Identifying the Topology of the Iranian Stock Market Network and Ranking its Groups

Samad Sedaghati, Ruhollah Farhadi*, Mir Feyz Fallahshams

Abstract--- *The stock market is a complex financial system with heterogeneous members which produces huge amounts of data. It is clear that analyzing this huge data and inferring practical results creates a significant competitive advantage for its participants. One method of analyzing financial market data expanded significantly after the global financial crisis is complex network-based analysis that considers the structure of interdependencies of a system's members. Therefore, the current study analyzes the Iranian stock market using the graph theory in mathematics. First, the correlation network of stock market groups is constructed in three time scales of daily, seasonal and annual, and then their topology will be compared. In the next stage, using the centrality indexes in the graph theory, the importance of each market group is calculated and the groups are ranked in the network. The results of this study have significant implications for market participants and regulators for making investment decisions, regulating and controlling risk*.

Keywords--- *complex network, graph theory, centrality indexes, stock market network, stock market topology.*

I. INTRODUCTION

The importance of innovation for success in financial markets is obvious and vast financial data in relevant areas can provide important information for creating innovation. Inferring and obtaining accurate results from raw data to gain a competitive advantage may be the ultimate goal of financial data analysis. In the financial sector, data is very valuable, but because of its complexity, effective and efficient analysis is still a challenge for the financial sector. In addition, analyzing the relationships between multiple data and finding the importance of each single data is a critical factor to reinforce the insight of financial sector actors about their investments. The stock market is also a vital part of finance, while comprehending the vast and complex collected stock data seems a challenge.

The study of complex systems has been an important area of research for many years. A complex system consists of many interacting components with chaotic and / or collective behavior. Similarly, financial markets can be thought of as complex systems, whose collective behavior has often been observed in reality, especially in times of crisis. In fact, the presence of important members in the market network creates significant heterogeneities in the markets that complicates its network structure and makes it difficult to comprehend it by traditional analytical tools. Traditionally, the individual behaviors of a financial market's members are examined using routine time series data, while achieving a systemic view of market developments will only be possible by considering the structure of members' interdependencies. This need to understand the complex stock market relationships has become more apparent since the recent global financial crisis.

A common method for analyzing complex systems with interdependencies is the graph theory in mathematics, which is used to analyze complex networks. With the development of financial-behavioral science, financial scholars have given more value to analysis based on the graph theory and complex networks by considering markets and financial-economic

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systems with a network structure, with the hope that some financial-economic phenomena, including cascading behaviors and contagion, can be explained by network analysis. As Billio et al. (2012) acknowledge, perhaps the most important advantage of this network analysis is the "increased chance of identifying black swans" in financial markets. And Allen and Babus (2008) also consider financial stability assessment as an important application of the network approach in financial market analysis.

Therefore, the present study analyses the Iranian stock market based on the analysis of complex networks and the graph theory. The purpose of this study is to identify the topological dynamics of the Iranian stock market network in short and long term periods and to recognize the groups with the highest and lowest relative importance in the Iranian stock market network. The importance of this research lies in the fact that the presence of heterogeneous members in stock markets produces complex behaviors in this financial market and it is unlikely that the traditional time series analysis could provide all the necessary information, especially for risk assessment and control, portfolio management or even regulation. The complexity of stock market interactions and interdependencies exacerbates the problems of risk contagion and information asymmetry, and may reduce the systemic risk adjustment. In addition, traditional analyzes are incapable of examining of networking, domino, and cascading effects on the financial system and their role in financial stability, contagion, and risk system determination. Therefore, there is a need to have a topological recognition and to identify important market groups, based on network relations in the Iranian stock market.

The rest of this article is organized as follows: The second section reviews the background of the research. In the third section, the research methodology for data analysis is introduced, and in the fourth section, the obtained results are examined. Finally, the fifth section is devoted to the study conclusions and inferred suggestions.

II. RESEARCH BACKGROUND

In foreign research and theoretically, articles such as Allen and Gale (2000), Elliott et al. (2014), Gabrieli (2011), Rogers and Veraart (2014), Acemoglu et al. (2015), Berndsen et al. (2005), **Rokni** et al. (2018) examined the topological structure of financial networks and their balances, and identified some risk factors and contagion in financial networks. The overall finding was that in a financial network, network dynamics over time, topological properties, number and intensity of network members' interactions, and cascading reactions (domino effect) were important in keeping financial stability, contagion, and types of risks. They also found the great importance of "Too-Big-to-Fail" and "Too-Connect-to-Fail" risks in systemic risk. In summary, these studies revealed that given the interactions and correlations in financial networks, any evaluation of financial markets will provide potentially unrealistic results if the network structure is not attended.

In addition to articles that have contributed to the theoretical advancement and mathematical tools of networks, there are some experimental studies that address the actual applications of network theory in financial sciences. Chi Tse et al. (2010) modeled the network of cross-correlations in the US stock market and found that this network had a scale free degree distribution, and a small number of shares in the financial industry influence a large number of stocks in other listed industries, while their developments fueled a wide range of changes in the market. Bech and Atalay (2010) evaluated the topology of the US interbank market network and reported that the network was sparse and the centrality indexes in the network were good predictors of interbank market interest rates. Knowing that the description of cash flow in the financial network is not accurate without considering the network structure, Bech et al. (2010) examined the relationship between interdependencies and liquidity in the Canadian banking system and proposed a better way to predict the maintenance of liquidity in banks.

Billiu et al. (2012) also used the Granger causality of a network for the monthly returns of hedging funds, banks, insurers and traders and found that the correlation between these four sectors was greatly increased, while systematic risk of the insurance and the financial industries were also augmented through a complex network of relationships. They also revealed the important role of banks in transmitting shocks in the financial system network. Majapa and Gossel (2016) examined the topography of the South African stock network during the global financial crisis. Their results showed a highly structured cluster and homogeneity of the network. They also observed that the largest number of interconnected nodes existed in the financial and resource industries, and the global financial crisis had reduced the network members' interactions. Xu et al. (2017) examined the stability and developments of the Italian interbank market, and after checking the effects of shocks on the network stability network, they concluded that the network had dynamic stability and decreases investment gain and deposit fluctuations could improve system flexibility. Lee et al. (2019) examined the global stock market turbulence network using financial network-based calculations and machine learning techniques, and finally, proposed investment strategies for portfolio management based on the results of network analysis.

