International Journal of Psychosocial Rehabilitation, Vol. 24, Issue 05, 2020
ISSN: 1475-7192

Transformation of Multiple Representations in Understanding Real-World Physics Problems

Mazlena Murshed, Fatin Aliah Phang* and
Muhammad Abd Hadi Bunyamin

Abstract--- *This paper aims to investigate the transformation of multiple representations (MRs) in explaining the physics related to real-world situations, for example, when a ball is thrown vertically upwards. The transformation of MR is investigated using one-to-one thinking-aloud problem-solving among five Form Four students from a few secondary schools in Johor, Malaysia. Later, a retrospective interview was carried out to understand the function and the meaning given by students towards the representation they made. Analyses of the students' solutions reveal how MRs is utilised to explain the characteristics of the situation given. The studies revealed that the majority of students employed texts, sketches, and symbols to illustrate the ideas of motion of the ball, the energy contained and the force in action. Backed by the results, it was suggested to use the transformation of multiple representations to improve students' understanding of the problem related to real-world situations.*

Keywords--- *Multiple Representations, Physics Situation, Real-World Situation, Physics Problem, Representation.*

I. INTRODUCTION

In physics, skill with multiple representations (MRs) is important, and prior work has shown that students of inexperienced physics struggle with the representations in problem-solving. There are various representations available which can be used in physics to make it less difficult and easier to understand [1]. MR refers to the use of more than one way of representing ideas, concepts and processes including oral, graphical and numerical forms [2–4]. MR has two types which are internal and external representations. In this study, MR is referring to the external representation used to understand the real-world situation. Various studies have shown that MR has the potential to facilitate learning [2, 5–7]. The use of MR can facilitate understanding of concepts, processes or relationships more and more quickly without the use of verbal explanations or experimental demonstrations [8].

Research in physics education shows that expert problem solvers often use visual representations, such as pictures, graphs, and sketches, to help them understand the problem before using equations to solve them quantitatively [9]. Studies in physics education further found that the achievement of student problem solving increased when more emphasis was placed on qualitative representation [10–12]. Qualitative representations can facilitate understanding of the problem or task, and so the problem can be adequately solved. Hence, students need to know and understand the use of representations to understand a problem before finding a solution.

The concepts of physics are abstract. However, it can be easily understood by some physicists in several representations [13]. The ability to switch between representations indicates that their knowledge structure consists of several representations that are directly related to the concept of physics. One of the ways to enhance

Mazlena Murshed, Ministry of Education, Pusat Pentadbiran Kerajaan Persekutuan, Putrajaya, Malaysia.
Fatin Aliah Phang, Centre for Engineering Education, Universiti Teknologi Malaysia, Johor, Malaysia. E-mail: p-fatin@utm.my*
Muhammad Abd Hadi Bunyamin, School of Education, Faculty of Social Sciences and Humanities, Universiti Teknologi, Malaysia, Johor, Malaysia.

DOI: 10.37200/IJPR/V24I5/PR2020248
Received: 14 Mar 2020 | Revised: 30 Mar 2020 | Accepted: 06 Apr 2020

students' knowledge structure is that they must know to represent physics concepts in different ways and complement each other [14].

According to Treagust et al. [15] furthermore, the construction of physics knowledge requires more than one representation or multiple representation (MR) formats to convey and support a concept's information. Also, abstract physics content can be visualised through MR. Besides, when using various sources of information, students are able to choose the sources they prefer to study [15]. Thus, the use of MR provides students with the opportunity to choose the right representation for them in improving problem-solving skills.

Waldrip, Prain, dan Carolan [16] and Kohl [17], stated that the ability to transform all the same concepts in different formats, including verbal, visual, mathematical, and visual representations is the ability to use MR. MR refers to the many ways in which a person conveys concepts and problems. However, students' ability to understand the problems presented is low, as previously stated [18]. The use of MR is vital to study in the context of problem-solving so that they have guidance in understanding the problem and solving it better. Therefore, there is a need for research on aspects of understanding problems where transformations between MR have been less addressed in previous studies.

II. METHODOLOGY

For this paper, the researcher focus on MRs as a different format of representations. In the analysis, the researcher presented vivid descriptions of student meaning-making processes to explain the complex relationship between various representations, the student's understanding of the situation and the nature of representations, and how MRs are transformed. Through this analysis, the researcher tried to demonstrate how Ainsworth's DeFT (Design, Function, Tasks) framework [19] has been adapted for educational researchers to understand more deeply the process of using and transforming MRs and to learn and communicate real-world situations.

This study is part of more extensive research exploring the transformation of MRs in improving to explain real-world situations. The analytical emphasis was the students' meaning-making of a concept of physics in a situation (a ball is thrown vertically upwards) over their use of MRs. The three research questions that led the analysis were:

1. What multiple representations do students embrace when explaining physics in real-world situations?
2. How does the transformation of MR enable students to understand and explain real-world situations?
3. Why was the transformation of MR adopted to facilitate understanding of physics real-world situations?

