

Hall effects on flow of a Prandtl fluid through a porous medium in a planar channel with peristalsis

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ABSTRACT— in this paper, the effect of Hall on the flow of Prandtl fluid through a porous medium in a planar channel under the assumption of long wavelength is investigated. A Closed form solutions are obtained for axial velocity and pressure gradient. The effects of various emerging parameters on the pressure gradient, time averaged volume flow rate and frictional force are discussed with the aid of graphs.

Keywords-- Hall, Newtonian fluid, Hartmann number, long wavelength, peristaltic pumping, Darcy number, Prandtl fluid, porous medium.

I INTRODUCTION

Many researchers considered the fluid to behave like a Newtonian fluid for physiological peristalsis including the flow of blood in arterioles. But such a model cannot be suitable for blood flow unless the non – Newtonian nature of the fluid is included in it. Peristaltic transport of non-Newtonian fluids in a tube was first studied by Raju and Devanathan (1972), by considering the blood as a power-law fluid. Few interesting studies dealing with the peristaltic flows of non-Newtonian fluids are given in (Bohme and Friedrich, 1983; Siddiqui et al., 1991; Subba Reddy et al., 2011). Recently, Akbar et al. (2012) have discussed the peristaltic flow of a Prandtl fluid in an asymmetric channel. Peristaltic flow of a Prandtl fluid in a symmetric channel under the effect of a magnetic field was investigated by Jyothi et al. (2012).

The basic perception regarding MHD is the magnetic field which induces the currents in conductive moving fluids which in results generates the forces on the fluid and also varies the magnetic field itself. It is well known that when any conductor comes into a magnetic field which in results creates a voltage, which is perpendicular to the current and field, this effect is known as Hall Effect. Hayat et al. (2007) have investigated the Hall effects on peristaltic flow of a Maxwell fluid in a porous medium. Effects of Hall and ion-slip currents on peristaltic transport of a couple stress fluid was analyzed by Abo-Eldahab et al. (2010).

Gad (2014) has studied the effects of Hall current on peristaltic transport with compliant walls. Eldabe (2015) have studied the Hall Effect on peristaltic flow of third order fluid in a porous medium with heat and mass transfer. Effect of hall and ion slip on peristaltic blood flow of Eyring Powell fluid in a non-uniform porous channel was studied by Bhatti et al. (2016). Shalini and Rajasekhar have investigated the effect of hall on peristaltic flow of a Newtonian fluid through a porous medium in a two-dimensional channel.

In view of these, we studied the effect of Hall on the peristaltic transport of a Prandtl fluid through a porous medium in a two-dimensional channel under the assumptions of long wavelength and low Reynolds number.

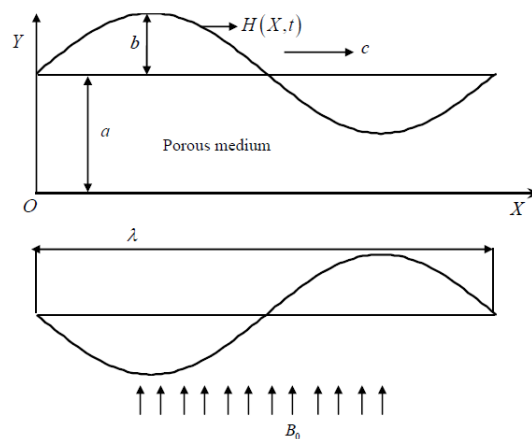
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Series solutions of axial velocity and pressure gradient are given by using regular perturbation technique when Prandtl number is small. The effects of various emerging parameters on the pressure gradient, pumping characteristics are studied in detail with the help of graphs.

II MATHEMATICAL FORMULATION

We consider the peristaltic transport of a conducting Prandtl fluid through a porous medium in a two dimensional channel of width $2a$ with heat transfer. The walls of the channel are flexible. A uniform magnetic field B_0 is applied in the transverse direction to the flow. The fluid is taken to be of small electrical conductivity, so that the magnetic Reynolds number is small and the induced magnetic field is neglected in comparison with the applied magnetic field. The flow is induced by periodic peristaltic wave of length λ and amplitude b with constant speed c along the channel walls.



The physical model of the channel is shown in Fig. 1.

Figure 1: The physical model

The equation of the wall is given by

$$Y = \pm H(X,t) = \pm a \pm b \sin \frac{2\pi}{\lambda} (X - ct) \quad (2.1)$$

Where t is the time, λ is the wavelength and (X,Y) are the Cartesian co-ordinates in laboratory frame of reference.

