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Radio Quotient Square Sum Labeling of a Graph

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Abstract--- *A radio quotient square sum labeling is a one to one mapping p from V (G) to N satisfying the condition d (u, v) +* $\lceil \frac{[p(u)]^2 + [p(v)]^2}{(p(u)+p(v))} \rceil$ *≥ 1 + dia (G) for all u, v ∈ V (G). The radio quotient square sum number of G, rqssn(G), is the maximum number assigned to any vertex of G. The radio quotient square sum number of G, rqssn (G) is the minimum value of rqssn (p) taken over all radio quotient square sum labeling p of G. In this paper we find the radio quotient square sum number of graphs with diameter three, gear graph, S(*$K_{m,n}$ *) and (*$W_m \Theta \overline{K_2}$*).*

Keywords--- *Radio Quotient Square Sum Labeling, Radio Quotient Square Sum Number, Gear Praph.*

AMS Subject Classification (2010)--- *05C78, 05C15*

I. INTRODUCTION

Throughout this paper we consider finite, simple, undirected and connected graphs. Let V (G) and E (G) respectively denote the vertex set and edge set of G. In 2001, Chartrand et al. [1] defined the concept of radio labeling of P. Radio labeling of graphs is inspired by restrictions inherent in assigning channel frequencies for radio transmitters. In [3] Selvarajan and Swapna Raveendran, introduced the notion of Quotient Square Sum Cordial Labeling and studied Quotient Square Sum Cordial Labeling of some standard graphs.. Ponraj et al. [2] introduced the notion of radio mean labeling of graphs and investigated radio mean number of some graphs. Now, we define radio Quotient square sum labeling. The symbol ⌈x⌉ stands for smallest integer greater than or equal to x.

Definition 1.1.[3]

Let G = (V, E) be a simple graph and p : V → {1, 2, ... | V |} be a bijection,

For each edge uv assigned the label 1 if $\lceil \frac{[p(u)]^2 + [p(v)]^2}{(p(u)+p(v))} \rceil$ is odd and 0 if it is even. f is called quotient square sum cordial labeling if $| e_p(0) - e_p(1)| \leq 1$,where $e_p(0)$ and $e_p(1)$ denote the number of edges labeled with 0 and labeled with 1 respectively. A graph which admits a quotient square sum cordial labeling is called quotient square sum cordial graph.

Definition 1.2 [1]

A Radio labeling of the graph *G* is a function p from the vertex set V (G) to Z^+ such that $|p(u) - p(v)| + d_G(u, v) \geq$ diam (G)+1 where diam (G) and d(u, v) are diameter and distance between u and v in graph G respectively. The radio number rn(G) of G is the smallest number *k* such that *G* has radio labeling with max { p(v) : v ∈ V (G)} = *k*.

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Definition 1.3

A radio quotient square sum labeling is a one to one mapping p from V (G) to N satisfying the condition d (u, v) + $\lceil \frac{[p(u)]^2 + [p(v)]^2}{(p(u) + p(v))} \rceil \geq 1$ + dia (G) for every u, v ∈ V (G). The span of a labeling p is the maximum integer that p maps to a vertex of G. The radio quotient square sum number of G, rqssn (G) is the lowest span taken over all radio quotient square sum labeling of the graph G.

Theorem 2.1

The radio quotient square sum number of the gear graph is $(2m + 1)$.

Proof

Let W_m =C_m + K_1 where C_m is the cycle and V (K_1) = {v}.

Let V (G_m) = V (W_m) ∪ $\{u_i;\ 1 \leq\ i\ \leq\ m\}$. Also dia ($G_3$) =3, now we have to prove that,

d (u, v) + $\lceil \frac{[p(u)]^2 + [p(v)]^2}{(p(u) + p(v))} \rceil \geq 4$ ------------------ (2.3)

Label the central vertex v with 1 and label the vertices of degree two on the rim by 2,3and 4, the remaining vertices of degree 3 by 5,6 and 7.