The researchers conducted the study among five form four students (purposively selected) and gathered multiple data sources, such as videos and field notes on think-aloud problem solving of real-world situation (TAP), retrospectives interviews with students (ProTRet) and document analysis on the worksheet of students' problem-solving in real-world situation to triangulate the results [20, 21]. The problem given was as shown below:

There is a girl throwing a ball vertically upwards. When reaching position X, the ball is falling down towards the ground. What is the velocity of the ball at position X? Describe the motion of the ball in this situation.

DOI: 10.37200/IJPR/V24I5/PR2020248
Received: 14 Mar 2020 | Revised: 30 Mar 2020 | Accepted: 06 Apr 2020

Through this study, the goal of the researcher was not to generalise a broad set of data [22]. Alternatively, by giving a detailed description of how specific experiences of students relate with their use of multiple representations. This study aimed to deliver intuition into how the researcher can accurately interpret the transformation of MR in students with respect to Ainsworth's analytical framework [20]. These findings intended to help researchers in physics education step forward in building a clearer framework on how to assist students to use different representations transformation.

After the problem solving of real-world situations using TAP, the researcher did one-to-one retrospective interviewed with the participants base on the situation answered. The think-aloud session took 10-15 minutes to complete. Then a semi-structured interview protocol (ProTRet) [20] was applied to explore and understand the transformation of MR. In the interviews, we sought to determine how students integrated the different representations to explain why they choose specific representations and how the transformation of MRs was being done. As a prop for the interview, the problem solving of real-world situation paper was presented in front of them to recall the memory and reasons behind the transformation of MR. Each interview session lasted between 15-20 minutes for this particular situation (a ball is thrown vertically upwards) and was video-recorded and later fully transcribed. Lastly, the problem-solving worksheets were examined to triangulate between data collected.

III. Results And Discussion

The results obtained were based on the real-world situation given during TAP and semi-structured interviews. During the TAP session, the first participant (P1), asked for permission from the researcher to do some sketching just after reading the situation given. The excerpt is as shown below:

Think aloud with P1 on 14 Nov 2018 (Line 14-17)

P1: I mean when the ball... Can I draw it?

R: (Researcher nodded)

P1: Use a pen to use a pencil?

R: Never mind.

P1 continues the solution by drawing a round shape, up arrow and down arrow. Based on the excerpts from this TAP, P1 tries to explain the situation in sketches. Figure 1.1 below shows the sketches, symbols, and texts of the situation.

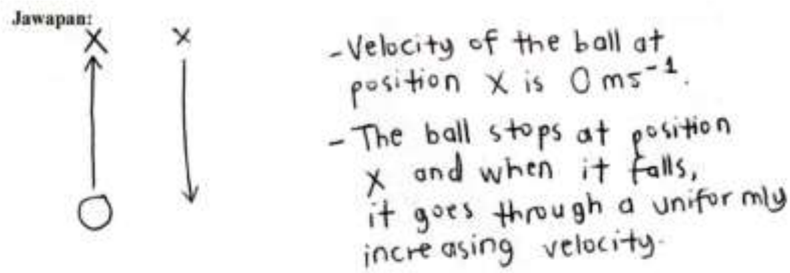


Figure 1.1: The Representation Developed by P1

A round shape is drawn to represent the thrown ball. Then the up arrow shows the movement of the ball

DOI: 10.37200/IJPR/V24I5/PR2020248
Received: 14 Mar 2020 | Revised: 30 Mar 2020 | Accepted: 06 Apr 2020

thrown upwards. P1 denotes the point X where the ball is positioned after being thrown up. Then, P1 draws the down arrow to indicate the ball's movement back down after being thrown upwards. The three sketches, which are round shape, up arrow, and down arrow, are intended to provide a clearer picture of the situation. According to P1, the sketches will help her think more and add whatever sketches she needs after sketching in the early stages. This finding shows that participants able to make the connection between representation with less effort as discovered from the previous study [23]. Here are excerpts from interviews with P1:

Interview with P1 on 14 Nov 2018 (Line 3-8)

R: I want to ask this question. Why you drew an arrow?

P1: Because when I draw, it looks like I can imagine how things are. If I didn't draw, I would dizzy so that I will dizzy.

R: Dizziness?

P1: Not dizzy, I can't imagine it without drawing it. It will be clear. I got the

Furthermore, according to P1, when a sketch is made for a situation, she can "see" the situation by merely looking at the sketch drawn in the beginning. P1 can also understand the situation, as described in the passage below.

Interview with P1 on 14 Nov 2018 (Line 11-14)

R: Based on this diagram? (point pen of the sketch on the sheet)

P1: That sounds like ok. If we draw that ball ok, he goes up. Then he stopped near it. After he stopped there, he fell back. I mean, it looks like we look back here oooo I see, (nodding), it's like that. I can see it.