We introduce a wave frame of reference (x, y) moving with velocity C in which the motion becomes independent of time when the channel length is an integral multiple of the wavelength and the pressure difference at the ends of the channel is a constant (Shapiro et al., 1969). The transformation from the fixed frame of reference (X, Y) to the wave frame of reference (x, y) is given by,

$$x = X - c t, y = Y, u = U - c, v = V \text{ and}$$

$$p(x) = P(X, t), \quad (2.2)$$

where (u, v) and (U, V) are the velocity components, p and P are pressures in the wave and fixed frames of reference, respectively.

The Constitutive equations for Prandtl fluid is given by (Patel and Timaol, 2010)

$$\tau = \frac{A \sin^{-1} \left(\frac{1}{C} \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial x} \right)^2 \right]^{\frac{1}{2}} \right)}{\left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial x} \right)^2 \right]^{\frac{1}{2}}} \frac{\partial u}{\partial y} \quad (2.3)$$

in which A and C are material constants of Prandtl fluid model

The equations governing the flow in wave frame of reference are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.4)$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\sigma B_0^2}{1+m^2} \quad (2.5)$$

$$(mv - (u+c)) - \frac{\mu}{k}(u+c)$$

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} - \frac{\sigma B_0^2}{1+m^2} \quad (2.6)$$

$$(m(u+c) + v) - \frac{\mu}{k}v$$

where ρ is the density, T is the temperature, α is the coefficient of linear thermal expansion of the fluid, k is thermal conductivity, ζ is the specific heat at constant volume, m is the Hall parameter, μ is the viscosity of the fluid and σ is the electrical conductivity.

The dimensional boundary conditions are

$$u = -c \quad \text{at} \quad y = H \quad (2.7)$$

$$\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = 0 \quad (2.8)$$

Introducing the following non-dimensional variables

$$\bar{x} = \frac{x}{\lambda}, \quad \bar{y} = \frac{y}{a}, \quad \bar{u} = \frac{u}{c}, \quad \bar{v} = \frac{v}{c\delta}, \quad \bar{p} = \frac{pa^2}{\mu c \lambda}, \quad \bar{t} = \frac{ct}{\lambda},$$

$$h = \frac{H}{a}, \quad \bar{\tau} = \frac{a\tau}{\mu c}, \quad \phi = \frac{b}{a}, \quad \delta = \frac{a}{\lambda}, \quad \theta = \frac{T-T_0}{T_1-T_0}, \quad \text{Pr} = \frac{\rho v \zeta}{k}, \quad \text{Ec} = \frac{c^2}{\zeta(T_1-T_0)}, \quad \text{Da} = \frac{k}{a^2}$$

where μ is the constant viscosity, in the Eqs. (2.4) – (2.6), we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.9)$$

$$\text{Re} \delta \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \delta \frac{\partial \tau_{xx}}{\partial x} +$$

$$\frac{M^2}{1+m^2} (m\delta v - (u+1)) - \frac{1}{\text{Da}} (u+1) \quad (2.10)$$

$$\text{Re} \delta^3 \left(u \frac{\partial v}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial y} + \delta^2 \frac{\partial \tau_{xy}}{\partial x} + \delta \frac{\partial \tau_{yy}}{\partial y} - \frac{\delta M^2}{1+m^2} (m(u+1) + \delta v) - \frac{\delta^2}{Da} v \quad (2.11)$$

where Da is the Darcy number, $M = aB_0 \sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number, Re is the Reynolds number and δ is the wave number.

Under the assumptions of long wave length ($\delta \ll 1$) and low Reynolds number ($\text{Re} \rightarrow 0$), the Equations

(2.10) - (2.11) become

$$\frac{\partial p}{\partial x} = \frac{\partial \tau_{xy}}{\partial y} - \left(\frac{M^2}{1+m^2} + \frac{1}{Da} \right) (u+1) \quad (2.12) \quad \frac{\partial p}{\partial y} = 0$$

(2.13)

$$\text{here } \tau_{xy} = \alpha \frac{\partial u}{\partial y} + \frac{\beta}{6} \left(\frac{\partial u}{\partial y} \right)^3, \quad \alpha = \frac{A}{\mu C} \quad \text{and} \quad \beta = \frac{Ac^2}{a^2 C^3}.$$

