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Applications of Novel Transformation for Solving Ordinary Differential Equations with Unknown Initial Conditions

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Abstract--- *In this paper, we apply the Novel transform to obtain the formulas of general solutions of ordinary differential equations with unknown initial conditions through utilizing the relationship between Novel and Laplace transforms.*

Keywords--- *Initial Conditions, Applications of Novel, Differential Equations*.

I. INTRODUCTION

Integral transformations are an abbreviated method for solving many types of equations that are difficult to solve by classical methods [6]. There are quite a few integral transforms applicable for solving differential equations like Laplace, Sumudu, Elzaki, Temimi, Melina, etc...[3,9,10,13]. Laplace transform was widely used in solving several types of differential equations such as ordinary, partial, integral, fractional, and other equations [5,8]. Sumudu transform is similar to Laplace transform [7]. Elzaki transform is a modified form of Sumudu and Laplace transform [4]. Recently, a new integral transform called the Novel transform's

where some definitions and properties of Novel transform are given in [11]. The Novel transform of the function $h(t)$ is defined as:

$$\Omega(s) = N_I\big(h(t)\big) = \frac{1}{s}\int_0^{\infty} e^{-st}\, h(t)\, dt, \text{ t>0, (1)}$$

where $h(t)$, $t > 0$, is a real function, $\frac{e^{-st}}{s}$ is the kernel function and N_I is the operator of NIT.

Authors in [12], used Novel transform to solve the differential equation arising in the heat- transfer problem. Novel transform is successfully used on different types of differential equations [1,2,11]. In this research, we apply the Novel transform to obtain a formula of general solutions of linear differential equations, whether known or unknown initial conditions. In section 2, we reviewed the properties of transforms and the important theorems for some functions through the relationship between Laplace and Novel transformations. In section 3, we achieved formulas of the general solution of differential equations of first, second, and higher-order. In section 4, we presented a formula of general solutions of the ordinary differential equation of order (n) without subjected to initial conditions. Finally, in the last section, we applied general formulas obtained in the previous section for solving some differential equations.

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II. BASIC DEFINITIONS AND PROPERTIES OF NOVEL TRANSFORM

Definition

The Laplace's transform is defined by: $F(s) = L_I(h(t)) = \int_0^{\infty} e^{-st}\, h(t)dt\,;\, t > 0$, (2)

where $h(t)$, $t > 0$, is a real function, e^{-st} is the kernel function, and L_I is the operator of Laplace transform. The Laplace and Novel transforms exhibit a duality relationship that expressed as :

$$\Omega(s) = \frac{1}{s}F(s).\ (3)$$

In detail, some functions are showed as follows:

∗For $h(t) = 1$ and $t > 0$, we have :

$$\mathrm{F(s) = L(1)} = \frac{1}{\mathrm{s}},$$

the Novel transform of $h(t)$ =1 is derived by the duality relationship

$$\Omega(s) = \frac{1}{s}F(s) = \frac{1}{s}L(1) = \frac{1}{s}\left(\frac{1}{s}\right) = \frac{1}{s^2}\ (4)$$

*For $h(t) = e^{rt}\ and\ t > 0$, we have: $\mathrm{F(s) = L(e^{rt})} = \frac{1}{\mathrm{s-r}}$,

the Novel transform of $h(t) = e^{rt}$ is derived by the duality relationship

$$\Omega(s) = F(s) = \frac{1}{s}L(e^{rt}) = \frac{1}{s(s-r)}\ (5)$$

*For $h(t) = t$ and t > 0,we have :

$$F(s) = L(t) = \frac{1}{s^2},$$

the Novel transform of $h(t) = t$ can be written as:

$$\Omega(s) = \frac{1}{s}F(s) = \frac{1}{s}L(t) = \frac{1}{s^3}\ (6)$$

*For $h(t)$ = $sin\, rt$,t > 0

$$F(s) = L(\sin rt) = \frac{r}{s^2+r^2},$$

the Novel transform of $h(t)$ =$sin\, rt$ can be written as :

$$\Omega(s) = \frac{1}{s}F(s) = \frac{1}{s}L(sin\, rt\ ) = \frac{r}{s(s^2+r^2)},\ (7)$$

* For $h(t)$ = cos rt,t >0

$$F(s) = L(cos\, rt) = \frac{s}{s^2+r^2},$$

the Novel transform of $h(t)$ =cos rt can be written as :

$$\Omega(s) = \frac{1}{s}F(s) = \frac{1}{s}L(cos\, rt) = \frac{1}{s^2+r^2}.\ (8)$$

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The following theorem represent Novel transform for derivative functions and the reader can return for proof to [11].