In a few internal studies, Sharifi Samani (2016) examined the topological features of the Tehran Stock Exchange and Securities Market Network, and determined the status of its minimum spanning tree from the beginning of 2011 to the end of 2019. He concluded that this tree could be used to analyze the Tehran Stock Market. The minimum spanning tree of indexes after the Joint Comprehensive Plan of Action (BARJAM) was different from that before BARJAM. It was also manifested that some indexes were more important due to the closer relationship with other sectors, and clustering, chart centrality, and the indexes proximity may change over time. Some indexes were clustered individually due to their small relationship with other ones, and had little importance in the network.

Ismail Pourmoghadam et al. (2019), using the complex network analysis of daily stock market data in the Tehran Stock Exchange during the years 2013-2016, created the Iranian stock market network with the threshold method and then examined the structural features of this network. Investigating the network's centrality and ranking several industries based on it showed that chemical production industries with relatively higher value added coefficient had the highest centrality, and the faster-growing economic sectors possessed a relatively higher centrality in the stock market. These authors confirmed effectiveness of using these kinds of analysis in gaining a deeper understanding the Iranian stock market.

III. DATA AND METHODOLOGY[1]

The stock market and any other phenomenon with a network-based approach is analyzed based on the graph theory in mathematics, and this research will follow the same methodology. The data analyzed in the current study are the prices index of 46 stock market groups with daily, seasonal and annual frequencies, which cover the first trading day from 2011 to 2019. In this section, the basic concepts of the graph theory and the graph assessment indexes will be reviewed. Finally, constructing stock market graphs using stock market data is described.

Basic concepts of the Graph Theory

In mathematical terminology, the graph theory refers to the study of graphs, or the pairwise relationships between the components of a set modeled by certain mathematical structures. A simple graph is shown by $G=(V,E)$ in which V is vertex or nodes, and E is a set of edges. A graph is formed by edges which connect nodes. Two nodes are connected if they have a common edge. The connection features of a graph is described by the adjacency matrix A. an adjacency matrix is a $n \times n$ matrix in which n shows the number of nodes in a graph. If a pair of nodes are connected by an edge, they are adjacent and the entry in the row *i* and column *j* in the adjacency matrix shown by a_{ij} will be 1, otherwise 0. If the adjacency matrix is symmetrical (or $a_{ij} = a_{ji}$), then, the graph is undirected. The sum of all a node's edges is called the

[1] . this section is largely written based on Kolaczyk and Csárdi (2014), Jackson (2010), and Naimzada et al. (2008)

node degree[2]. The node degree gives important information about the node, like identifying the importance of a certain node compared to the others.

The path between two nodes is a sequence of adjacent nodes in which each pair of consecutive nodes is connected by an edge. Additionally, each node cannot be crossed more than once. So, the length of a path is calculated by the number of edges that must be traveled to reach the node *j* from the node *i*. The shortest path between two nodes, and the length of that path is called the "geodetic path" and "geodetic distance", respectively. The number of shortest paths, N_{ij}, between a pair of nodes are obtained from the adjacency matrix. If the path between two nodes is straight, i.e., $d_{ij} = 1$ then $a_{ij} = 1$. The number of paths with length d is identified by:

$$N_{ij}^{(d)} = \left[A^d \right]_{ij} \tag{1}$$

In which $\left[\ldots \right]_{ij}$ shows the ij entry of the matrix A . This relationship is true both for the directed and undirected graphs. The distance between i and j nodes is a path with the shortest d in which $N_{ij}^{(d)} > 0$. A first-degree matrix is a $\mathrm{n} \times \mathrm{n}$ matrix in which the diametrical entries correspond to the degree of the i^{th} node and the non-diametrical entries are 0. In a directed graph, the total number of edges is:

$$\left| E \right| = \sum_{i=1}^{N} k_i^{in} = \sum_{i=1}^{N} k_i^{out} \tag{2}$$

And the mean of a graph degree is:

$$k = \frac{\left| E \right|}{N} \tag{3}$$

The density of a graph is calculated through dividing the mean of degree by the maximum possible number of edges:

$$\delta = \frac{k}{N-1} \tag{4}$$

In fully connected graphs, where all nodes are accessible from another node, the diameter of the graph can be measured as the longest-shortest path on all pairs of nodes $\left(n_i, n_j \right)$. The diameter of a graph is actually the longest path between the two nodes with the farthest distance. The mean of the path length d is the mean of distance between all pairs of nodes in a graph. In a directed graph with N nodes:[3]

$$d = \frac{1}{N.\left(N-1 \right)} \sum_{i \neq j}^{N} d\left(n_i, n_j \right) \tag{5}$$

In a "weighted graph", a value is assigned each edge which specifies the importance or weight of that edge. The adjacency matrix of this graph is called the "weighted adjacency matrix", and in contrast to the previous graph, its entries

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[2] . In contrast to a directed graph, there is a graph in which the edges are held from one node to another and the adjacency matrix is not necessarily symmetric. In other words, if an edge has connected the node *i* to the node *j*, the opposite is not necessarily true. However, in a directed graph it is possible to have an edge between the node *j* to the node *i*. In a directed graph, the entry-degree k_i^{in} and the exit-degree k_i^{out}, which manifest the entering and exiting number of edges to the node *i*, should be clarified. The sum of these two agrees will be the total node.

[3] . for an undirected graph, the right side is multiplied by 2.

do not include 1 and 0, but can have any value. Graph components are graph subsets with properties connected to each node and any other node in this subset. If the whole graph forms a component, it is a fully connected graph. The non-connected graph is constructed by two or more components. The size of a component is determined by the number of its nodes, and the component with the highest number of grades is called the "biggest component".