The corresponding boundary conditions in wave frame of reference are given by

$$u = -1 \quad \text{at} \quad y = h = 1 + \phi \cos 2\pi x, \quad (2.14)$$

$$\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = 0. \quad (2.15)$$

Equations (2.12), (2.13) indicate that p is independent of y . Therefore Eq. (2.12) can be rewritten as

$$\frac{dp}{dx} = \alpha \frac{\partial^2 u}{\partial y^2} + \frac{\beta}{6} \frac{\partial}{\partial y} \left[\left(\frac{\partial u}{\partial y} \right)^3 \right] - \left(\frac{M^2}{1+m^2} + \frac{1}{Da} \right) (u+1) \quad (2.16)$$

The volume flow rate q in a wave frame of reference is given by

$$q = \int_0^h u dy. \quad (2.17)$$

The instantaneous flux $Q(X, t)$ in the laboratory frame is

$$Q(X, t) = \int_0^h u dy = \int_0^h (u+1) dy = q + h. \quad (2.18)$$

The time average flux over one period $T \left(= \frac{\lambda}{c} \right)$ of the peristaltic wave is

$$\bar{Q} = \frac{1}{T} \int_0^T Q dt = \int_0^1 (q+h) dx = q + 1. \quad (2.19)$$

III SOLUTION

The Eq. (2.16) is non-linear and its closed form solution is not possible. Hence, we linearize this equation in terms of $\alpha (\ll 1)$. So we expand u, p and q as

$$u = u_0 + \beta u_1 + O(\beta^2)$$

$$\begin{aligned} p &= p_0 + \beta p_1 + O(\beta^2) \\ q &= q_0 + \beta q_1 + O(\beta^2) \end{aligned} \quad (3.1)$$

Substituting (3.1) in the Equation (2.16) and in the boundary conditions (2.14) - (2.15) and equating the coefficients of like powers of β to zero and neglecting the terms of β^2 and higher order, we get the following equations:

3.1 System of order zero (β^0)

$$\alpha \frac{\partial^2 u_o}{\partial y^2} - M_1^2 u_o = \frac{dp_0}{dx} + M_1^2 \quad (3.2)$$

$$\text{here } M_1^2 = \frac{M^2}{1+m^2} + \frac{1}{Da}$$

with the corresponding boundary conditions are

$$u_o = -1 \quad \text{at } y = h = 1 + \phi \cos 2\pi x, \quad (3.3)$$

$$\frac{\partial u_o}{\partial y} = 0 \quad \text{at } y = 0. \quad (3.4)$$

3.2 System of order one (β)

$$\alpha \frac{\partial^2 u_1}{\partial y^2} - M_1^2 u_1 = \frac{dp_1}{dx} - \frac{1}{6} \frac{\partial}{\partial y} \left(\frac{\partial u_o}{\partial y} \right)^3 \quad (3.5)$$

with the corresponding boundary conditions are

$$u_1 = 0 \quad \text{at } y = h = 1 + \phi \cos 2\pi x \quad (3.6)$$

$$\frac{\partial u_1}{\partial y} = 0 \quad \text{at } y = 0. \quad (3.7)$$

3.3 Solution of order zero (β^0)

Solving Eq. (3.2) together with the boundary conditions (3.4) and (3.5), we get

$$u_o = \frac{1}{M_1^2} \frac{dp_o}{dx} \left[\frac{\cosh \Omega y}{\cosh \Omega h} - 1 \right] - 1 \quad (3.8)$$

Here $\Omega = M_1 / \sqrt{\alpha}$.

The volume flow rate q_0 in the moving coordinate system is given by

$$q_0 = \int_0^h u_o dy = \frac{1}{M_1^2} \frac{dp_o}{dx} \left[\frac{\sinh \Omega h}{\Omega \cosh \Omega h} - h \right] - h \quad (3.9)$$

From Eq. (3.10), we have

$$\frac{dp_o}{dx} = \frac{M_1^2 (q_0 + h) \Omega \cosh \Omega h}{(\sinh \Omega h - \Omega h \cosh \Omega h)} \quad (3.10)$$