Based on the observation and analysis of TAP data for P1, the MR transformation performed is the text of the given problem, sketch, symbol, and text for explanation of the solution made. Generally, the MR transformation process performed by P1 for the first problem is, as shown in Figure 1.2 below:

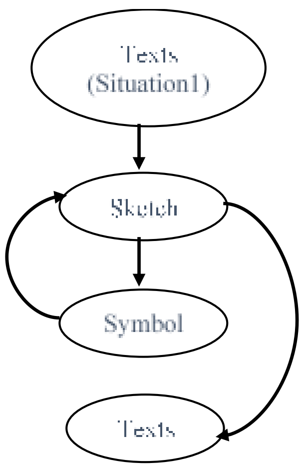


Figure 1.2: Transformation of MR for P1

Based on the observation during the think-aloud and the analysis of the TAP transcript, P2 only relies on the same representation, which is texts. After reading aloud the situation given, P2 explained the motion of the ball

DOI: 10.37200/IJPR/V24I5/PR2020248
Received: 14 Mar 2020 | Revised: 30 Mar 2020 | Accepted: 06 Apr 2020

as shown by the excerpt below:

Think aloud with P2 on 19 Nov 2018 (Line 11-17)

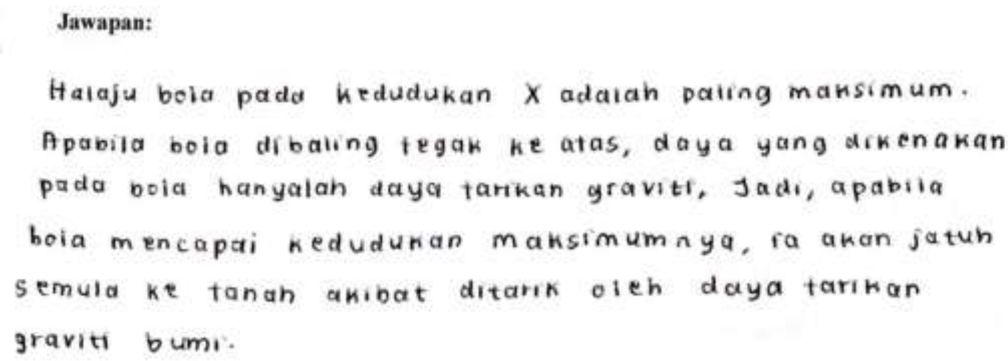
X is ... err mmm what ... err mmm the maximum height of the ball to go, and then the ball moves, the ball moves... errr ... the ball from the girl's hand when he rolls up ... err mmm …… .. When the ball goes past the girl's hand, err..it is just ... there is, err..the force that is applied to the ball is only from the gravitational pull. So when the ball reaches the point x, which is the maximum that the ball can reach, it will ... that gravity will pull it down again.

From the reading, she directly explains the situation by writing down the description of the motion of the ball, as shown by the excerpt below:

Think aloud with P2 on 19 Nov 2018 (Line 23-27)

The velocity of the ball at the x position is maximum (write an explanation). When the ball is thrown perpendicular to the force applied to the ball, it is just a gravitational pull. So when the ball reaches its maximum position, it will fall back to the ground as it is attracted by the gravitational pull of the earth.

As for the proof, Figure 1.3 below shows that she explain the motion of the ball by using texts.


Jawapan:

Halaju bola pada kedudukan X adalah paling maksimum. Apabila bola dibaling tegak ke atas, daya yang dikenakan pada bola hanyalah daya tarikan graviti. Jadi, apabila bola mencapai kedudukan maksimumnya, ia akan jatuh semula ke tanah akibat ditarik oleh daya tarikan graviti bumi.

Figure 1.3: P2 Use Texts to Represent an Explanation for Situation 1

To summarise the representations used by P2 in this situation, the researcher able to identify that only one representation used by her. The situation given was in text form only, and she was giving out the solution by explaining using text also. Figure 1.4 shows the transformation of MR done by P2.

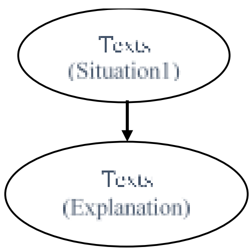


Figure 1.4: The Transformation of MR for P2

The analysis of the transcript for P3 shows that she is sketching a round shape. Then she sketched an arrow pointing downwards and labeling with symbol X. The excerpt of this action is as shown below:

Think aloud with P3 on 21 Nov 2018 (Line 10-17)

DOI: 10.37200/IJPR/V24I5/PR2020248
Received: 14 Mar 2020 | Revised: 30 Mar 2020 | Accepted: 06 Apr 2020

P3: Arr..when he reached x, he…err (draw a circle and down arrow then x). Emm ... (silent) ... (reread question).