Graph Assessment Indexes

Degree Distribution: We often want to see how many nodes in a graph have the same degree. The scatter diagram shows all nodes degree in the degree distribution. If there are N nodes in a graph, and of N_k them have *k* degree, the degree distribution is:

$$\mathrm{P}(k) = \frac{N_k}{N} \tag{6}$$

The degree distribution can be considered as the probability of random selection of a node with a *k* degree. Many calculations in the graph theory require a degree distribution. The mean of a graph degree can be obtained by:

$$k = \sum_{k=0}^{\infty} k\mathrm{P}(k) \tag{7}$$

Clustering: Dense handles with high cohesive nodes form clusters. The clustering coefficient is related to the number of triangles in the graph. The local clustering coefficient of the node i is as follows:

$$Cl(i) = \frac{umber\ of\ closed\ triangles\ connected\ to\ the\ node\ i}{Number\ of\ triangles\ centered\ on\ the\ node\ i} \tag{8}$$

A triangle is a single node connected to other two nodes by edges. The local clustering coefficient may be an index of the graph local density. The higher the density of interconnections among a node neighborhoods, the higher the clustering coefficient will be. This is the clustering coefficient of a directed graph:

$$Cl_d(i) = \frac{E_i}{k_i(k_i - 1)} \tag{9}$$

In this equation, E_i is the number of edges between k_i of the node i neighbors. The local clustering coefficient is $0 \le Cl(i) \le 1$, in which $Cl(i) = 0$ means that there is no edge among the node i neighbors, while $Cl(i) = 1$ indicates that each neighbor is connected to the other. For nodes with 0 and 1 degree, the clustering coefficient is 0. Global clustering coefficient refers to the mean of local clustering coefficients in a graph:

$$Cl_g = \frac{1}{N}\sum_i Cl(i) \tag{10}$$

This index refers to the possibility of a connection between two randomly selected neighboring nodes. In random graphs, the global clustering coefficient is generally low and tends to 0 with as the graph size increase.

Centrality: Some nodes are important due to their topological positions in the graph. This feature of a graph is called "centrality". However, the centrality is specified by various indexes. The strength centrality, or node strength, in weighted graphs is measured by the total weight of the node's edges. For the node i, the strength centrality is

$$C_s(i) = \sum_{j=1}^{N} a_{ij} w_{ij} \tag{11}$$

Here, w_{ij} is the edge's weight between nodes i and j.

Closeness centrality measures the number of steps it takes to get from one particular node to another in a graph. For the i node:

$$C_c(i) = \sum_{j \neq i}^{N} \frac{1}{d_{ij}} \tag{12}$$

In this equation, d_{ij} is the shortest distance between i and j nodes. If the i node is directly connected to all nodes, the above index is 1.

A node with high closeness centrality connects faster to other nodes, while a node with fewer connections has a lower closeness centrality. A node that has more connections with other nodes may possess a better position, so that by increasing the connections, the possibility of meeting a node's needs improves and its dependence to other specific nodes decreases. Also, the node's access to resources in the graph expands and it may accept the role of a market maker, medium, and broker.

Betweenness centrality measures the number of times a node is placed in the path between two nodes (for example, to reach the j node starting from the k node, the i node must be crossed):

$$C_b(i) = \sum_{j,k \neq i} \frac{d_{ijk}(i)}{d_{jk}} \tag{13}$$

Here, d_{jk} shows the shortest paths from j to k, and $d_{ijk}(i)$ is the number of paths that crosses i. A node with high betweeness centrality is outstanding since it can observe or control the information flow in a graph.

A node enjoyed with both a high betweeness centrality and geodetic path is highly powerful. Since in weighted graphs, the betweeness centrality is measured based on all paths among nodes, not just geodetic ones, it is called "Flow betweenness centrality" and measures the share of a node in the maximum current flows in the graph.

Eigenvector centrality measures the importance of a node based on the importance of the nodes connected to it. In other words, the centrality of the nodes connected to a specific node determines its importance in the graph. The eigenvector centrality of the node is obtained by the sum of the eigenvector centrality of its neighbors, which is in the form of a matrix as follows:

$$(1 - \mathrm{L})\overrightarrow{C_E} = 0 \tag{14}$$

In the above equation, 1 is the $n \times n$ identity matrix, L is the adjacency matrix, and $\overrightarrow{C_E}$ is the $n \times 1$ vector of the eigenvector centrality of nodes. In fact, nodes with high eigenvector centrality are those connected to many nodes, which themselves are connected to many other nodes.

Graphs Constructing Method

As mentioned, in order to study the Iranian stock market according to the graph theory, first, the weighted adjacency matrix must be constructed through calculating the mutual correlation between the groups' returns. We calculate this adjacency matrix for three time-periods: daily, seasonal and annual; therefore 3 daily, seasonal, and annual graphs will be constructed.

For this, the time series data of the price index for 46 groups in the Iran Stock Market with daily, seasonal and annual frequencies are extracted, and their returns are calculated. In the next step, the mutual correlations of all these groups for these three time-periods are obtained. As a result, three matrices with 46 rows and columns of correlation coefficients are generated for three time scales of daily, seasonal and annual frequencies.

Then the adjacency matrix for daily, seasonal and annual graphs is constructed based on the mutual correlation of returns in the main groups of in the Iranian Stock Market. For this, the critical value $2.58 \times \frac{1}{\sqrt{T}}$ is used to specify the significance of mutual correlations with 99% confidence level (*T* is the sample size) (Krehbiel, 2004). The sample size is 1780 in the daily, 28 in the seasonal, and 7 in the annual data. Therefore, the critical value is 0.0611 for the daily scale, 0.4875 for the seasonal scale, and 0.9121 for the annual scale. Then, if the mutual correlation coefficient for both groups *i* and *j* is greater than these values, the correlation is statistically significant at the 99% confidence level, otherwise, 0 will be replaced with that component in the adjacency matrix (the row *i* and column j). It means that no edge exists between these two groups (nodes) corresponding to that component. Also, the main diameter of the adjacency matrix must be 0 since a group adjacency to itself is meaningless. Performing the same operation for all the market groups leads to constructing the adjacency matrix based on the correlation matrix. After constructing the adjacency matrix, the graphs are formed and ready to be analyzed.

IV. Results

In the first step of graph analysis, the stock market network topology for the three daily, seasonal and annual graphs will be examined using the introduced indexes. The results of the calculations based on these indexes are reported in Table 1.