3.4 Solution of order one (β)

Solving the Equation (3.6) by using the Equation (3.9) and the boundary conditions (3.7) and (3.8) to get

$$u_1 = \frac{1}{M_1^2} \frac{dp_1}{dx} \left[\frac{\cosh \Omega y}{\cosh \Omega h} - 1 \right] + \frac{\Omega A_1^3}{8M_1^6} \left(\frac{dp_0}{dx} \right)^3 \quad (3.11)$$

$$\left[\frac{y \sinh \Omega y}{2\alpha\Omega} - \frac{\cosh 3\Omega y}{8M_1^2} - A_2 \cosh \Omega y \right]$$

Where $A_1 = \frac{\Omega}{\cosh \Omega h}$,

$$A_2 = \left(\frac{h \sinh \Omega h}{2\alpha\Omega} - \frac{\cosh 3\Omega h}{8M_1^2} \right) \frac{1}{\cosh \Omega h},$$

and the volume flow rate q_1 is given by

$$q_1 = \int_0^h u_1 dy = \frac{1}{M_1^2 \Omega \cosh \Omega h} \frac{dp_1}{dx} \quad (3.12)$$

$$\left[\sin h\Omega h - \Omega h \cosh \Omega h \right] + A_3 \left(\frac{dp_0}{dx} \right)^3$$

where $A_3 = \frac{\Omega A_1^3}{8M_1^6} \left[\frac{h \cosh \Omega h}{2\alpha\Omega^2} - \frac{\sinh \Omega h}{2\alpha\Omega^3} - \frac{\sinh 3\Omega h}{24\Omega M_1^2} - \frac{A_2 \sinh \Omega h}{\Omega} \right]$

From Eq. (3.13), we have

$$\frac{dp_1}{dx} = \frac{M_1^2 \Omega \cosh \Omega h \left[q_1 - A_3 \left(\frac{dp_0}{dx} \right)^3 \right]}{\sinh \Omega h - \Omega h \cosh \Omega h} \quad (3.13)$$

Substituting Equations (3.11) and (3.13) into the second Equation of (3.1) and using the relation $\frac{dp_0}{dx} = \frac{dp}{dx} - \beta \frac{dp_1}{dx}$ and neglecting terms greater than $O(\beta)$, we get

$$\frac{dp}{dx} = \frac{\Omega M_1^2 \cosh \Omega h}{\left[\sinh \Omega h - \Omega h \cosh \Omega h \right]} \quad (3.14)$$

$$\left[q + h - \beta A_3 \left\{ \frac{M_1^2 (q + h) \Omega \cosh \Omega h}{\sinh \Omega h - \Omega h \cosh \Omega h} \right\}^3 \right]$$

The dimensionless pressure rise per one wavelength in the wave frame is defined as

$$\Delta p = \int_0^1 \frac{dp}{dx} dx \quad (3.15)$$

Note that, as $Da \rightarrow \infty$ our results coincide with results of Subba Narasimhudu (2017).

IV DISCUSSIONS AND RESULTS

Fig. 2 shows the variation of axial pressure gradient $\frac{dp}{dx}$ with Prandtl fluid parameter β for $\alpha = 1.2, m = 0.2$,

$Da = 0.1, M = 1, \phi = 0.5$ and $\bar{Q} = -1$. It is observed that, the axial pressure gradient $\frac{dp}{dx}$ increases on increasing β .

The variation of axial pressure gradient $\frac{dp}{dx}$ with Prandtl fluid parameter α for $m = 0.2, Da = 0.1, M = 1, \beta = 0.01, \phi = 0.5$ and $\bar{Q} = -1$ is shown in Fig. 3. It is noted that, the axial pressure gradient $\frac{dp}{dx}$ increases with an increase in α .

Fig. 4 depicts the variation of axial pressure gradient $\frac{dp}{dx}$ with m for $\alpha = 1.2, Da = 0.1, M = 1, \beta = 0.01, \phi = 0.5$ and $\bar{Q} = -1$. It is found that, the axial pressure gradient $\frac{dp}{dx}$ decreases with increasing m .

The variation of axial pressure gradient $\frac{dp}{dx}$ with Hartmann number M for $\alpha = 1.2, m = 0.2, Da = 0.1, \beta = 0.01, \phi = 0.5$ and $\bar{Q} = -1$ is depicted in Fig. 5. It is observed that, the axial pressure gradient $\frac{dp}{dx}$ increases with an increase in M .

Fig. 6 illustrates the variation of axial pressure gradient $\frac{dp}{dx}$ with Darcy number Da for $\alpha = 1.2, m = 0.2, M = 1, \beta = 0.01, \phi = 0.5$ and $\bar{Q} = -1$. It is found that, the axial pressure gradient $\frac{dp}{dx}$ decreases with increasing Da .

