}\Lambda_{i2} - 2.2807\Lambda_{i1}\Lambda_{i3} + 0.1903\Lambda_{i2}\Lambda_{i3}$$

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Appendix A

$$\frac{\partial Ln\, L^*}{\partial \theta_0} = \frac{n}{\sigma} - \frac{(b+1)}{\sigma}\sum_{i=1}^{n}(\alpha_i - \beta_i S_{(i)}) = 0$$

$$\sum_{i=1}^{n}\beta_i\left((y_{[i]} - G_0\lambda_{[i]0}) - \xi\right) - \left(\sum_{i=1}^{n}(\alpha_i - (b+1)^{-1})\right)\tilde{\sigma} = 0$$

$$G_0 = \sum_{i=1}^{n}\beta_i y_{[i]}\left(\sum_{i=1}^{n}\beta_i\lambda_{[i]0}\right)^{-1} - \sum_{i=1}^{n}\beta_i\xi\left(\sum_{i=1}^{n}\beta_i\lambda_{[i]0}\right)^{-1} - \left(\sum_{i=1}^{n}\alpha_i - (b+1)^{-1}\right)\tilde{\sigma}\left(\sum_{i=1}^{n}\beta_i\lambda_{[i]0}\right)^{-1}$$

$$\frac{\partial Ln\, L^*}{\partial \theta_j} = \sum_{i=1}^{n}\sum_{j=1}^{q}\frac{\Lambda_{ij}}{\tilde{\sigma}} - \frac{(b+1)}{\tilde{\sigma}}\sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}(\alpha_i - \beta_i S_{(i)}) = 0$$

$$\sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}\beta_i\left((y_{[i]} - G_\ell\lambda_{[i]\ell}) - \xi\right) - \sum_{i=1}^{q}\sum_{j=1}^{q}\Lambda_{ij}(\alpha_i - (b+1)^{-1})\tilde{\sigma} = 0 \qquad , \quad (1 \le \ell \le q)$$

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$$G_\ell = \sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}\beta_i y_{[i]}(\sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}\beta_i\lambda_{[i]}\ell)^{-1} - \sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}\beta_i\xi\sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}\beta_i\lambda_{[i]}\ell)^{-1} -$$

$$\sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}(\alpha_i-(b+1)^{-1})\tilde{\sigma})(\sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}\beta_i\lambda_{[i]}\ell)^{-1}$$

$$\frac{\partial Ln\, L^*}{\partial\theta_{jj}} = \sum_{i=1}^{n}\sum_{j=1}^{q}\frac{\Lambda_{ij}{}^2}{\tilde{\sigma}} - \frac{(b+1)}{\tilde{\sigma}}\sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}{}^2(\alpha_i-\beta_i S_{(i)}) = 0$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}{}^2\beta_i\left((y_{[i]}-G_\ell\lambda_{[i]\ell})-\xi\right) - \left[\sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}{}^2(\alpha_i-(b+1)^{-1})\tilde{\sigma}\right] = 0 \quad , \quad (q \le \ell \le 2q)$$

$$G_\ell = \sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}{}^2\beta_i y_{[i]}\left(\sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}{}^2\beta_i\lambda_{[i]_\ell}\right)^{-1} - \sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}{}^2\beta_i\xi\left(\sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}{}^2\beta_i\lambda_{[i]_\ell}\right)^{-1} -$$

$$\left(\sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}{}^2(\alpha_i-(b+1)^{-1})\tilde{\sigma}\right)\left(\sum_{i=1}^{n}\sum_{j=1}^{q}\Lambda_{ij}{}^2\beta_i\lambda_{[i]_\ell}\right)^{-1}$$