Table 1. Network topology indexes

Index	Period		
	Daily	Seasonal	Annual
Number of nodes	46	46	46
Number of edges	2068	2006	454
Mean of degree	44.94	43.60	9.86
Diameter of graph	2	2	8
Density of graph	0.999	0.969	0.219
Mean of the path length	1.001	1.03	2.65
Number of components	1	1	6
Global clustering coefficient	0.999	0.973	0.644

The number of nodes and edges refer to the number of stock market groups, and significant correlations between the groups' returns, respectively. The mean of degree is obtained by dividing the number of edges by the number of groups. The diameter of a graph is the sum of all edges between two nodes with the farthest distance. Dividing the mean of degree by the maximum possible number of edges connected to a node (46 minus 1) creates the graph density. The mean of path length is obtained through dividing the sum of all the shortest paths between two nodes by the total number of a graph's possible edges (multiplying the number of nodes by the number of nodes minus one). The number of components is equal to the number of connected components of a graph. Finally, the global clustering coefficient is obtained from the mean of local clustering coefficients (number of triangular relations of each node) of all the nodes. Given Table 1, it is inferred that:

- By moving from the daily period to time periods with less frequency (seasonal and annual), the number of correlations between the groups of the Iranian Stock Market is reduced, since the number of edges is 2068 for daily, 206 for seasonal, and 454 for annual correlation network.
- By moving from the daily period to time periods with less frequency (seasonal and annual), each group's mean of mutual correlation is reduced (from 44.94 to 9.86). In other words, the correlation between various groups' returns is greatly reduced in the long run.
- The maximum distance among the stock market groups in daily and seasonal periods is 2. That is, if the groups are not directly connected, they are correlated with up to one common group (medium) with a very small distance. However, in the annual period, the maximum distances is 8 nodes, meaning that in the annual, compared to the daily and seasonal frequencies, the distance between the nodes is multiplied by 4, and the mutual correlation is significantly decreased. In the daily and seasonal graphs, the mean of the path length is very close to 1, while it is 2.56 in the annual graph, which confirms the mentioned findings.
- In the daily period, the density of the stock market correlation network is very high and close to 1, and the network is almost a complete graph. However, moving towards the annual period, the density is reduced to nearly one-fifth.
- Since the correlation network in the Iranian stock market groups is fully connected in the daily and seasonal periods, all groups are affected by the network evolutions (the number of components is equal to 1). On the other hand, the graph in the annual period have 6 components, meaning that there are 6 components in a graph which are not correlated while the members of each component are inter-correlated.
- In the daily, seasonal, and annual graphs, the probable correlation between two groups is 99.9%, 97.3% and 64.4%, respectively since the values of global clustering coefficient corresponds to these probabilities and it manifest the possibility of a connection between two nodes in the graph.

Figure 1 also shows the relative frequency of the mentioned graphs' edges. It is obvious that the annual graph has fewer edges with wider scattering than the seasonal and daily graphs. In fact, as we move from the daily to the seasonal and annual periods, we see less mutual correlations and more common scatterings in the Iranian stock market. Therefore, In the long run, mutual correlations become heterogeneous and their frequencies in the stock market groups diversify. So, it can be acknowledged that the probability of information transmission in short-term periods is higher and faster.

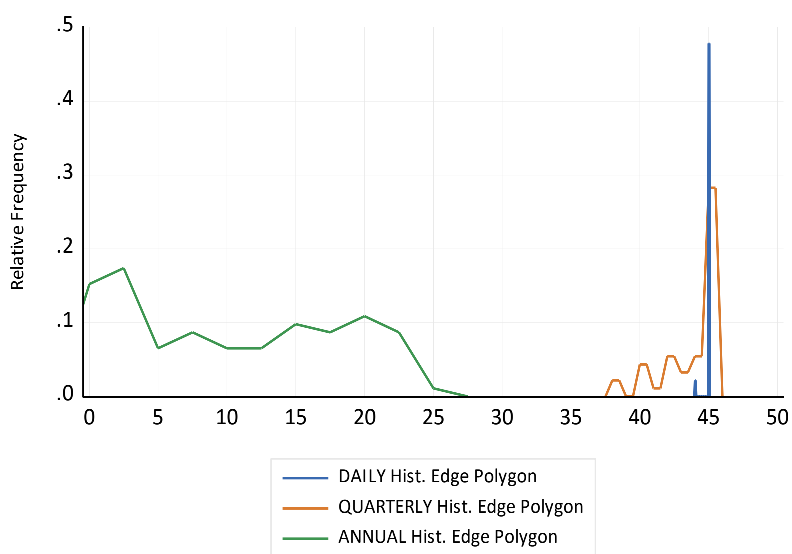

Figure 1. Relative frequency of the groups' mutual correlations in the Iranian stock market

Examining the topological properties of the correlation graph in the Iranian stock market groups in daily, seasonal and annual frequencies reveal that at the daily level, the groups' mutual correlation is very high and all groups are affected by any individual evolutions. This is true for the seasonal level, but the heterogeneity in groups' correlations grows slightly. Finally, at the annual level, the returns' correlation in the stock market groups varies to the extent that it changes the network's topology. Now, the network loses its connectness property and turns into a graph with 6 components without any correlations, while the number of group's correlations is also reduced. Therefore, it may be concluded that in very short periods of time, any (informational or operational) shock imposed on the stock market groups will directly affect most other groups with a significant transformation speed. In contrast, as we move toward long-term periods, the mutual correlations diminish and even the shocks endured by some groups will have no effect on the others. In the next section, the Iranian stock market groups are ranked by measuring the centrality indexes and individual importance to identify main groups in the above mentioned graphs. Ranking groups based on their importance in the network helps to find those who could disseminate more information in the market and influence its future behaviors.