The variation of axial pressure gradient $\frac{dp}{dx}$ with ϕ for $\alpha = 1.2, m = 0.2, Da = 0.1, \beta = 0.01, M = 1$ and $\bar{Q} = -1$ is shown in Fig. 7. It is observed that, the axial pressure gradient $\frac{dp}{dx}$ increases by increasing ϕ .

Fig. 8 shows the variation of pressure rise Δp with time averaged flux \bar{Q} for different values of β with $\alpha = 1.2, m = 0.2, Da = 0.1, M = 1$ and $\phi = 0.5$. It is observed that, the time averaged flux \bar{Q} increases with increasing β in the pumping region ($\Delta p > 0$), while it decreases with increasing β in both the free-pumping ($\Delta p = 0$) and co-pumping ($\Delta p < 0$) regions. Further, it is observed that, the pumping is more for Prandtl fluid than that of Newtonian fluid ($\alpha = 1, \beta = 0$).

The variation of pressure rise Δp with time averaged flux \bar{Q} for different values of α with $\beta = 0.01, m = 0.3, Da = 0.1, M = 1$ and $\phi = 0.5$ is depicted in fig 9. It is noticed that, the time averaged

flux \bar{Q} increases with increasing α in the pumping region, while it decreases with increasing α in both the free-pumping and co-pumping regions.

Fig. 10 depicts the variation of pressure rise Δp with time averaged flux \bar{Q} for different values of m with $\beta = 0.01$, $\alpha = 1.2$, $Da = 0.1$, $M = 1$ and $\phi = 0.5$. It is found that, the time averaged flux \bar{Q} decreases with increasing m in the pumping region and increases in both the free-pumping and co-pumping regions with increasing m .

The variation of the pressure rise Δp with \bar{Q} for different values of M with $\beta = 0.1$, $m = 0.2$, $Da = 0.1$, $M = 1$ and $\phi = 0.5$ is depicted in Fig. 11. It is observed that, in the pumping region, the \bar{Q} increases with increasing M , while it decreases in both the free-pumping and co-pumping regions with increasing M .

Fig. 12 illustrates the variation of pressure rise Δp with time averaged flux \bar{Q} for different values of Da with $\beta = 0.1$, $m = 0.2$, $Da = 0.1$, $M = 1$ and $\alpha = 1.5$. It is noted that, the time averaged flux \bar{Q} decreases with increasing Da in the pumping region, while it decreases with increasing Da in both the free pumping and the co-pumping regions.

The variation of pressure rise Δp with time averaged flux \bar{Q} for different values of ϕ with $\beta = 0.1$, $m = 0.2$, $Da = 0.1$, $M = 1$ and $\alpha = 1.5$ is shown in Fig. 13. It is noted that, the time averaged flux \bar{Q} increases with increasing ϕ in both the pumping and free-pumping regions, while it decreases with increasing ϕ in the co-pumping region for chosen $\Delta p(<0)$.

In this chapter, we studied the effect of Hall on peristaltic flow of a Prandtl fluid through a porous medium in a tow-dimensional channel under the assumptions of long wavelength and low Reynolds number. Series solutions of axial velocity and pressure gradient are given by using regular perturbation technique when Prandtl number is small. It is observed that, the axial pressure gradient increases with increasing β, M, α or ϕ , whereas it decreases with increasing m or Da . In the pumping region, time averaged flux \bar{Q} increases with increasing β, M, α or ϕ , whereas decreases with increasing. Also, it is observed that, the pumping is more for Prandtl fluid than that of Newtonian fluid.

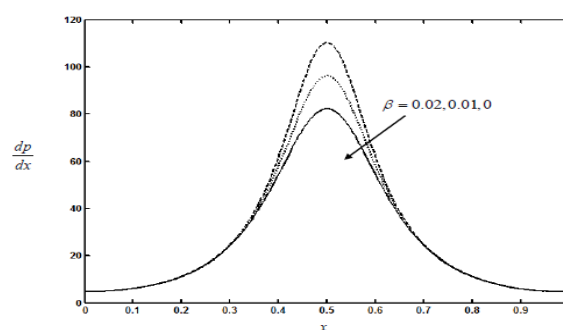


Figure 2: The variation of the axial pressure gradient $\frac{dp}{dx}$ with β for $\alpha = 1.2$, $m = 0.2$, $Da = 0.1$, and $\bar{Q} = -1$.
 $M = 1$, $\phi = 0.5$