Theorem

If $h(t), \acute{h}(t), \ldots., h^{n-1}(t)$ are continuous for

(t > 0) and of exponential order as $t \to \infty$, also $h^{(n)}(t)$ is continuous,

$\mathrm{L}\{h(t)\} = \mathrm{F}(s)$, it follows that

$$L\{h^{(n)}(t)\} = s^n F(s) - \sum_{k=0}^{n-1} s^{n-2-k}\, h^{(k)}(0). \ (9)$$

III. Formulas Of The General Solution For The Differential Equation

a- Consider the first-order ordinary differential equation

$$a_1 \acute{y} + a_2 y = f(t)\,; t > 0, (10)$$

with the initial condition *y(0)= β,*

where a_1, a_2 *and* β are constants and f (t) is an integrable function.

Applying Novel transform of the equation (10), we have:

$$a_1 N_I(\acute{y}) + a_2 N_I(y) = N_I(f(t))$$

$$a_1 s N_I(y) - \frac{a_1}{s} y(0) + a_2 N_I(y) = N_I(f(t))$$

$$N_I(y) = \frac{\beta a_1 + s N_I(f(t))}{a_1 s(s + \frac{a_2}{a_1})}, (11)$$

the inverse Novel transform leads to the solution.

b - The second-order linear ordinary differential equation has the general form

$$a_1 \acute{\acute{y}} + a_2 \acute{y} + a_3 y = f(t), t > 0. (12)$$

With the initial conditions y(0)= β_1 and $\acute{y}(0) = \beta_2$,

where,a_1, a_2, a_3, β_1 and β_2 are constants and f (t) is an integrable function.

Applying Novel transform of the equation (12), we have:

$$a_1 N_I(\acute{\acute{y}}) + a_2 N_I(\acute{y}) + a_3 N_I(y) = N_I(f(t))$$

$$a_1 s^2 N_I(y) - a_1 y(0) - \frac{a_1}{s} \dot{y}(0) + a_2 s N_I(y) - \frac{a_2}{s} y(0) + a_3 N_I(y) = N_I(f(t))$$

$$a_1 s^2 N_I(y) - a_1 \beta_1 - \frac{a_1}{s} \beta_2 + a_2 s N_I(y) - \frac{a_2}{s} \beta_1 + a_3 N_I(y) = N_I(f(t))$$

$$N_I(y) = \frac{s a_1 \beta_1 + a_2 \beta_1 + a_1 \beta_2 + s N_I(f(t))}{s(a_1 s^2 + a_2 s + a_3)}. (13)$$

Taking the inverse of the Novel transform of equation (13), we get the solution of equation (12).

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c- The third-order linear ordinary differential equation has the general form.

$$a_1\acute{\acute{\acute{y}}} + a_2\acute{\acute{y}} + a_3\acute{y} + a_4 y = f(t),\ t>0 \quad (14)$$

with the initial conditions $y(0) = \beta_1, \acute{y}(0) = \beta_2$ and $\acute{\acute{y}}(0) = \beta_3$,

where, $a_1, a_2, a_3, a_4, \beta_1, \beta_2$ and β_3 are constants and f (t) is an integrable function.

Applying Novel transform of the equation (14), we have:

$$a_1 N_I\left(\acute{\acute{\acute{y}}}\right) + a_2 N_I\left(\acute{\acute{y}}\right) + a_3 N_I(\acute{y}) + a_4 N_I(y) = N_I(f(t))$$

$$a_1 s^3 N_I(y) - a_1 s y(0) - a_1 \dot{y}(0) - \frac{a_1}{s}\ddot{y}(0) + a_2 s^2 N_I(y) - a_2 y(0) - \frac{a_2}{s}\dot{y}(0) + a_3 s N_I(y) - \frac{a_3}{s} y(0) + a_4 N_I(y) = N_I(f(t))$$

$$a_1 s^3 N_I(y) - a_1 s\beta_1 - a_1\beta_2 - \frac{a_1}{s}\beta_3 + a_2 s^2 N_I(y) - a_2\beta_1 - \frac{a_2}{s}\beta_2 + a_3 s N_I(y) - \frac{a_3}{s}\beta_1 + a_4 N_I(y) = N_I(f(t))$$

$$N_I(y) = \frac{a_1\beta_1 s^2 + s[a_1\beta_2 + a_2\beta_1 + N_I(f(t))] + a_1\beta_3 + a_2\beta_2 + a_3\beta_1}{s[a_1 s^3 + a_2 s^2 + a_3 s + a_4]}, \quad (15)$$

the inverse Novel transform leads to the general solution of equation (14).