As mentioned in Section 3, the centrality indexes assess the graph theory. By examining the degree centrality for each node in the network, the centrality of that node in terms of the number of its connections and correlations (node's edges) with other nodes in the network is identified. In Table 2, the stock market groups are ranked based on their weighted degree centrality in daily, seasonal, and annual graphs[4]. The term "weighted" in this centrality shows that in addition to the number of edges, the weight of each edge is also included in the calculation. As can be seen, in the daily graph, the investment group, in the seasonal graph, the insurance group, and in the annual graph, the cement group, have the first ranking in terms of degree centrality in the Iranian stock market. To say differently, these three groups have the most and strongest connections and correlations with other stock market groups in the these time periods. Conversely, as the rank of a group decreases, so does the number and severity of its correlations. An implication of this ranking is that changes in returns of lower-ranked groups lead to changes in returns of more groups. In addition, if a shock occurs for one of these groups, the lower the group's rank, the more probable that the shock will spread to more groups. It should also be noted that in the annual graph, wood, paper products, oil products, software and services, and coal groups do not have a significant correlation with other groups in the stock market; therefore they are independent and do not fall in the rankings.

Table 2. Ranking the Iranian stock market groups based on the weighted degree centrality

Ranking	Daily	Seasonal	Annual
1	Investment	Insurance	Cement, lime and gypsum
2	Other non-metallic mineral products	Other non-metallic mineral products	Transportation by railways
3	Insurance	Medicinal products	Insurance
4	Mass construction, real estate	Cement, lime and gypsum	Other non-metallic mineral products
5	Cement, lime and gypsum	Mass construction, real estate	Medicinal products
6	Medicinal products	Ceramic and tiles	Sweets
7	Leasing	Dairy products	Cleaning products

[4] . The weighted degree centrality in a node equals to the sum of the node's edges multiplied by the weights of those edges; the edge's weight is size of correlation coefficient ($C_s(i) = \sum_{j=1}^{N} a_{ij} w_{ij}$). For example, if a node has two edges with the same weight 0.928, the mean of the weight degree is 1.85. The nodes are ranked from the highest to lowest based on their weighted degree centrality. Since the nodes' importance will be determined by comparing their scores, not by their individual scores, the ranking process was performed.

Ranking	Daily	Seasonal	Annual
8	Automobile parts	Engineering activities	Industrial contracting
9	Machines	Machines	Ceramic and tiles
10	Banks and credit institutions	Transportation by railways	Hardware and equipment
11	Dairy products	Investment	Automobile parts
12	Rubber and plastic	Banks and credit institutions	Dairy products
13	Iron and steel	Sweets	Electric machines
14	Metal products	Industrial equipment	Metal products
15	Software and services	Agricultural products	Engineering activities
16	Electric machines	Telecommunication	Textile
17	Engineering activities	Leasing	Home appliance
18	Transportation by railways	Metal products	Agricultural products
19	Ceramic and tiles	Automobile parts	Banks and credit institutions
20	Hardware and equipment	Electric machines	Industrial equipment
21	Fertilizers and nitrogen compounds	Hardware and equipment	Mass construction, real estate
22	Various chemicals	Home appliance	Machines
23	Telecommunication	Fertilizers and nitrogen compounds	Drinks
24	Metallic minerals	Drinks	Ports and shipping
25	Non-iron precious metals products	Paper products	Investment
26	Automobile	Excavation	Printing and publishing
27	Excavation	Rubber and plastic	Telecommunication
28	Paper products	Road transportation	Rubber and plastic
29	Industrial equipment	Sugar	Excavation
30	Agricultural products	Telecommunication equipment	Leasing
31	Coal	Retailing, excluding motor vehicles	Telecommunication equipment
32	Sugar	Printing and publishing	Iron and steel
33	Sweets	Oil products	Metallic minerals
34	Cleaning products	Other food products	Fertilizers and nitrogen compounds
35	Drinks	Automobile	Other food products
36	Oil products	Iron and steel	Road transportation
37	Other food products	Ports and shipping	Various chemicals
38	Home appliance	Software and services	Non-iron precious metals products
39	Industrial contracting	Cleaning products	Automobile
40	Road transportation	Various chemicals	Retailing, excluding motor vehicles
41	Printing and publishing	Coal	Sugar
42	Retailing, excluding motor vehicles	Textile	
43	Wood	Metallic minerals	
44	Ports and shipping	Non-iron precious metals products	
45	Telecommunication equipment	Wood	
46	Textile	Industrial contracting	

Other indexes for assessing a graph included betweenness, closeness, clustering, and eigenvector centrality. The results of ranking the Iranian stock market groups in terms of these indexes in the seasonal and annual graphs are reported in Tables 3 and 4[5].

The betweenness centrality measures a node's importance in the flow of information and communication in a network, and identifies important nodes that establish a connection between different groups of nodes[6]. A node with higher betweenness centrality is located between the shortest paths among several nodes. Consequently, this node controls the flow of information to more nodes, and has closer interactions with all the network's nodes. The higher the betweenness centrality of a node, the more it is possible that the network will lose it cohesiveness if that node is removed. Given this index in the seasonal graph, the Insurance and Industrial contracting groups have the first and last ranking, respectively. However, in the annual graph, these rankings belong to the Agriculture products, and Sugar groups, respectively.

The index of closeness centrality, as its name implies, shows how close one node is to the other nodes. The higher the score of a node in terms of this index, the faster the information will be transmitted from it to other nodes. This node can be connected to other nodes in the shortest possible distance and benefits from more powerful access power in the network[7]. According to this index in the seasonal graph, the Insurance group ranks first, and the Industrial contracting group ranks last. In the annual graph, however, the first rank belongs to the Cement group and the last rank belongs to the Sugar group.

The clustering centrality index manifests the tendency of a node's neighbors in the network tend to be correlated. As a result, a high clustering score shows dense connections and correlations among a node's neighbors, as well as, its great power to influence them. Based on this index in the seasonal graph, the Industrial contracting group is ranked first, and the Sugar group is ranked last[8]. In the annual graph, however, the first rank belongs to the Investment group and the last rank belongs to the Sugar group.

The eigenvector centrality index points to the importance of a node's connections and correlations. That is, the more important the nodes associated with a given node in the network, the higher the eigenvector centrality of that note. In fact, the higher the eigenvector centrality score of a node, the more nodes with eigenvector centrality score it is connected to. In terms of this index in the seasonal graph, the Insurance, and Non-iron precious metals products groups ranks the first and last, respectively.[9] But, in the annual graph, these ranks belong to the Cement, and Sugar groups, respectively.