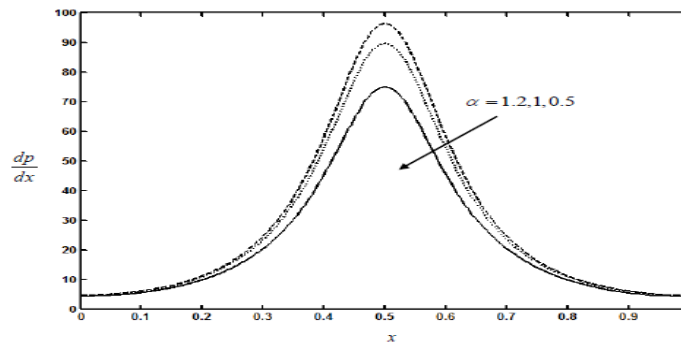


Figure 3: The variation of the axial pressure gradient $\frac{dp}{dx}$ with α for $m = 0.2$, $M = 1$, $\beta = 0.01$, $Da = 0.1$, $\phi = 0.5$ and $\bar{Q} = -1$.

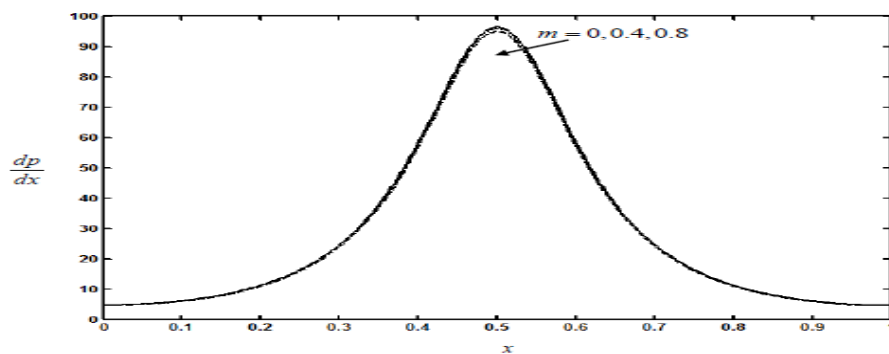


Figure 4: The variation of the axial pressure gradient $\frac{dp}{dx}$ with m for $\alpha = 1.2$, $M = 1$, $\beta = 0.01$, $Da = 0.1$, $\phi = 0.5$ and $\bar{Q} = -1$.

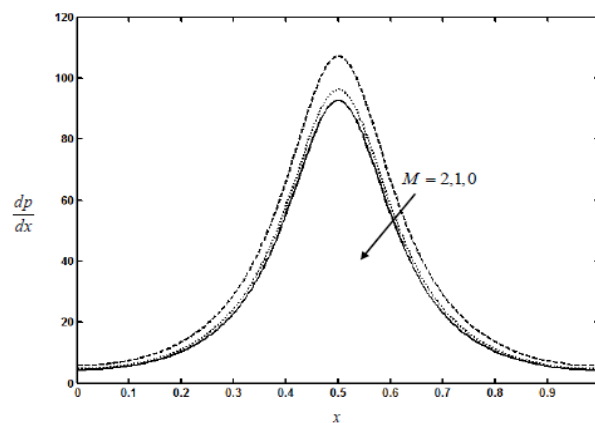


Figure 5: The variation of the axial pressure gradient $\frac{dp}{dx}$ with M for $\alpha = 1.2$, $m = 0.2$, $\beta = 0.01$, $Da = 0.1$, $\phi = 0.5$ and $\bar{Q} = -1$.

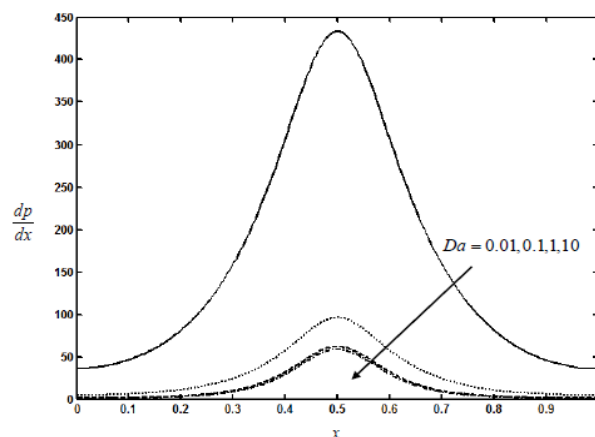


Figure 6: The variation of the axial pressure gradient $\frac{dp}{dx}$ with Da for $\alpha = 1.2$, $m = 0.2$, $\beta = 0.01$, $M = 1$, $\phi = 0.6$ and $\bar{Q} = -1$.