R: Keep talking

P3: Hmm..(nodding)...

Ball vertically upward ... Describe the movement of the ball in this situation (reread the question). Until X, it falls. So here (pointing arrow), it has a high potential

After that, P3 continued to write an explanation of the movement of the ball for the given situation. The following is an excerpt from the TAP transcript on the use of text in explaining the situation.

Think aloud with P3 on 21 Nov 2018 (Line 22-25)

Arr ... when you get to the top.

It potential… the velocity of the ball at X, zero because it is motionless (while writing an explanation).

It ... wants to move down. The ball begins to slow down as compared when moving upwards. (in writing).

The task given in situation 1 is that students are assigned to determine the velocity of the ball at position X and to describe the motion of the ball. The solution of the task for situation 1 done by P3 is shown in Figure 1.5.

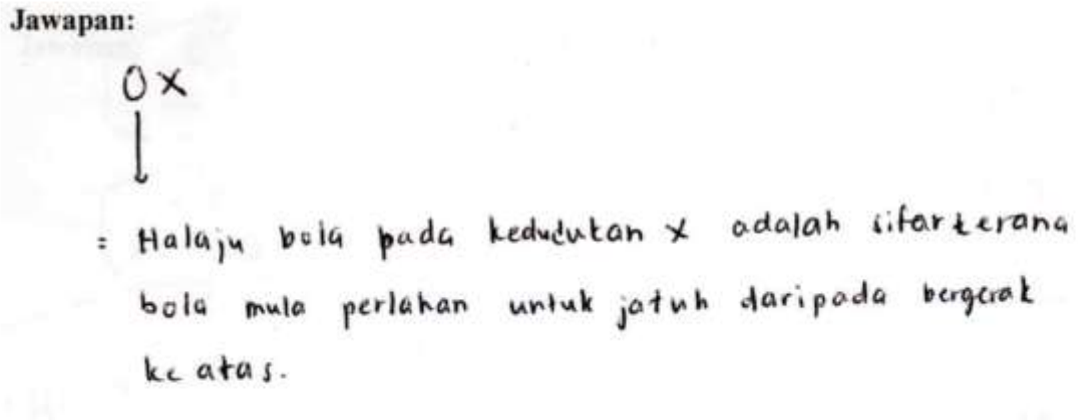


Figure 1.5: The Representation Created by P3 to do the Task

The round shape sketched represents the ball thrown, while the down arrow shows the ball falling after reaching point X. X is considered to be the maximum point reached by throwing the ball upwards. The following excerpt from the ProTret transcript explains the representation function built by P3.

Interview with P3 on 21 Nov 2018 (Line 24-33)

R: Understand that question, ok. When did you draw the ball with that down arrow, what is it for?

P3: The circle shows the ball, X means it is at the topmost as it moved.

R: Where's mention that X is at the topmost?

P3: Because the question said this, (point to the worksheet) when he reached X, the ball fell.

R: But the situation didn't say it was the maximum.

P3: Oooo. But it said the ball fell to the ground.

DOI: 10.37200/IJPR/V24I5/PR2020248
Received: 14 Mar 2020 | Revised: 30 Mar 2020 | Accepted: 06 Apr 2020

R: How do you know it's the maximum?

P3:It says she threw up (move right-hand to indicate throwing upwards). At the top, it meant to be upright (move left hand showing up) Then it said what … when the ball

The following is a compilation of the transformation of the MR carried out by P3 to resolve the problem from situation 1. Texts, sketches, and symbols are the different formats of representations involved.

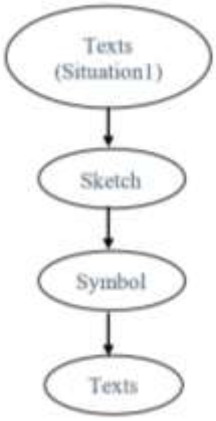


Figure 1.5: Transformation of MR for P3

Further data analysis was performed to understand the MR transformation of P4. After reading the situation given during the think-aloud session, P4 does some sketching on the solution space on the worksheet. The TAP transcript below shows the transformation of MR that was fostered for situation 1.

Think aloud with P4 on 21 Nov 2018 (Line 10-13)

So this ball, from the girl's hand (drawing hands).

It goes up (draws an up arrow).

Up to the position X, then it falls back (drawing down arrow).

So the velocity in that X is zero ms-1 (while writing 0 ms-1).

Based on the think-aloud, it is observed that P4 state the velocity of the ball by writing down an equation showing that the velocity at X is 0 ms-1. Then P4 describes the motion of the ball by writing the explanation in texts or words. The following is proof of the transformation of MR developed by P4.