$$\frac{\partial Ln\, L^*}{\partial\theta_{jk}} = \sum_{i=1}^{n}\sum_{j=1}^{q-1}\sum_{k=j+1}^{q}\frac{\Lambda_{ij}\Lambda_{ik}}{\tilde{\sigma}} - \frac{(b+1)}{\tilde{\sigma}}\sum_{i=1}^{n}\sum_{j=1}^{q-1}\sum_{k=j+1}^{q}\Lambda_{ij}\Lambda_{ik}(\alpha_i-\beta_i S_{(i)}) = 0$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{q-1}\sum_{k=j+1}^{q}\Lambda_{ij}\Lambda_{ik}\beta_i\left((y_{[i]}-G_\ell\lambda_{[i]_\ell})-\xi\right) - \sum_{i=1}^{n}\sum_{j=1}^{q-1}\sum_{k=j+1}^{q}\Lambda_{ij}\Lambda_{ik}(\alpha_i-(b+1)^{-1})\tilde{\sigma} = 0 \;, \left(2q \prec \ell \prec k \le p\right)$$

$$G_\ell = \sum_{i=1}^{n}\sum_{j=1}^{q-1}\sum_{k=j+1}^{1}\Lambda_{ij}\Lambda_{ik}\beta_i y_{[i]}\left(\sum_{i=1}^{n}\sum_{j=1}^{q-1}\sum_{k=j+1}^{q}\Lambda_{ij}\Lambda_{ik}\beta_i\lambda_{[i]_\ell}\right)^{-1} - \sum_{i=1}^{n}\sum_{j=1}^{q-1}\sum_{k+j=1}^{q}\Lambda_{ij}\Lambda_{ik}\beta_i\xi\left(\sum_{i=1}^{n}\sum_{j=1}^{q-1}\sum_{k=j+1}^{q}\Lambda_{ij}\Lambda_{ik}\beta_i\lambda_{[i]_\ell}\right)^{-1} -$$

$$\sum_{i=1}^{n}\sum_{j=1}^{q-1}\sum_{k=j+1}^{q}\Lambda_{ij}\Lambda_{ik}(\alpha_i-(b+1)^{-1}\tilde{\sigma}(\sum_{i=1}^{n}\sum_{j=1}^{q-1}\sum_{k=j+1}^{q}\Lambda_{ij}\Lambda_{ik}\beta_i\lambda_{[i]_\ell})^{-1}$$

$$\frac{\partial Ln\, L^*}{\partial\sigma} = \frac{-n}{\tilde{\sigma}} + \frac{1}{\tilde{\sigma}}\sum_{i=1}^{n}S_{(i)} - \frac{(b+1)}{\tilde{\sigma}}\sum_{i=1}^{n}S_{(i)}(\alpha_i-\beta_i S_{(i)}) = 0$$

$$n\tilde{\sigma}^2 + (b+1)\sigma\sum_{i=1}^{n}(\alpha \;\; -(b+1)^{-1}\left(e_{(i)}-\xi\right) - (b+1)\sum_{i=1}^{n}\beta_i\left(e_{(i)}-\xi\right)^2 = 0$$

Solving the equation using constitution method:

$$\tilde{\sigma} = \left(-B+\sqrt{B^2+4nC}\right)\Big/\left(2\sqrt{n(n-p)}\right)$$

$$B = (b+1)\sum_{i=1}^{n}\Delta_i\left[y_{[i]}-\sum_{i=0}^{p}G\ell\lambda_{[i]}\ell-\xi\right] \qquad , \qquad \Delta_i = \alpha_i-(b+1)^{-1}$$

$$C = (b+1)\sum_{i=1}^{n}\beta_i\left[y_{[i]}-\sum_{i=0}^{p}G\ell\lambda_{[i]}\ell-\xi\right]^2$$

$$\lambda_{[i]0} = 1 \qquad , \qquad \lambda_{[i]_\ell} = \Lambda_{[i]_\ell} \qquad (1 \le \ell \le q)$$

$$\lambda_{[i]_\ell} = \Lambda_{[i]_\ell}{}^2 \qquad \left(q \le \ell \le 2q\right) \qquad , \qquad \lambda_{[i]_\ell} = \Lambda_{[i],\ell}\, k \qquad (2q < \ell < k \le p)$$

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