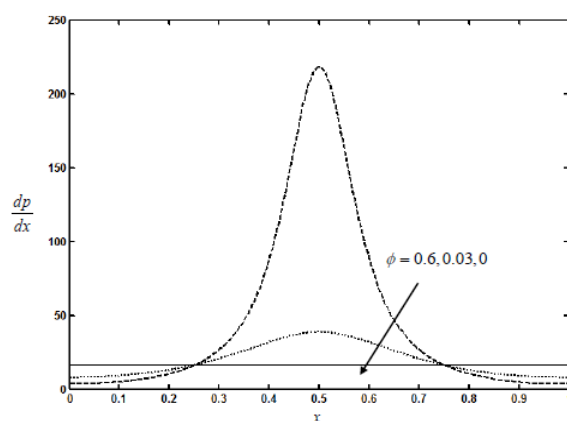


Figure 7: The variation of the axial pressure gradient $\frac{dp}{dx}$ with ϕ for $\alpha = 1.2$, $m = 0.2$, $\beta = 0.01$, $Da = 0.1$, $M = 1$ and $\bar{Q} = -1$.

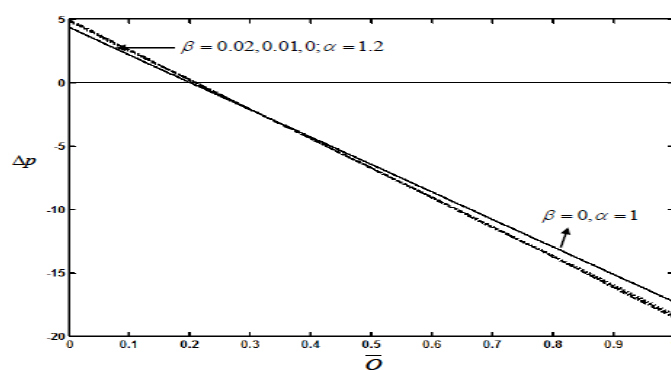


Figure 8: The variation of the pressure rise Δp with \bar{Q} for different values of β with $\alpha = 1.2$, $m = 0.2$, $Da = 0.1$, $M = 1$ and $\phi = 0.5$.

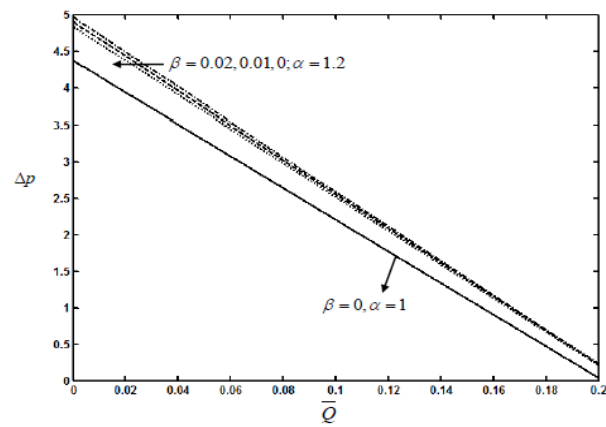


Figure 8(a): Expansion of Fig. 8.

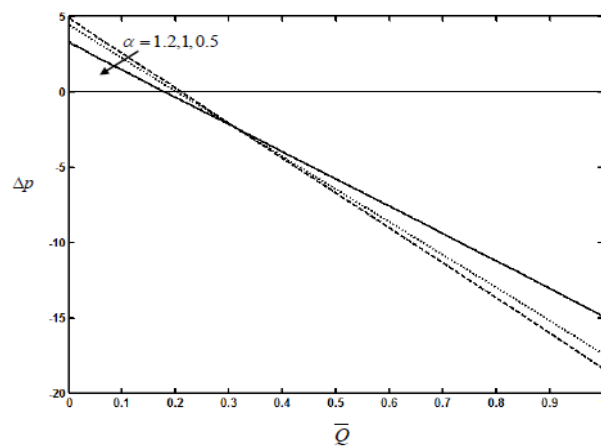


Figure 9: The variation of the pressure rise Δp with \bar{Q} for different values of α with $\beta = 0.01$, $m = 0.2$, $Da = 0.1$, $M = 1$ and $\phi = 0.5$.