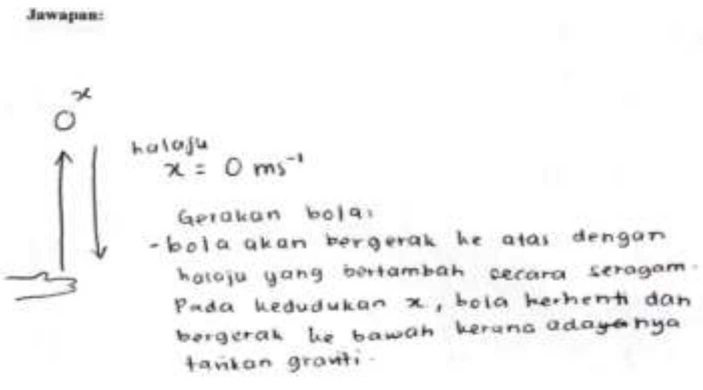


Figure 1.6: The Representations Created by P4

DOI: 10.37200/IJPR/V24I5/PR2020248
Received: 14 Mar 2020 | Revised: 30 Mar 2020 | Accepted: 06 Apr 2020

P4 constructs particular representations for solving assigned task consists of text, sketches, and mathematical equations. The design is intended to get a sense of the situation that has been read. According to P4, without a sketch, it would have been difficult for her to see the motion. P4 believes that the sketch can help her visualise the situation. The following is an excerpt from the retrospective interview transcript stating the function of that sketch.

Interview with P3 on 21 Nov 2018 (Line 6-16)

R: Ok. Question one, after you read the question, then you draw.

P4: Read? (While taking the question paper to read)

R: Eh, no ... no. It's during the time you answer this question. At the time you answer the questions, you draw…eh.. sketch a hand. What is it for? That hand that you sketched?

P4: To get an illustration...

R: Then, what?

P4: Just for fun.

R: Just for fun? You want to draw it or why? If you didn’t draw, you cannot see it?

P4: (nodding her head).

P4 drew sketches from the text provided for the given situation, and then expressed velocity using mathematical equations. She uses some text to describe the ball's motion. At the same time, when expressing mathematical equations, P4 also uses symbols to write equations. The following is a summary of the transformation of the MR carried out by P4 to complete a given task.

As for P5, while reading the given situation, he made a gesture to show the action of throwing the ball up and down with his hands. An excerpt from the TAP transcript shows the participant using hand gestures while reading the situation.

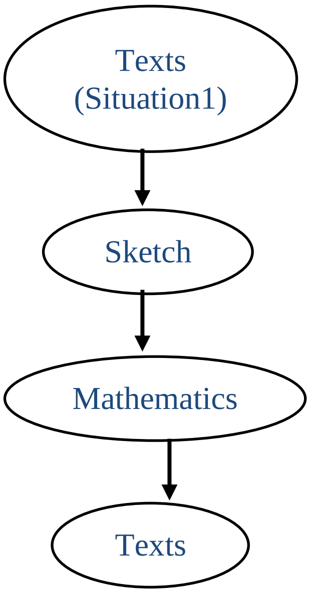


Figure 1.7: Transformation of MR for P4

DOI: 10.37200/IJPR/V24I5/PR2020248
Received: 14 Mar 2020 | Revised: 30 Mar 2020 | Accepted: 06 Apr 2020

Think aloud with P5 on 20 Nov 2018 (Line 3-5)

A girl throws the ball vertically from the bottom upwards (hands indicate movement of the ball upwards), when it reaches the position x, the ball begins to fall to the ground (hand indicates the movement of the ball downwards).

After reading the given situation, P5 continues to sketch a round shape with the up and down arrows. P5 also uses the X symbol to indicate the position of the ball. Also, P5 uses a short form of words or symbols explicitly designed to represent the movement of the ball before with the "seb" symbol and after with the "sel" symbol.

Think aloud with P5 on 20 Nov 2018 (Line 8-12)

When at below there is no velocity (drawing a round shape), when it is thrown vertically upwards, velocity ... there is a velocity because there is a change of distance, there is a time (Draw an up arrow). Go up ... it is at the position X(write X) it'll start to fell down ... gravity pull. From this X, it will start to fall (draw downward arrow). Before (writing "seb"), after (writing "sel").

Then, P5 creates the task assigned by merely writing an equation to show the velocity of the ball at position X. The excerpt of the think-aloud transcript below shows that P5 was writing the equation.

Think aloud with P5 on 20 Nov 2018 (Line 13-17)

P5: Hmmm ... at X this velocity is zero (scratching the forehead). Zero velocity and when it has a tendency to… when he comes down, it has a velocity ba..ba..back.

R: Keep talking, even if you're writing this.

P5: Hmm… velocity is equal to zero (write velocity = zero).

The following figures are representations of the sketches, symbols, and equations that P5 has built for situation 1.