[5] . Since the daily graph is a complete graph, the stock market groups have similar rankings in terms of betweenness centrality, closeness, clustering, and special value indexes. Therefore, no separated table is included.

[6] . To calculate the betweenness centrality, first, two nodes in a network are selected. Then, the number of times the node *i* is placed between the shortest path (geodetic path) between these two nodes is counted and divided by the shortest path between the two mentioned nodes. The number of fractions is $(n-1)(n-2)/2$ (in which, n refers to the network's nodes), and the sum of these fractions will be the score of betweenness centrality for the node *i*.

[7] . To calculate the closeness centrality, the shortest paths between the node i and other nodes are summed and divided by $(n-1)$ (n is the number of the network's nodes). Then, the obtained number is reversed. This is the score of closeness centrality for the node i.

[8] . To calculate the clustering centrality (local clustering coefficient), the number of edges between k neighbors of the node i with degree k will be divided by $k(k-1)$ (k shows the degree of the node i). Then, the obtained number is multiplied by 2 for the undirected graph. This new number is the clustering centrality (coefficient) for the node i.

[9] . The eigenvector centrality index is calculated in 4 steps: 1. All nodes will be given the score 1; 2. The score of each node is recalculated as the weighted sum of the scores of all its neighboring nodes; 3. The nodes' scores are normalized through dividing by the largest obtained value; 4. Steps 2 and 3 are repeated until no changes occur in the scores.

Table 3. Ranking the Iranian stock market groups based on the centrality indexes in the seasonal graph

Ranking	Betweenness	Closeness	Clustering	Eigenvector
1	Insurance	Insurance	Industrial contracting	Insurance
2	Other non-metallic mineral products	Other non-metallic mineral products	Non-iron precious metals products	Other non-metallic mineral products
3	Medicinal products	Medicinal products	Textile	Medicinal products
4	Cement, lime and gypsum	Cement, lime and gypsum	Cleaning products	Cement, lime and gypsum
5	Mass construction, real estate	Mass construction, real estate	Wood	Mass construction, real estate
6	Ceramic and tiles	Ceramic and tiles	Iron and steel	Ceramic and tiles
7	Dairy products	Dairy products	Metallic minerals	Dairy products
8	Engineering activities	Engineering activities	Ports and shipping	Engineering activities
9	Machines	Machines	Various chemicals	Machines
10	Transportation by railways	Transportation by railways	Printing and publishing	Transportation by railways
11	Investment	Investment	Other food products	Investment
12	Banks and credit institutions	Banks and credit institutions	Coal	Banks and credit institutions
13	Sweets	Sweets	Oil products	Sweets
14	Industrial equipment	Industrial equipment	Road transportation	Industrial equipment
15	Agricultural products	Agricultural products	Telecommunication equipment	Agricultural products
16	Telecommunication	Telecommunication	Automobile	Telecommunication
17	Leasing	Leasing	Software and services	Leasing
18	Metal products	Metal products	Rubber and plastic	Metal products
19	Automobile parts	Automobile parts	Retailing, excluding motor vehicles	Automobile parts
20	Electric machines	Electric machines	Drinks	Electric machines
21	Hardware and equipment	Hardware and equipment	Insurance	Hardware and equipment
22	Home appliance	Home appliance	Other non-metallic mineral products	Home appliance
23	Fertilizers and nitrogen compounds	Fertilizers and nitrogen compounds	Medicinal products	Fertilizers and nitrogen compounds
24	Paper products	Paper products	Cement, lime and gypsum	Paper products
25	Excavation	Excavation	Mass construction, real estate	Excavation
26	Sugar	Sugar	Ceramic and tiles	Sugar
27	Retailing, excluding motor vehicles	Retailing, excluding motor vehicles	Dairy products	Road transportation
28	Drinks	Drinks	Engineering activities	Telecommunication equipment
29	Rubber and plastic	Rubber and plastic	Machines	Rubber and plastic
30	Road transportation	Road transportation	Transportation by railways	Retailing, excluding motor vehicles
31	Telecommunication	Telecommunication	Investment	Drinks

Ranking	Betweenness	Closeness	Clustering	Eigenvector
	equipment	equipment		
32	Automobile	Automobile	Banks and credit institutions	Printing and publishing
33	Software and services	Oil products	Sweets	Oil products
34	Oil products	Printing and publishing	Industrial equipment	Automobile
35	Other food products	Software and services	Agricultural products	Ports and shipping
36	Coal	Other food products	Telecommunication	Various chemicals
37	Printing and publishing	Coal	Leasing	Coal
38	Various chemicals	Various chemicals	Metal products	Other food products
39	Ports and shipping	Ports and shipping	Automobile parts	Software and services
40	Metallic minerals	Iron and steel	Electric machines	Iron and steel
41	Iron and steel	Metallic minerals	Hardware and equipment	Textile
42	Wood	Wood	Home appliance	Cleaning products
43	Cleaning products	Cleaning products	Fertilizers and nitrogen compounds	Wood
44	Textile	Textile	Paper products	Metallic minerals
45	Non-iron precious metals products	Non-iron precious metals products	Excavation	Industrial contracting
46	Industrial contracting	Industrial contracting	Sugar	Non-iron precious metals products

Table 4. Ranking the Iranian stock market groups based on the centrality indexes in the annual graph

Ranking	Betweenness	Closeness	Clustering	Eigenvector
1	Agricultural products	Cement, lime and gypsum	Investment	Cement, lime and gypsum
2	Fertilizers and nitrogen compounds	Transportation by railways	Road transportation	Transportation by railways
3	Various chemicals	Insurance	Other food products	Medicinal products
4	Metallic minerals	Other non-metallic mineral products	Printing and publishing	Industrial contracting
5	Iron and steel	Sweets	Drinks	Other non-metallic mineral products
6	Insurance	Medicinal products	Rubber and plastic	Insurance
7	Cement, lime and gypsum	Agricultural products	Telecommunication	Cleaning products
8	Other non-metallic mineral products	Cleaning products	Telecommunication equipment	Sweets
9	Medicinal products	Ceramic and tiles	Home appliance	Ceramic and tiles
10	Machines	Dairy products	Engineering activities	Automobile parts
11	Transportation by railways	Industrial contracting	Textile	Dairy products
12	Ports and shipping	Mass construction, real estate	Metal products	Hardware and equipment
13	Mass construction, real estate	Hardware and equipment	Banks and credit institutions	Metal products

