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ADAPTIVE ESTIMATION ALGORITHMS FOR THE STATE OF NONLINEAR DYNAMIC SYSTEMS

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Abstract--- *The article discusses the construction of adaptive state estimation algorithms for nonlinear dynamic systems. The tasks of adaptive state estimation, adaptive identification of parameters and constructing an adaptive observer are considered. The conditions of global stability of an adaptive observer are given. The above algorithms for adaptive state estimation, adaptive parameter identification, and the construction of an adaptive observer of a nonlinear dynamic system make it possible to increase the accuracy of estimating the state vector and thereby the quality indicators of control processes.*

Key words--- *nonlinear dynamic system, adaptive state estimation, adaptive identification of parameters, adaptive observer.*

I. Introduction

The solution of many practical problems in a number of technical areas - navigation and radar [1-3], communications and telecommunications [4,5], management of technological facilities [6-8], design of technically optimal systems [9-11], etc., leads to the use of filtration and control systems with linear and nonlinear stochastic models. The theory of such systems is well developed for conditions when all the properties of the models are fully known. If these properties are not known or are subject to sharp, unforeseen changes, adaptive systems can give an acceptable solution [9,11-13], while adaptation includes both the detection [14-16] and the assessment [11,17-21] of changes in models to optimize the system. This task is especially difficult in real conditions of a priori uncertainty and unforeseen variability of the characteristics of models, in the most general case including: intrinsic dynamic properties of an object, characteristics of executive bodies, parameters of external perturbations, laws or modes of functioning of measuring instruments, and interference parameters during measurements. Under these conditions, the introduction of adaptation and control of the functioning of the system is advisable in relation to significant model violations, which cannot be considered as simple interfering factors and the assessment of which will significantly improve the quality of the system as a whole.

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Thus, the reason for the instability of optimal algorithms is their informational uncertainty: we proceed from the fact that all the assumptions underlying the receipt of optimal algorithms are accurate, while in reality they are approximate. Therefore, in the formation of estimation algorithms, it is necessary to take into account the available, albeit of a very general nature, a priori information, and in this way to eliminate a priori uncertainty, and on this basis to synthesize adaptive controllers in the presence of uncertain perturbations. Using adaptive filtering concepts will allow synthesizing effective control algorithms for objects that do not require complete a priori knowledge of the control object and the conditions of its functioning [22-24].

II. Formulation of the problem

Let a nonlinear dynamic system be described as follows:

$$\begin{cases} \dot{z} = f(z, u, p), \\ y = z_1, \end{cases} \tag{1}$$

where $u(t) \in D_u \subseteq R$ – measurable input, possibly limited in subspace D_u of space R, $y(t) \in R$ – measurable output, $z(t) \in R^n$ – state vector, $\rho(t) \in D_p \subseteq R^q$ – vector of unknown limited time-varying parameters, and $f(\cdot)$ is a smooth vector field on a smooth n-dimensional manifold.

Parameters $\rho(t)$ may be possibly unknown functions $z(t)$,and can be interpreted as unknown time-varying parameters. A priori it may be known that the restriction $\rho(t)$ is in subspace D_ρ in space R^q. System structure, i.e. function $f(\cdot)$, known from physical laws or from user observations, i.e. known a priori.

Most common conditions $z_i(t)$ and some unknown parameters $\rho_i(t)$ in (1) have a purely physical meaning. In this case (1) is called a given physical system(GPS)[3, 5, 23].The following tasks are most often encountered:

Problem 1. Non-autonomous estimation of immeasurable states $z_i(t)$ GPSbased on input / output data. This problem is called the adaptive state assessment problem.

Task 2. Non-autonomous estimation of some physical parameters $\rho_i(t)$ GPS based on input / output data. This problem is called the adaptive parameter identification problem.

Task 3. Constructing an adaptive observer for a non-autonomous state assessment in an equivalent state space model. This problem is called the task of constructing an adaptive observer. This task differs from the task alone in that it does not require knowledge of physical values z_i GPS; in this problem, state estimates can be used to construct, for example, a feedback controller.

Problem 2 makes sense only when the GPS is parametrically identifiable, while tasks 1 and 3 require that, in addition, for al $u(t) \in D_u$ and all $\rho(t) \in D_\rho$ GPS would be locally observable.

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III. Decision

The general method for solving these three problems is to supplement state $z(t)$ vector of parameters $\rho(t)$ and to construct an extended Kalman filter [18,23].