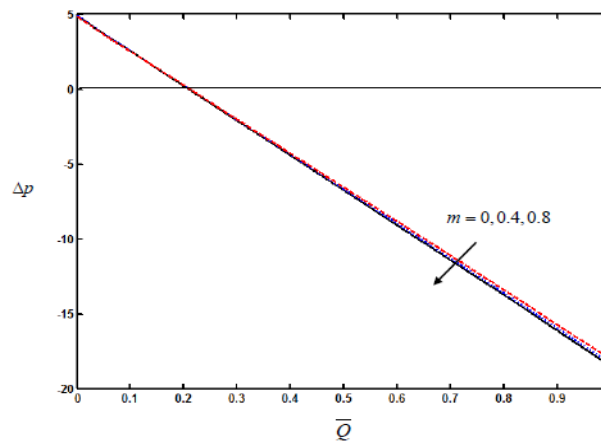


Figure 10: The variation of the pressure rise Δp with \bar{Q} for different values of m with $\beta = 0.01$, $\alpha = 1.2$, $Da = 0.1$, $M = 1$ and $\phi = 0.5$.

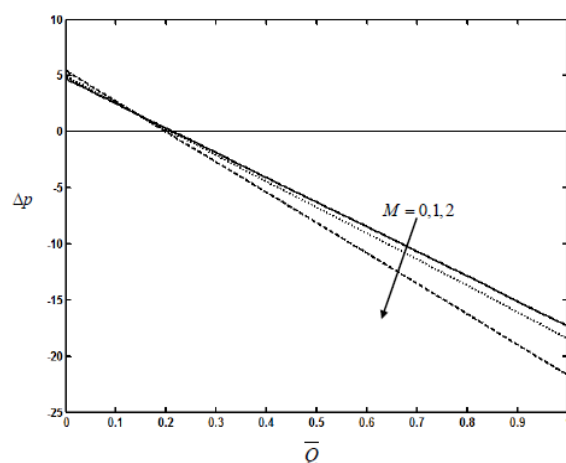


Figure 11: The variation of the pressure rise Δp with \bar{Q} for different values of M with $\beta = 0.01$, $m = 0.2$, $Da = 0.1$, $\alpha = 1.2$ and $\phi = 0.5$

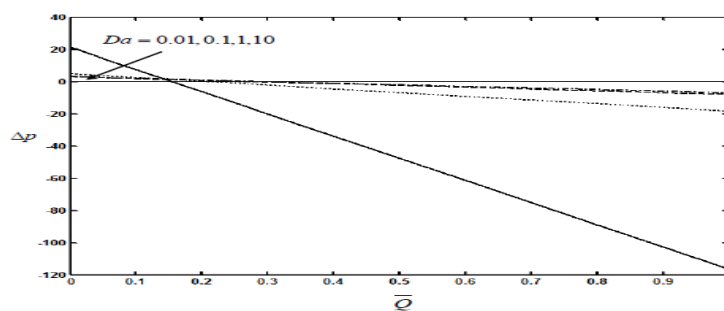


Figure 12: The variation of the pressure rise Δp with \bar{Q} for different values of Da with $\beta = 0.01$, $m = 0.2$, $M = 1$, $\alpha = 1.2$ and $\phi = 0.5$.

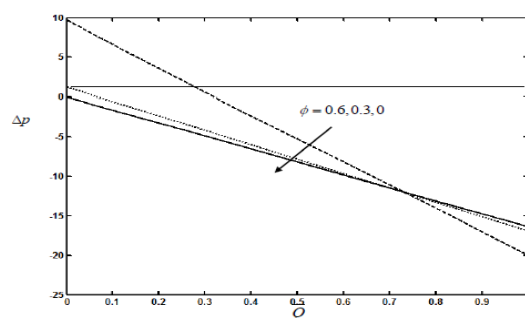


Figure 13: The variation of the pressure rise Δp with \bar{Q} for different values of ϕ with $\beta = 0.01$, $m = 0.2$, $Da = 0.1$, $M = 1$ and $\alpha = 1.2$.

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