Similarly, general formula can be found to solve differential equations of order (n) by Novel transformation.

$$a_0 y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + \cdots + a_n y = f(t),\ t>0 \quad (16)$$

with the initial conditions

$$y(0) = \beta_1, \acute{y}(0) = \beta_2, \ldots, y^{(n-2)}(0) = \beta_{n-1}\ and\ y^{(n-1)}(0) = \beta_n,$$

where, $a_0, a_1, a_2, \ldots, a_n, \beta_1, \beta_2, \ldots, and\ \beta_n$ are constants and f (t) is an integrable function,

by taking Novel transform for both sides, we get :

$$a_0 N_I\left(y^{(n)}\right) + a_1 N_I\left(y^{(n-1)}\right) + a_2 N_I\left(y^{(n-2)}\right) + \cdots + a_n N_I(y) = N_I(f(t))\ t>0$$

$$a_0\left[s^n N_I(y) - s^{n-2} y(0) - s^{n-3}\acute{y}(0) - \cdots - \frac{1}{s} y^{(n-1)}(0)\right] + a_1\left[s^{n-1} N_I(y) - s^{n-3} y(0) - \cdots - \frac{1}{s} y^{(n-2)}(0)\right] + \ldots + a_n N_I(y) = N_I\big(f(t)\big),$$

Substituting initial condition, yields:

$$a_0 s^n N_I(y) - a_0\beta_1 s^{n-2} - a_1\beta_2 s^{n-3} - \cdots - \frac{a_0}{s}\beta_n + a_1 s^{n-1} N_I(y) - a_1\beta_1 s^{n-3} - \cdots - \frac{a_1}{s}\beta_{n-1} + \cdots + a_n N_I(y) = N_I(f(t))$$

$$[a_0 s^n + a_1 s^{n-1} + \cdots + a_n] N_I(y) = N_I(f(t)) + a_0\beta_1 s^{n-2} + (a_0\beta_2 + a_1\beta_1) s^{n-3}$$

$$+ (a_0\beta_3 + a_1\beta_2) s^{n-4} + \ldots + \frac{a_0\beta_n + a_1\beta_{n-1}}{s},$$

let k(s) is polynomial of degree (n-1) conditions all the terms of initial conditions in the right side, then :

$$[a_0 s^n + a_1 s^{n-1} + \cdots + a_n] N_I(y) = N_I(f(t)) + k(t)$$

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$$N_I(y) = \frac{N_I(f(t))+k(t)}{[a_0s^n+a_1s^{n-1}+\cdots+a_n]}, (17)$$

the inverse Novel transform leads to the general solution of equation (16).

IV. To Solve Ordinary Differential Equations of Order (n) Without Using Any Initial Conditions, Using Novel's Transformation

We will apply the Novel transformation to obtain solutions of the ordinary differential equations with constant coefficients and formed by the general as follows:

$$a_0y^{(n)} + a_1y^{(n-1)} + a_2y^{(n-2)} + \cdots + a_ny = f(t), \text{ t>0 } (18)$$

$y(0), \acute{y}(0), \ldots, y^{(n-2)}(0)$ *and* $y^{(n-1)}(0)$,are unknown conditions,

where,$a_0, a_1, a_2, \ldots, and\ a_n$, are constants, f (t) is an integrable function.

we apply the Novel transform to both sides, we get:

$$a_0N_I(y^{(n)}) + a_1N_I(y^{(n-1)}) + a_2N_I(y^{(n-2)}) + \cdots + a_nN_I(y) = N_I(f(t)) \text{ t>0}$$

$$a_0\left[s^nN_I(y) - s^{n-2}y(0) - s^{n-3}\,\acute{y}(0) - \cdots - \frac{1}{s}y^{(n-1)}(0)\right] + a_1\left[s^{n-1}N_I(y) - s^{n-3}y(0) - \cdots - \frac{1}{s}y^{(n-2)}(0)\right] +$$
$$\ldots + a_nN_I(y) = N_I(f(t)),$$

where $y(0), \acute{y}(0), \ldots, y^{(n-2)}(0)$ *and* $y^{(n-1)}(0)$,are unknown conditions.

$$[a_0s^n + a_0s^n + \cdots + a_0s^n]\ N_I(y) = N_I(f(t)) + a_0y(0)s^{n-2} + (a_0\acute{y}(0) + a_1y(0))s^{n-3}$$
$$+(a_0\acute{\acute{y}}(0) + a_1\acute{y}(0))s^{n-4} + \ldots + \frac{a_0y^{(n-1)}(0)+a_1\,y^{(n-2)}(0)}{s},$$

let z(s) is polynomial of degree (n-1), which represent $N_I(f(t))$ and terms initial conditions in the right sides.