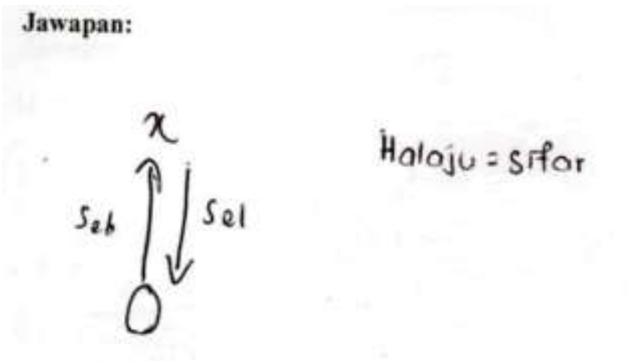


Figure 1.8: Representation that was Built by P5

Through interviews conducted with P5, the researcher discovered that the purpose of the project was to describe the situation that was read and understood. Also, the purpose of the sketch is to enable others to understand the situation by merely looking at his sketch. Therefore, the function of the sketch is to represent and inform others of the situation.

Interview with P5 on 20 Nov 2018 (Line 62-70)

R: Ok. Then why are you drawing this?

DOI: 10.37200/IJPR/V24I5/PR2020248
Received: 14 Mar 2020 | Revised: 30 Mar 2020 | Accepted: 06 Apr 2020

P5: Ok, for me to imagine this ball, in her hand (take pen to throw). It threw up, to the maximum, then it'll fall back. (put down pen)

R: What for are you drawing?

P5: For me to understand

R: Just now, you said you understand that question?

P5: So if anyone else even looks at it, he or she will understand.

R: Ooo ... you would like the teacher to understand? Hahaha

P5: Haaa… hahaha

During the TAP session, P5 forgot about the explanation that he should do. Then, the researcher asks him to explain the motion of the ball. According to P4, the explanation should be longer words (texts) because he had to explain in detail. The researcher wants to have a short or straightforward response. Then, he decided to answer it in equation format, as shown in Figure 1.9 below.

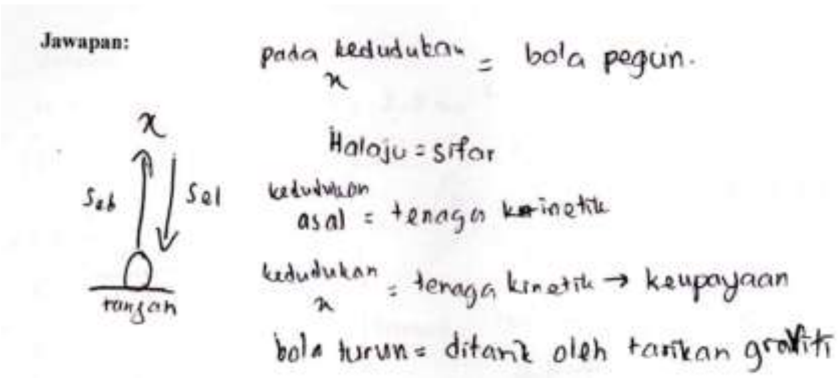


Figure 1.9: The Explanation was Written in Equation Format Done P5

Figure 1.10 below describes the MR transformations that P5 has built for Situation 1 and responds to the assigned task. P5 has created the gesture, sketches, symbols and equations for that situation.

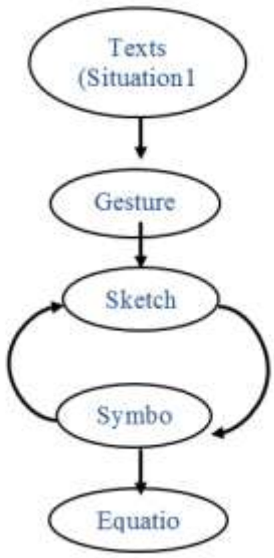


Figure 1.10: Transformation of MR for P5

DOI: 10.37200/IJPR/V24I5/PR2020248
Received: 14 Mar 2020 | Revised: 30 Mar 2020 | Accepted: 06 Apr 2020

As a summary of this finding, the transformations undertaken by all study participants included text, sketches and symbols, and equations. It is based on the DeFT framework, which focuses on the aspects of design, function, and tasks [19]. The tasks associated with cognitive in which students may need to choose a suitable representation or create one for themselves, which may offer advantages due to new cognitive tasks [19]. Students should be aware of the format of the representation they are presenting and be able to convey the intended information of the representative being constructed. Because of this, students need to select appropriate representations for the task at hand [19].

In fact, detailed analysis shows that all participants are aware of the relationship between position, velocity, and acceleration in this kinematics case. They were able to indicate that the energy produced and force gain by the ball and relate with velocity, acceleration, and position, respectively. The transformation of MR created for this situation were texts, symbols, sketches, and equations. However, none of the participants exhibits graph as representation to explain the motion of the ball. The graph-representation was seldom-used by students due to the difficulties to transform in problem-solving because it has many details to be considered such as axis, label, the data and the relationship between variables [19, 24].