$\lceil \frac{[p(u)]^2 + [p(v)]^2}{(p(u) + p(v))} \rceil \geq 2$ for every pair of vertices u and v, $u \neq v$.

Thus we obtain the inequality d (u, v) + $\lceil \frac{[p(u)]^2 + [p(v)]^2}{(p(u) + p(v))} \rceil \geq 4$

Now dia (G_m) = 4 for $m\ \geq 4$, we have to prove that,

d (u, v) + $\lceil \frac{[p(u)]^2 + [p(v)]^2}{(p(u) + p(v))} \rceil \geq 5$ ------------------ (2.3a)

Label central vertex v with $(2m + 1)$, label the vertices of degree two on the rim by $(m + 1)$, $(m + 2)$, $(m + 3)$, ... $2m$ and label any three vertices of degree 3 and distance between them is 4 on the rim by 1,2 and 3. Label the remaining vertices by 4,5,…m.

If either p(u) ≥ 4 or p(v) ≥ 4, then $\lceil \frac{[p(u)]^2 + [p(v)]^2}{(p(u) + p(v))} \rceil \geq 4$ and hence (2.3a) trivially holds.

Let 1 ≤ p (u), p(v) ≤ 3, distance between them is 4. Hence (2.3a) holds.

Theorem 2.2

The radio quotient square sum number of the subdivision of complete bipartite graph is $(m + n\ + mn)$.

Proof

Complete bipartite graph $K_{m,n}$ contains $(m + n)$ vertices , (mn) edges and

$E(K_{m,n})$={ $u_i\ v_j;\ 1 \leq i \leq m$,$1 \leq j \leq\ n\}$. Subdivision of complete bipartite graph is denoted by S($K_{m,n}$) and contains $(m + n\ + mn)$ vertices and 2(mn) edges. Let the subdivided vertices of S(($K_{m,n}$) be $\{w_i$;$1 \leq i \leq mn\}$.

dia(S ($K_{m,n}$))= $\begin{cases} 2, if\ m = n = 1 \\ 4, otherwise \end{cases}$

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For $m = n = 1$, we have to prove that d(u, v) + $\lceil \frac{[p(u)]^2 + [p(v)]^2}{(p(u) + p(v))} \rceil \geq 3$ ---------------- (2.4a).

Label the vertex of degree two by 3 and remaining vertices by 1 and 2. Clearly $\lceil \frac{[p(u)]^2 + [p(v)]^2}{(p(u) + p(v))} \rceil \geq 2$ for every pair of vertices u and v , u≠ v. The distance between every pair of vertices is at least 1, therefore d(u, v) + $\lceil \frac{[p(u)]^2 + [p(v)]^2}{(p(u) + p(v))} \rceil \geq 3$. Thus (2.5a) holds.

Define a function p : V(S ($K_{m,n}$)) → N by $p(v_i) = i$, $1 \leq i \leq$ m,

$p(u_j)$ = ($m + j$), $1 \leq j \leq$ n and $p(w_k)$ = ($m + n + k$), $1 \leq k \leq$ mn.

dia(S ($K_{m,n}$)) = 4, we have to prove that

d(u, v) + $\lceil \frac{[p(u)]^2 + [p(v)]^2}{p(u) + p(v))} \rceil \geq 5$ for all $u, v \in V(S\ (K_{m,n}))$ ------------ (2.4b)

Case 1: For the pair (v_i, w_k) *and* $d(v_i w_k) = 1$, $1 \leq i \leq$ m and $1 \leq k \leq$ mn.

$d(v_i, w_k) + \lceil \frac{[p(v_i)]^2 + [p(w_k)]^2}{(p(v_i) + p(w_k))} \rceil \geq 1 + 4$ = 1 + diam (S ($K_{m,n}$)) .

Case 2: For the pair (u_i, w_k) and $d(u_i w_k) = 1$, $1 \leq i \leq$ n and $1 \leq k \leq$ mn.