Ranking	Betweenness	Closeness	Clustering	Eigenvector
14	Sweets	Automobile parts	Dairy products	Electric machines
15	Cleaning products	Electric machines	Ports and shipping	Engineering activities
16	Ceramic and tiles	Metal products	Automobile parts	Textile
17	Industrial equipment	Textile	Medicinal products	Home appliance
18	Electric machines	Engineering activities	Industrial contracting	Banks and credit institutions
19	Dairy products	Banks and credit institutions	Electric machines	Agricultural products
20	Automobile parts	Home appliance	Hardware and equipment	Industrial equipment
21	Industrial contracting	Machines	Ceramic and tiles	Drinks
22	Hardware and equipment	Industrial equipment	Cleaning products	Mass construction, real estate
23	Banks and credit institutions	Drinks	Mass construction, real estate	Machines
24	Metal products	Excavation	Sweets	Printing and publishing
25	Excavation	Telecommunication	Other non-metallic mineral products	Investment
26	Textile	Rubber and plastic	Industrial equipment	Telecommunication
27	Engineering activities	Printing and publishing	Transportation by railways	Ports and shipping
28	Leasing	Investment	Excavation	Rubber and plastic
29	Home appliance	Ports and shipping	Leasing	Excavation
30	Drinks	Fertilizers and nitrogen compounds	Cement, lime and gypsum	Telecommunication equipment
31	Rubber and plastic	Telecommunication equipment	Insurance	Leasing
32	Telecommunication equipment	Leasing	Machines	Road transportation
33	Telecommunication	Road transportation	Agricultural products	Fertilizers and nitrogen compounds
34	Printing and publishing	Retailing, excluding motor vehicles	Fertilizers and nitrogen compounds	Retailing, excluding motor vehicles
35	Investment	Other food products	Retailing, excluding motor vehicles	Other food products
36	Other food products	Various chemicals	Various chemicals	Automobile
37	Road transportation	Automobile	Automobile	Various chemicals
38	Producing non-iron precious metals	Metallic minerals	Metallic minerals	Iron and steel
39	Automobile	Iron and steel	Iron and steel	Metallic minerals
40	Retailing, excluding motor vehicles	Producing non-iron precious metals	Producing non-iron precious metals	Producing non-iron precious metals
41	Sugar	Sugar	Sugar	Sugar

In sum, given that the centrality indexes are diverse, a suitable solution for the final ranking of the Iranian stock market groups in the network can be calculating the mean scores in the 5 centrality indexes and then ranking the groups based on this mean score. This kind of ranking is reported in Table 5. This table reveals that in the daily period, the

Investment group, in the seasonal period, the Insurance group, and in the Annual period, the Cement group are the most important ones in the Iranian stock market correlation network.[10]

Table 5. Ranking the Iranian stock market groups based on the mean of centrality indexes

Ranking	Daily	Seasonal	Annul
1	Investment	Insurance	Cement, lime and gypsum
2	Other non-metallic mineral products	Other non-metallic mineral products	Transportation by railways
3	Insurance	Medicinal products	Medicinal products
4	Mass construction, real estate	Cement, lime and gypsum	Other non-metallic mineral products
5	Cement, lime and gypsum	Mass construction, real estate	Insurance
6	Medicinal products	Ceramic and tiles	Agricultural products
7	Leasing	Dairy products	Sweets
8	Automobile parts	Engineering activities	Cleaning products
9	Machines	Machines	Industrial contracting
10	Banks and credit institutions	Transportation by railways	Ceramic and tiles
11	Dairy products	Investment	Dairy products
12	Rubber and plastic	Banks and credit institutions	Automobile parts
13	Iron and steel	Sweets	Hardware and equipment
14	Metal products	Industrial equipment	Metal products
15	Software and services	Agricultural products	Electric machines
16	Electric machines	Telecommunication	Engineering activities
17	Engineering activities	Leasing	Textile
18	Transportation by railways	Metal products	Home appliance
19	Ceramic and tiles	Automobile parts	Banks and credit institutions
20	Hardware and equipment	Electric machines	Mass construction, real estate
21	Fertilizers and nitrogen compounds	Hardware and equipment	Industrial equipment
22	Various chemicals	Home appliance	Drinks
23	Telecommunication	Fertilizers and nitrogen compounds	Investment
24	Metallic minerals	Paper products	Machines
25	Producing non-iron precious metals	Excavation	Printing and publishing
26	Automobile	Sugar	Ports and shipping
27	Excavation	Drinks	Telecommunication
28	Paper products	Rubber and plastic	Rubber and plastic
29	Industrial equipment	Road transportation	Telecommunication equipment
30	Agricultural products	Telecommunication equipment	Road transportation
31	Coal	Retailing, excluding motor vehicles	Excavation
32	Sugar	Printing and publishing	Other food products
33	Sweets	Oil products	Leasing
34	Cleaning products	Automobile	Fertilizers and nitrogen compounds
35	Drinks	Other food products	Various chemicals

[10] . It is necessary to re-emphasize that in the annual graph, wood, paper products, oil products, software and services, and coal groups do not have a significant correlation with other groups in the stock market; therefore they are independent and do not fall in the rankings.

36	Oil products	Ports and shipping	Metallic minerals
37	Other food products	Various chemicals	Iron and steel
38	Home appliance	Software and services	Retailing, excluding motor vehicles
39	Industrial contracting	Coal	Automobile
40	Road transportation	Iron and steel	Producing non-iron precious metals
41	Retailing, excluding motor vehicles	Cleaning products	Sugar
42	Wood	Textile	
43	Ports and shipping	Metallic minerals	
44	Telecommunication equipment	Wood	
45	Printing and publishing	Producing non-iron precious metals	
46	Textile	Industrial contracting	

V. CONCLUSION

This study was performed by the aim of recognizing the network topology of mutual correlation in network in Iranian stock market groups, and to identify the groups with the most and least importance in the market network in the daily, seasonal and annual periods. Given the complexity and mutual connections among members of a financial market, computational indexes of the graph theory in mathematics were used to analyze these relations.