We will consider nonlinear systems in the form of (1), which can be transformed using a time-invariant nonlinear smooth transformation [5,12]:

$$\begin{bmatrix} x \\ \theta \end{bmatrix} = T(z, p, c_2, ..., c_n) \tag{2}$$

into the next equivalent form, which we will call the canonical form of the adaptive observer (CFAO):

$$\begin{cases} \dot{x}(t) = Rx(t) + \Omega(\omega(t))\theta(t) + g(t) \\ y(t) = x_1(t). \end{cases} \tag{3}$$

In the expressions (2) and (3):

$x(t) \in R^n$ – state vector of the same size as $z(t)$;

$\theta(t) \in R^m$ - vector of unknown time-varying parameters that can be estimated offline;

$\omega(t) \in R^s$ there is a vector of known functions $u(t)$ and $y(t)$, ie. $\omega(t) = [u(t), y(t), y^2(t), \sin y(t)]$;

$\Omega(\omega(t))$ there is $n \times m$ -matrix, all elements of which have the form $\Omega_{ij}(\omega(t)) = \alpha_{ij}^{\tau}\omega(t)$ for a known constant, possibly equal to zero, and the vectors $\alpha_{ij} \in R^s$;

R- known constant $n \times m$ -shaped matrix:

$$R = \left[\begin{array}{c:c} 0 & k^T \\ \hdashline 0 & \\ \cdot & F(c_2, c_3, \ldots, c_n) \\ \cdot & \\ 0 & \end{array}\right], \; k^T \stackrel{\Delta}{=} [k_2, ..., k_n], \tag{4}$$

where $k_2, ..., k_n$ - known constants, a $F(c_2, ..., c_n)$ – constant $(n-1) \times (n-1)$ -a matrix whose eigenvalues can be freely determined by the correct choice of constant parameters $c_2, ..., c_n$.

In general, $F = \mathrm{diag}(-c_2, ..., -c_n)$ while $c_i > 0$;

$g(t) \in R^n$ -vector of known functions of time;

$T(\cdot) \in R^{n+m}$ - continuous smooth conversion from (z, ρ) on (x, θ), parameterized $n-1$ parameters $c_2, ..., c_n$

For system (2), it is necessary to describe an adaptive observer and obtain sufficient conditions so that the global stability system GPS (1) can be guaranteed global stability [20,21]. For this, the solution of Problem 3 is necessary. If the transformation of T in (2) is such that the inverse transformation

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$$z = H_1(x, \theta, c_2, ..., c_n)$$

exists, is single and continuous for all $u \in D_u$,then the solution to Problem 3 is simultaneously the solution to Problem 1. If the inverse transformation

$$p = H_2(x, \theta, c_2, ..., c_n)$$

exists, is single and continuous for all $u \in D_u$, we can obtain a solution to problem 2.

For the system described by (3), we consider the following adaptive observer [5,12]:

$$\begin{cases} \dot{\hat{x}}(t) = R\hat{x}(t) + \Omega(\omega(t))\hat{\theta}(t) + g(t) + \begin{bmatrix} c_1\tilde{y}(t) \\ V(t)\dot{\hat{\theta}}(t) \end{bmatrix} & (5) \\ \hat{y}(t) = \hat{x}_1(t), \ \tilde{y}(t) \overset{\Delta}{=} y(t) - \hat{y}(t), & (6) \end{cases}$$

where c_1- arbitrary positive constant, and, $c_2, ..., c_n$ are chosen in such a way that the eigenvalues $F(c_2, ..., c_n)$ are in the open left half-plane.

Adaptation of parameters:

$$\dot{\hat{\theta}}(t) = \Gamma\varphi(t)\tilde{y}(t), \tag{7}$$

where Γ - arbitrary positive definite matrix selected as $\Gamma = \text{diag}(v_1, ..., v_n), \ v_i > 0$.

Auxiliary filter: $V(t)$ as $(n-1)\times m$ -matrix, a $\varphi(t)$ while m -vector. These matrix and vector represent the solution for the following auxiliary filter:

$$\dot{V}(t) = FV(t) + \overline{\Omega}(\omega(t)), \ V(0) = 0 \tag{8}$$

$$\varphi(t) = V^T(t)k + \overline{\Omega}_1^T(\omega(t)), \tag{9}$$

where Ω_1 - first lines $\Omega(\omega(t))$, a $\overline{\Omega}$ –otherlines,

$$\Omega \overset{\Delta}{=} \begin{bmatrix} \Omega_1 \\ \overline{\Omega} \end{bmatrix}.$$

Remember that F and k –submatrixes *R*, defined (4), $\Omega(\omega(t))$ and $g(t)$ -clear functions $y = x_1$ is measurable.

It is noteable $\overline{\Omega}(\omega(t))$ may contain null elements. If, in addition, F is diagonal, then the corresponding elements of V (t) are identically equal to zero, and the solution of (8), (9) is greatly simplified.