$d(u_i, w_k) + \lceil \frac{[p(u_i)]^2 + [p(w_k)]^2}{(p(u_i) + p(w_k))} \rceil \geq 1 + 4 = 5$.

Case 3: For the pair $(w_{k,} w_l)$ *and* $d(w_{k,} w_l) = 2$, $1 \leq k, l \leq$ mn .

$d(w_{k,} w_l) + \lceil \frac{[p(w_k)]^2 + [p(w_l)]^2}{(p(w_k) + p(w_l))} \rceil \geq 2 + 6 > 5$.

Case 4: For the pair (v_i, u_j), $1 \leq i \leq$ m *and* $1 \leq j \leq$ n.

$d(v_i, u_j) + \lceil \frac{[p(v_i)]^2 + [p(u_j)]^2}{(p(v_i) + p(u_j))} \rceil \geq 2 + \lceil \frac{i^2 + (m+j)^2}{m+i+j} \rceil \geq 2 + 3 = 5$.

Case5: For the pair (u_i, w_k) *and* $d(u_i w_k) = 3$, $1 \leq i \leq$ n and $1 \leq k \leq$ mn.

$d(u_i, w_k) + \lceil \frac{[p(u_i)]^2 + [p(w_k)]^2}{(p(u_i) + p(w_k))} \rceil \geq 3 + 6 > 5$.

Case 6: For the pair (v_i, w_k) and $d(u_i w_k) = 3$, $1 \leq i \leq$ m and $1 \leq k \leq$ mn.

$d(v_i, w_k) + \lceil \frac{[p(v_i)]^2 + [p(w_k)]^2}{(p(v_i) + p(w_k))} \rceil \geq 3 + 6 > 5$.

Case 7: For the pair (v_i, v_j), $i \neq j$ *and* $1 \leq i, j \leq$ m.

$d(v_i, v_j) + \lceil \frac{[p(v_i)]^2 + [p(v_j)]^2}{(p(v_i) + p(v_j))} \rceil \geq 4 + \lceil \frac{i^2 + j^2}{i+j} \rceil \geq 5$.

Case 8: For the pair (u_i, u_j), $i \neq j$ *and* $1 \leq i, j \leq$ n.

$d(u_i, u_j) + \lceil \frac{[p(u_i)]^2 + p(u_j)]^2}{(p(u_i) + p(u_j))} \rceil \geq 4 + \lceil \frac{(m+i)^2 + (m+j)^2}{2m+i+j} \rceil \geq 5$.

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Case9: For the pair $(w_{k,}w_l)$ and $d(w_{k,}w_l) = 4$, $1 \le k, l \le$ mn .

$d(w_{k,}w_l) + \lceil \frac{[p(w_{k,})]^2 + [p(w_{l,})]^2}{(p(w_{k,}) + p(w_{l,}))} \rceil \ge 4 + 6 > 5.$

Hence (2.5b) holds.

Theorem 2.3

The radio quotient square sum number of $(W_m \Theta \overline{K_2}) = (3m + 1)$, m ≥ 3 .

Proof.

Let u, $v_1 v_2 \dots v_m$ be the vertices of the wheel path W_m of n vertices.

let x_i and y_i be the vertices of $\overline{K_2}$, which are joined to the vertex v_i of the wheel W_m, $1 \le i \le$ m. The resultant graph is $(W_m \Theta \overline{K_2})$ .The graph $(W_m \Theta \overline{K_2})$ contains $4m$ edges and $(3m + 1)$ vertices. Dia $(W_m \Theta \overline{K_2})$ = 4.

Define a function p : $V((W_m \Theta \overline{K_2})) \to N$ by $p(v_i) = (2m + i)$, $1 \le i \le$ m,

$p(x_i) = i$, $1 \le i \le$ m, $p(y_i) = (m + i)$, $1 \le i \le$ m and $p(u) = (3m + 1)$.

Now it is enough to prove that

$d(u, v) + \lceil \frac{[p(u)]^2 + [p(v)]^2}{(p(u) + p(v))} \rceil \ge 5.$ ------------------------------(2.5)

Case 1: For the pair (v_i, v_j), $i \ne j$ and $1 \le i, j \le$ m.