It was discovered that in the short term, the Iranian stock market network is very complex and intertwined, and any changes in a network member (a stock market group) affect a large number of other members. However, as we move towards longer periods, the network's complexity decreases. Also, rankings of the Iranian stock market groups indicate that several groups including Insurance, Medicinal products, Investment, Other non-metallic mineral products, Mass construction and real estate, Cement, machinery and Ceramic and tiles have the most relevance and impact on other stock market groups, both in the short and long term, and are outstanding in terms of transferring information in the market. In contrast, those like the Sugar, Coal, Textiles or Wood groups have the least impact and slow information transition in the market.

Implications and suggestions of this study may be more useful for investors and participants in the Iranian stock market, especially analysts and portfolio management institutions in the risk management filed. However, market's watchers and regulators can also use these results to control and manage market stability. It is suggested to attend the groups' impact on the market in selecting stock market groups to form a portfolio. Ranking various groups based on the centrality indexes identifies both the groups with the most and least powerful impact on the market, and those with the minimum impactability from other market members.

For forming a portfolio, it is recommended to select groups with different rankings according to the time scale of the target trading strategy in order to reduce both the risk of portfolio concentration and unsystematic risk, and to control the risk of contagion among groups. Additionally, in analyzing the effects of changes and economic shocks on the stock market, it is better to attend groups' rankings. For example, if information is published about an important group in the stock market, it will be disseminated to other groups through its mutual correlations. The less relevant the information published to other important groups, the less the stock market as a whole will be affected.

Finally, the market's watcher and regulator must consider the importance of each group in the stock market and its influence on the market behavior to successfully regulate the market and to control its stability and risk. Deviation of individual groups from the facts, herd behaviors, and trading errors of market's participants, if occurred in important market groups with significant contagion to other market's groups, may have more devastating effects on the whole market. The results of this study reflect notable implications in terms of this effect and unfold the importance of using a network approach in analyzing the stock market data.

REFERENCES

[1] Acemoglu, D., Ozdaglar, A., & Tahbaz-Salehi, A. (2015). Systemic risk and stability in financial networks. American Economic Review, 105(2), 564-608.
[2] Allen, F., & Babus, A. (2009). Networks in finance. The network challenge: strategy, profit, and risk in an interlinked world, 367.
[3] Allen, F., & Gale, D. (2000). Financial contagion. Journal of political economy, 108(1), 1-33.
[4] Battiston, S., Caldarelli, G., May, R. M., Roukny, T., & Stiglitz, J. E. (2016). The price of complexity in financial networks. Proceedings of the National Academy of Sciences, 113(36), 10031-10036.
[5] Bech, M. L., & Atalay, E. (2010). The topology of the federal funds market. Physica A: Statistical Mechanics and its Applications, 389(22), 5223-5246.
[6] Bech, M. L., Chapman, J. T., & Garratt, R. J. (2010). Which bank is the “central” bank?. Journal of monetary economics, 57(3), 352-363.
[7] Berndsen, R. J., Leon, C., & Renneboog, L. (2018). Financial stability in networks of financial institutions and market infrastructures. Journal of Financial Stability, 35, 120-135.
[8] Billio, M., Getmansky, M., Lo, A. W., & Pelizzon, L. (2012). Econometric measures of connectedness and systemic risk in the finance and insurance sectors. Journal of financial economics, 104(3), 535-559.
[9] Chi, K. T., Liu, J., & Lau, F. C. (2010). A network perspective of the stock market. Journal of Empirical Finance, 17(4), 659-667.
[10] Elliott, M., Golub, B., & Jackson, M. O. (2014). Financial networks and contagion. American Economic Review, 104(10), 3115-53.
[11] Gabrieli, S. (2011). The microstructure of the money market before and after the financial crisis: a network perspective. CEIS Tor Vergata Research Paper Series, 9(1), 181.
[12] Georg, C. P. (2013). The effect of the interbank network structure on contagion and common shocks. Journal of Banking & Finance, 37(7), 2216-2228.
[13] Ismail Pourmoghadam, Hadi., Mohammadi, Teymour., Feqhi Kashani, Mohammad. & Shakeri, Abbas. (2019). Section growth and centrality in the Iranian stock market: Application of complex network analysis. Journal of Economic Research and Policies. 27 (90).
[14] Jackson, M. O. (2010). Social and economic networks. Princeton university press.
[15] Kolaczyk, E. D., & Csárdi, G. (2014). Statistical analysis of network data with R (Vol. 65). New York, NY: Springer.
[16] Krehbiel, T. C. (2004). Correlation coefficient rule of thumb. Decision Sciences Journal of Innovative Education, 2(1), 97-100.
[17] Lee, T. K., Cho, J. H., Kwon, D. S., & Sohn, S. Y. (2019). Global stock market investment strategies based on financial network indicators using machine learning techniques. Expert Systems with Applications, 117, 228-242.
[18] Majapa, M., & Gossel, S. J. (2016). Topology of the South African stock market network across the 2008 financial crisis. Physica A: Statistical Mechanics and its Applications, 445, 35-47.
[19] Naimzada, A. K., Stefani, S., & Torriero, A. (Eds.). (2008). Networks, topology and dynamics: Theory and applications to economics and social systems (Vol. 613). Springer Science & Business Media.
[20] Rogers, L. C., & Veraart, L. A. (2013). Failure and rescue in an interbank network. Management Science, 59(4), 882-898.
[21] Roukny, T., Battiston, S., & Stiglitz, J. E. (2018). Interconnectedness as a source of uncertainty in systemic risk. Journal of Financial Stability, 35, 93-106.
[22] Sharifi Samani, Farshad. (2016). Topological features of the stock network in the Tehran Stock Exchange (Case study of BARJAM effect). Master Thesis, Yazd University of Science and Art.
[23] Xu, R., Wong, W-K, Chen, G., & Huang, Sh. (2017). Topological Characteristics of the Hong Kong Stock Market: A Test-based P-threshold Approach to Understanding Network Complexity, Scientific Reports volume 7, Article number: 41379.