Definable $\tilde{x} \overset{\Delta}{=} x - \hat{x}, \ \ \tilde{\theta} \overset{\Delta}{=} \theta - \hat{\theta}$ and introduce the following auxiliary error vector:

$$\tilde{x}^* \overset{\Delta}{=} \tilde{x} - \begin{bmatrix} 0 \\ V\tilde{\theta} \end{bmatrix}.$$

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Using (3) and (5) - (9), one can imagine the following error of the system [17,23]:

$$\begin{bmatrix} \dot{\tilde{x}}^* \\ \dot{\tilde{\theta}} \end{bmatrix} = \left[\begin{array}{cc:c} R^* & & \varphi^T \\ & & 0 \\ \hdashline -\Gamma\varphi & 0 & 0 \end{array}\right] \begin{bmatrix} \tilde{x}^* \\ \tilde{\theta} \end{bmatrix} + \left[\begin{array}{c} 0\,.\,.\,0 \\ -V \\ \hdashline I_m \end{array}\right] \dot{\theta},$$

where

$$R^* \overset{\Delta}{=} \left[\begin{array}{c:c} -c_1 & k^T \\ \hdashline 0 & \\ \cdot & F(c_2, \ldots, c_n) \\ \cdot & \\ 0 & \end{array}\right].$$

Noteable that $\dim V(t) = (n-1)\times m$.

We also note that F is a constant matrix whose eigenvalues are completely determined by the parameters $c_2, \ldots, c_n$.

Recall that F can often be $\operatorname{diag}(-c_2, \ldots, -c_n)$, where $c_i > 0$ and all different.

On that case, if $\hat{\theta} = \theta$, mistake is $\tilde{x}$ there is a solution to a linear time-invariant equation whose poles are completely determined by the parameters $c_1, \ldots, c_n$.

Let GPS be a stationary observable bilinear system described as:

$$\begin{cases} \dot{z}(t) = M(p_M)z(t) + u(t)N(p_N)z(t) + K(p_K)u(t) \\ y(t) = z_1(t), \end{cases} \tag{10}$$

Where *M* and *N* usual $n\times m$ -matrixes, a*K* –general *n*-a vector that depends on constant but unknown vectors of physical parameters p_M, p_N and p_K respectively, and where $u(t) \in S_\Delta$ for some Δ.

Then there exists a constant non degenerate matrixT_1[22,23]such that with $\varsigma = T_1 z$ GPS system (10) is equivalent:

$$\begin{cases} \dot{\varsigma}(t) = \left[\begin{array}{c:c} -a_1 & I_{n-1} \\ \vdots & \\ \hdashline -a_n & 0 \cdots 0 \end{array}\right] \varsigma(t) + \begin{bmatrix} b_1(\varsigma) \\ \vdots \\ b_n(\varsigma) \end{bmatrix} u(t) = A\varsigma(t) + b(\varsigma)u(t) \\ y(t) = \varsigma_1(t), \end{cases} \tag{11}$$

where

$$A = T_1 M T_1^{-1},\ b(\varsigma) = B\varsigma + T_1 K \tag{12}$$

and

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$$B = T_1 N T_1^{-1} = \begin{bmatrix} b_{11} & 0 & \dots & 0 \\ b_{21} & b_{22} & \dots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ b_{n1} & \dots & \dots & b_{nn} \end{bmatrix}. \quad (13)$$

Expressions (11) - (13) are a special case of the following “observer form” ”:

$$\dot{z} = \left[\begin{array}{c:c} -a_1(z,t) & I_{n-1} \\ \vdots & \\ -a_n(z,t) & 0 \cdots 0 \end{array}\right] z + \begin{bmatrix} -b_1(z,t) \\ \vdots \\ -b_n(z,t) \end{bmatrix} u_1 .$$

$$y = z_1 .$$

Thus, (11) can be transformed into CFAO (3), and x are defined as

$$\theta(t) = \overline{T}^{-1} \begin{bmatrix} \begin{bmatrix} t_1 \\ 0 \end{bmatrix} - a \\ b(x) \end{bmatrix}, \quad x(t) = \overline{T}^{-1} \varsigma(t) ,$$

where $a \overset{\Delta}{=} (a_1, ..., a_n)^T$ и

$$b(x) = B\overline{T}x + T_1 K .$$

Now you can use the adaptive observer for this CFAO. Let the GPS be given by equations (10). Then (10) can be transformed into CFAO using the matrix transformation *T*. Then it can be shown [5,22] that the adaptive observer (5) - (9) for this system is globally stable.

IV. Conclusion

The above algorithms for adaptive state estimation, adaptive parameter identification, and the construction of an adaptive observer of a nonlinear dynamic system make it possible to increase the accuracy of estimating the state vector and thereby the quality indicators of control processes.

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