$d(v_i, v_j) + \lceil \frac{[p(v_i)]^2 + [p(v_j)]^2}{(p(v_i) + p(v_j))} \rceil \ge 1 + \lceil \frac{(2m+i)^2 + (2m+j)^2}{4m+i+j} \rceil$

$\ge 2m + 2 \ge 5.$

$= 1 + \text{diam } (W_m \Theta \overline{K_2}).$

Case 2: For the pair $(x_{i,}x_j)$, $i \ne j$ and $1 \le i, j \le$ m.

$d(x_i, x_j) + \lceil \frac{[p(x_i)]^2 + p(x_j)]^2}{(p(x_i) + p(x_j))} \rceil \ge 3 + \lceil \frac{i^2 + j^2}{i+j} \rceil$

$\ge 3 + 2 = 5.$

Case 3: For the pair $(y_{i,}y_j)$, $i \ne j$ and $1 \le i, j \le$ m.

$d(y_i, y_j) + \lceil \frac{[p(y_i)]^2 + [p(y_j)]^2}{(p(y_i) + p(y_j))} \rceil \ge 3 + \lceil \frac{(m+i)^2 + (m+j)^2}{2m+i+j} \rceil$

$\ge 3 + m \ge 5$

Case 4: For the pair (v_i, x_j), $i \ne j$ and $1 \le i, j \le$ m.

$d(v_i, x_j) + \lceil \frac{[p(v_i)]^2 + [p(x_j)]^2}{(p(v_i) + p(x_j))} \rceil \ge 1 + \lceil \frac{(2m+i)^2 + j^2}{2m+i+j} \rceil$

$\ge 2m + 1 \ge 5.$

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Case 5: For the pair (v_i, y_j), $i \neq j$ and $1 \leq i, j \leq$ m.

$$d(v_i, y_j) + \lceil \frac{[p(v_i)]^2 + [p(y_j)]^2}{(p(v_i) + p(y_j))} \rceil \geq 1 + \lceil \frac{(2m+i)^2 + (m+j)^2}{3m+i+j} \rceil$$

$$\geq \left(\frac{5}{3}m\right) + 1 \geq 5.$$

Case 6: For the pair (x_i, y_j), $i \neq j$ and $1 \leq i, j \leq$ m.

$$d(x_i, y_j) + \lceil \frac{[p(x_i)]^2 + [p(y_j)]^2}{(p(x_i) + p(y_j))} \rceil \geq 2 + \lceil \frac{i^2 + (m+j)^2}{m+i+j} \rceil$$

$$\geq m + 2 \geq 5.$$

Case 7: For the pair (u, v_j), $1 \leq j \leq$ m.

$$d(u, v_j) + \lceil \frac{[p(u)]^2 + [p(v_j)]^2}{(p(u) + p(v_j))} \rceil \geq 1 + \lceil \frac{(3m+1)^2 + (2m+j)^2}{5m+1+j} \rceil$$

$$\geq \left(\frac{13}{5}m\right) + 1 \geq 5.$$

Case 8: For the pair (u, x_j), $1 \leq j \leq$ m.

$$d(u, x_j) + \lceil \frac{[p(u)]^2 + [p(x_j)]^2}{(p(u) + p(x_j))} \rceil \geq 2 + \lceil \frac{(3m+1)^2 + j^2}{3m+1+j} \rceil$$

$$\geq 3m + 2 \geq 5.$$

Case 9: For the pair (u, y_j), $1 \leq j \leq$ m.

$$d(u, y_j) + \lceil \frac{[p(u)]^2 + [p(y_j)]^2}{(p(u) + p(y_j))} \rceil \geq 2 + \lceil \frac{(3m+1)^2 + (m+j)^2}{4m+1+j} \rceil$$

$$\geq \left(\frac{10}{4}m\right) + 1 \geq 5$$

Hence (2.5) holds.

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