Besides, most of the participants created sketches to represent the situation. The purpose behind this is to visualise and make the situation clear to them and also to enable them a better understanding. Sketches are depictive representations that are object or a process in a concrete or abstract form [25]. Sketches also portray the physical properties of the subjects. This data is in contrast to the finding of the study [26] which stated that students rarely draw diagrams when solving problems. Problem-solving often requires students to recognise or create at least one appropriate representation of a given task [25]. Hence, represent it in a different format will provide extensive information which, leads to proper solutions [13–15], [19].

The majority of participants just focus on the use of texts in describing the motion. Thus, it can be concluded that for kinematics, where students have better understanding and experience, the texts representation used to structure the task given which mention by the word "describe". This finding is aligned with Fredlund, Linder, Airey, & Linder [28] study, stated that the, which is the most relevant representation for a particular task depends partly on those aspects of the problem that are disciplinary relevant.

The researcher also observed that there was an engagement between participants and assigned tasks while using representations to understand the situations and tasks provided. Hence, MR is crucial in engagement between students and the learning materials [29]. Although the researcher present in detail the findings from a participant that we found particularly interesting, other participants showed similar patterns. The participants construct a particulate representation of the situation. They also transform the MR, which is almost the same among them for this situation. This finding shows that the transformation of MR was explicitly made to make them understand the problem [17, 29, 30] and able to explain the motion of the ball by using texts. None of them explain the motion of the ball by using a graphical representation, which is difficult to be done by students in high school or tertiary education [31, 32].

Our analysis shows that the nature of the student's work shows that their transformation of MR interlinked primary sources; such as the book and the teacher and the students' experience in daily life and also with their previous knowledge. They were able to describe the motion in detail because they have already experienced it. As per cognitive constructivism, individuals construct their knowledge of the world, so that people build their

DOI: 10.37200/IJPR/V24I5/PR2020248
Received: 14 Mar 2020 | Revised: 30 Mar 2020 | Accepted: 06 Apr 2020

mental models to make sense of their experiences and observations [34]. As a result, this finding shows that participants build representations on the grounds of their existing knowledge and experience as part of their knowledge development.

IV. CONCLUSION

As a conclusion, it was found that students tend to use three formats of representation and transform between them. They were texts, sketches, symbols, and equations. Most of the participants transform the text provided by the situation into sketches so that they can visualise and depict more explicit sketches with the addition of the symbol. The transformation of MR provides clear meaning for them to understand better. Based on the finding, the researcher suggested that students should re-representing a concept or an idea using multiple representation strategies to enhance the transformation of MR during learning and problem solving and a better understanding of content knowledge.

ACKNOWLEDGMENT

The project is supported by the Ministry of Education under the Fundamental Research Grant Scheme (FRGS) vot number R. J130000.7853.5F140.

REFERENCES

[1] N. S. A. Rahim, F. A. Phang, and J. Pusppanathan, “Action research to improve students’ problem solving using multiple modes of representation,” *Int. J. Recent Technol. Eng.,* vol. 7, no. 6, pp. 953–957, 2019.

[2] S. Ainsworth, “The functions of multiple representations,” *Comput. Educ.,* vol. 33, pp. 131–152, 1999.

[3] J. A. Dahlan and D. Juandi, “Analisis Representasi Matematik Siswa Sekolah Dasar Dalam Penyelesaian Masalah Matematika Kontekstual,” *J. Pengajaran Mat. dan Ilmu Pengetah. Alam,* vol. 16, no. 1, p. 128, 2011.

[4] B. Waldrip, V. Prain, and P. Sellings, “Explaining Newton’s laws of motion: Using student reasoning through representations to develop conceptual understanding,” *Instr. Sci.,* vol. 41, no. 1, pp. 165–189, 2013.

[5] Hikmat, Liliasari, and Rusdiana, “How is Physics Teacher Use Multiple Representation,” *in MSCEIS* 2015, 2015, p. 67.

[6] M. A. Rau and P. G. Matthews, “How to make ‘more’ better? Principles for effective use of multiple representations to enhance students’ learning about fractions,” *ZDM,* vol. 49, pp. 531–544, 2017.

[7] M. Won, H. Yoon, and D. F. Treagust, “Students’ Learning Strategies With Multiple Representations: Explanations of the Human Breathing Mechanism,” *Sci. Educ.,* 2014.

[8] M. Opfermann, A. Schmeck, and H. E. Fischer, “Multiple Representations in Physics and Science Education – Why Should We Use Them?,” *in Multiple Representations in Physics Education,* vol. 10, D. F. Treagust, R. Duit, and H. E. Fischer, Eds. Cham, Switzerland, 2017, pp. 1–22.

[9] A. Van Heuvelen and X. Zou, “Multiple representations of work–energy processes,” *Am. J. Phys.,* vol. 69, no. 2, pp. 184–194, 2001.