$$N_I(y) = \frac{\mathrm{z(s)}}{s[a_0s^n+a_1s^{n-1}+\cdots+a_n]H(s)}. (19)$$

Where $H(s)$ is a polynomial of s, and it is represented a denominator of Novel transform for the function $f(t)$. $H(s)$ is also a polynomial of s which has a degree less than the degree of the product $(s[a_0s^n + a_1s^{n-1} + \cdots + a_n])$ with $H(s)$ and is not necessary to know the terms of $\mathrm{z(s)}$, but we denoted it only by this symbol. The second step of supplementary solution, taking inverse Novel to transform to both sides of equation (19).

$$y(t) = N_I^{-1}[\frac{\mathrm{z(s)}}{s[a_0s^n + a_1s^{n-1} + \cdots + a_n]H(s)}]$$

$$y(t) = A_1\varphi_1(t) + A_2\varphi_2(t) + A_3\varphi_3(t) + \cdots + A_k\varphi_k(t)\ ;\ n < k. (20)$$

Now, we note that general solution of equation (18), has number of constants (k) greater than the order of equation (18). Therefore, we eliminate extra constants by substituting equation (20) in equation (18).

V. Application

In this section, the validation and use fullness of Novel transform are showed by obtaining the general solution

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of the following equations:

Example (1)

Consider the first order differential equation

$$\acute{y} + 2y = t, t > 0 \text{ (21)}$$

with the initial condition y (0)= β, β is unknown constant,

by using formula (11), we have:

$$N_I(y) = \frac{\beta + s(\frac{1}{s^3})}{s(s+2)},$$

using partial fractions

$$N_I(y) = \frac{\beta + \frac{1}{4}}{s(s+2)} + \frac{\frac{1}{2}}{s^3} - \frac{\frac{1}{4}}{s^2},$$

take inverse Novel transform on both sides, we get:

$$y(t) = \left(\beta + \frac{1}{4}\right) e^{-2t} + \frac{1}{2}t - \frac{1}{4}$$

Example(2)

Consider the second-order differential equation

$$\acute{\acute{y}} + 2\acute{y} + 5y = e^{-t} \sin t\,;\ t > 0 \text{ (22)}$$

with the initial conditions $y(o) = \beta_1$and $\acute{y}(0) = \beta_2$,

such that $\beta_1 and\ \beta_2$ are unknown constants.

By applying formula (13), we have:

$$N_I(y) = \frac{s\beta_1 + 2\beta_1 + \beta_2 + s(\frac{1}{s((s+1)^2+1)})}{s(s^2+2s+5)}$$

$$N_I(y) = \frac{1}{S[(S+1)^2+1][(s+1)^2+4]} + \frac{\beta_1}{[(s+1)^2+4]} + \frac{2\beta_1 + \beta_2}{s[(s+1)^2+4]},$$

By using partial fractions, we have:

$$N_I(y) = \frac{\frac{1}{3}}{S[(S+1)^2+1]]} - \frac{\frac{1}{3}}{s[(s+1)^2+4]} + \frac{\beta_1}{[(s+1)^2+4]} + \frac{2\beta_1 + \beta_2}{s[(s+1)^2+4]},$$

taking the inverse of the Novel transformation of to both sides,

$$y(t) = \frac{1}{3}e^{-t} \sin t + \beta_1 e^{-t} \cos 2t + \left(\beta_1 + \frac{1}{2}\beta_2 - \frac{1}{6}\right) e^{-t} \sin 2t$$

$$y(t) = \frac{1}{3}e^{-t} \sin t + \beta_1 e^{-t} \cos 2t + \beta e^{-t} \sin 2t\ ;\ \beta = \beta_1 + \frac{1}{2}\beta_2 - \frac{1}{6},$$

which represent a general solution of original equation, where $\beta\ and\ \beta_1$ are constants.

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Example (3)

Consider the third-order differential equation

$$y''' - 3y'' + 3y' - y = t^2 e^t \; ; t > 0, (23)$$

with the initial conditions $y(0) = \beta_1, y'(0) = \beta_2 \; and \; y''(0) = \beta_3$,

where $\beta_1, \beta 2, and \; \beta_3$ are unknown constants.

By formula (15), we have:

$$\mathrm{N_I(y)} = \frac{\beta_1 s^2 + s\left(\beta_2 - 3\beta_1 + \frac{2}{(s-1)^3}\right) + \beta_3 - 3\beta_2 - 3\beta_1}{s(s^3 - 3s^2 + 3s - 1)},$$

after simple calculation, we get:

$$N_I(y) = \frac{\beta_1 s^2 + (\beta_2 - 3\beta_1)s + (\beta_3 - 3\beta_2 - 3\beta_1)}{s(s-1)^3} + \frac{2}{s(s-1)^6}.$$

Using partial fractions, we have :

$N_I(y) = \frac{\beta_1}{s(s-1)} + \frac{\beta_2 - \beta_1}{s(s-1)^2} + \frac{\beta_3 - 2\beta_2 - 5\beta_1}{s(s-1)^3} + \frac{2}{s(s-1)^6}$. Take inverse Novel transform on both sides, yields :

$$y(t) = \beta_1 e^t + (\beta_2 - \beta_1)te^t + \frac{1}{2}(\beta_3 - 2\beta_2 - 5\beta_1)t^2 e^t + \frac{1}{60}t^5 e^t$$

$y(t) = \beta_1 e^t + \beta_1^* te^t + \frac{1}{2}\beta_2^* t^2 e^t + \frac{1}{60}t^5 e^t; \;\; \beta_1^* = \beta_2 - \beta_1, \beta_2^* = \beta_3 - 2\beta_2 - 5\beta_1$, which represent a general solution of original equation, where β_1^* and β_2^* are constants.

Example (4)

To solve the equation:

$$y^{(4)} + y^{(3)} = 2sin\, t \; ; \; t > 0, (24)$$

with the initial conditions $y(0) = \beta_1, y'(0) = \beta_2, y''(0) = \beta_3, and \; y'''(0) = \beta_4$,

where $\beta_1, \beta_2, \beta_3$, and β_4 are unknown constants.

Apply formula (17) where n=4, we have:

$$\mathrm{N_I(y)} = \frac{\frac{2}{s(s^2+1)} + \beta_1 s^2 + (\beta_2 + \beta_1)s + (\beta_3 + \beta_2) + \frac{\beta_4 + \beta_3}{s}}{s^4 + s^3},$$

and by simple calculation, we get:

$$N_I(y) = \frac{2}{s^4(s^2+1)(s+1)} + \frac{s^3\beta_1 + (\beta_1 + \beta_2)s^2 + (\beta_3 + \beta_2)s + \beta_3 + \beta_4}{s^4(s+1)}$$

Using partial fractions, we have:

$$N_I(y) = \frac{s+1}{s(s^2+1)} - \frac{(1+\beta_4)}{s(s+1)} + \frac{(\beta_1 + \beta_4)}{s^2} + \frac{(\beta_2 - \beta_4 - 2))}{s^3} + \frac{(\beta_3 + \beta_4 + 2)}{s^4},$$

take inverse Novel transform on both sides, we get:

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$$y(t) = \cos t + \sin t \ - (1+\beta_4)e^{-t} + (\beta_1+\beta_4) + (\beta_2-\beta_4-2)t + \frac{1}{2}(\beta_3+\beta_4+2)t^2$$

$$y(t) = \cos t + \sin t - \alpha e^{-t} + \alpha_1 - \alpha_2 t + \frac{1}{2}\alpha_3 t^2 \ ;$$

$$\alpha = 1+\beta_4,\ \alpha_1 = \beta_1+\beta_4, \alpha_2 = \beta_2-\beta_4-2 \text{ and } \alpha_3 = \beta_3+\beta_4+2,$$

which represent a general solution of original equation, where $\alpha, \alpha_1, \alpha_2\ and\ \alpha_3$ are constants.

Example(5)

Solve the equation

$$\acute{\acute{y}} - 3\acute{y} + 2y = 4e^{3t} \ (25)$$

Without subjected to initial conditions, taking Novel transform to both sides and apply the formula (19), where n=2, we have:

$$N_I(y) = \frac{\mathrm{z(s)}}{s[s^2-3s+2]H(s)},$$

where z(s) and $H(s)$ are polynomial of s,

After simplification, and using partial fractions, we get:

$N_I(y) = \frac{A}{s(s-3)} + \frac{B}{s(s-2)} + \frac{C}{s(s-1)}$; such that A,B and C are constants,

take inverse Novel to transform on both sides, we get:

$$y(t) = Ae^{3t} + Be^{2t} + C\mathrm{e}^{\mathrm{t}} \ (26)$$

The above equation has three arbitrary constants, but the general solution of equation (25) must contain only two constants. To eliminate the extra constants, we derivative the solution (26) and substitute in equation (25), we find the value of A=2, therefore, the general solution of (25) is:

$$y(t) = 2e^{3t} + Be^{2t} + Ce^t$$

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