[10] J. I. Heller and F. Reif, “Prescribing Effective Human Problem-Solving Processes : Problem Description in Physics Linked references are available on JSTOR for this article : Prescribing Effective Human Problem- Solving Processes : Problem Description in Physics,” *Cogn. Instr.,* vol. 1, no. 2, pp. 177–216, 1984.

[11] D. Hestenes, “Toward a modeling theory of physics instruction a),” *Am. J. Phys.,* vol. 55, no. May, pp. 440–454, 1987.

[12] A. Van Heuvelen, “Learning to Think Like a Physicist: A Review of Research-based Instructional Strategies,” *Am. J. Phys.,* vol. 59, no. 10, pp. 891–897, 1991.

[13] D. E. Meltzer, “Relation between students’ problem-solving performance and representational format,” *Am. J. Phys.,* vol. 73, no. 898, pp. 463–478, 2005.

[14] A. Maries, “Role of Multiple Representations in Physics Problem Solving,” University of Pittsburgh, 2014.

[15] D. F. Treagust, R. Duit, and H. E. Fischer, Multiple Representations in Physics and Science Education, vol. 10. Springer, 2017.
[16] B. Waldrip, V. Prain, and J. Carolan, "Learning Junior Secondary Science through Multi-Modal Representations," *Electron. J. Sci. Educ. Preview Publ.*, vol. 11, no. 1, 2006.
[17] P. B. Kohl, "Towards an Understanding of How Students Use Representations in Physics Problem Solving," 2007.
[18] F. Reif, Applying Cognitive Science to Education: Thinking and Learning in Scientific and Other Complex Domains. London, England: MIT Press, 2008.
[19] S. Ainsworth, "DeFT: A conceptual framework for considering learning with multiple representations," *Learn. Instr.*, vol. 16, pp. 183–198, 2006.
[20] L. Cohen, L. Manion, and K. Morrison, Research methods in Education, 7th Editio. Oxon: Routledge, 2011.
[21] S. B. Merriam, Qualitative research: *A guide to design and implementation, Second. San Francisco,CA.: Jossey-Bass*, 2009.
[22] R. K. Yin, "Validity and generalization in future case study evaluations," 2013.
[23] M. A. Rau, V. Aleven, and N. Rummel, "Making connections among multiple graphical representations of fractions: sense-making competencies enhance perceptual fluency, but not vice versa," *Instr. Sci.*, vol. 45, no. 3, pp. 331–357, 2017.
[24] M. Hill, "Growth in students' representational ability while at university," *University of Sydney,* 2011.
[25] R. Hettmannsperger et al., "Developing conceptual understanding in ray optics via learning with multiple representations," *Z Erziehungswiss*, vol. 19, pp. 235–255, 2016.
[26] M. Snetinova and Z. Koupilova, "Students' Difficulties in Solving Physics Problems," *Proc. 21st Annu. Conf. Dr. Students - WDS 2012,* pp. 93–97, 2012.
[27] T. Fredlund, C. Linder, J. Airey, and A. Linder, "Unpacking physics representations: Towards an appreciation of disciplinary affordance," *Phys. Rev. Spec. Top. - Phys. Educ. Res.*, vol. 10, no. 2, 2014.
[28] J. Forsman, R. Moll, and C. Linder, "Extending the theoretical framing for physics education research: An illustrative application of complexity science," *Phys. Rev. Spec. Top. - Phys. Educ. Res.*, vol. 10, no. 2, 2014.
[29] T. Fredlund, J. Airey, and C. Linder, "Exploring the role of physics representations:an illustration example from students sharing knowledge about refraction," *Eur. J. Phys.*, vol. 33, no. 3, pp. 657–666, 2012.
[30] W. Schnotz and M. Bannert, "Construction and interference in learning from multiple representation," *Learn. Instr.*, vol. 13, no. 2, pp. 141–156, 2003.
[31] V. Prain and B. Waldrip, "An exploratory study of teachers' and students' use of multi-modal representations of concepts in primary science," *Int. J. Sci. Educ.*, vol. 28, no. 15, pp. 1843–1866, 2006.
[32] S. den Eynde, P. van Kampen, W. Van Dooren, and M. De Cock, "Translating between graphs and equations: The influence of context, direction of translation, and function type," *Phys. Rev. Phys. Educ. Res.*, vol. 15, no. 2, Jul. 2019.
[33] L. Bollen, P. Van Kampen, C. Baily, M. Kelly, and M. De Cock, "Student difficulties regarding symbolic and graphical representations of vector fields," *Phys. Rev. Phys. Educ. Res.*, vol. 13, no. 2, 2017.
[34] T. Bond, "Piaget's Learning Theories," *in Encyclopedia of the Sciences of Learning,* 2012, pp. 2634–2636.

DOI: 10.37200/IJPR/V24I5/PR2020248
Received: 14 Mar 2020 | Revised: 30 Mar 2020 | Accepted: 06 Apr 2020