SOLID BODY OSCILLATIONS UNDER THE INFLUENCE OF GROUP VIBRATION LOADS

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ABSTRACT--*The article considers oscillations of a dissipative mechanical system with a finite number of degrees of freedom when exposed to group vibration loads. As an example, a dissipative mechanical system with two degrees of freedom is considered. The considered bodies are mounted on viscoelastic supports whose rigidity is described by the integral Boltzmann – Walter relations.The resulting integro-differential equations are solved by the method of complex amplitudes of the theory of oscillations (or analytically).Investigated before the resonance, resonance and after resonance regions of the mechanical system under consideration. Found by the condition in the form of inequality when self-synchronization occurs. With sufficient maximum possible values of the vibrational moments, self-synchronization can take place even in the case when the partial angular velocities differ significantly from one another. Knowing the distribution law of a random variable, we can calculate the probability of inequality, that is, the probability of the presence of self-synchronization. It has been established that for sufficient maximum possible values of the vibrational moments, self-synchronization can take place even in the case when the partial angular velocities differ significantly from one another. It was also determined that the amplitude of the basement oscillations decreases by a factor of five compared with the amplitude of oscillations in the absence of self-synchronization. Thus, the mathematical formulation and methods for solving the problem of dynamic stability of a viscoelastic mechanical system with a finite number of degrees of freedom are developed. Comparison of calculated values with known results. Taking into account the viscous properties of the material has a noticeable effect on the areas of dynamic stability. Viscous properties play a stabilizing factor for parametric oscillations of mechanical systems. Geometrically, the sizes of the regions of dynamic instability corresponding to the main parametric resonances are reduced and shifted above the abscissa axis.*

Keywords*-- vibrations, vibration load, angular velocities, resonance, group foundations, mode stability.*

I. INTRODUCTION

Under certain conditions, the shifts are such that the imbalances caused by the operation of individual machines are mutually compensated. It is this case that is considered below. This idea of using the phenomenon of self-synchronization in the design of group foundations for unbalanced machines was proposed in [1,2,3]. For the practical use of this principle, the following two conditions must be met:

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1.There must exist and be stable a regime of synchronous rotation of the shafts of machines with such a combination of phases, in which the imbalances of the machines are mutually compensated, and the machines are considered exactly the same, that is, the scatter of their parameters is not taken into account due to inaccuracies in the manufacture and assembly of parts of the vibrators and their engines.

2.This mode of synchronous movement of machines must be stable, that is, insensitive to the deviations of machine parameters from their nominal values that are always available in practice, which may involve a mismatch in synchronous movement.

The stability of the synchronous mode is ensured by a sufficiently strong vibrational coupling between the machines, that is, the vibrational interaction of the machines through the foundation compensates for the harmful effect of the variation in the parameters of the machines.

The problem of damping foundation vibrations for two machines that develop unbalanced forces of constant direction (for example, compressors) is described in [4]. In solving this problem, it was assumed that the foundation не oscillations are not interconnected in x and φ coordinates [5,6,7].

This article summarizes the results of solving this problem for a more complex and practically more realistic case, taking into account the relationship of horizontal and rotational vibrations of the foundation installed on viscoelastic supports [8,9,10].

II. PROBLEM STATEMENT AND SOLUTION METHODS

Let two identical machines with progressively moving unbalanced masses according to a harmonic law be mounted on a common foundation, which rests on an elastic foundation and can perform plane motion (Fig. 1). The action lines of unbalanced forces do not pass through the center of gravity of the foundation with O1 machines. Machines are driven by an asynchronous motor, so that the vibrations of unbalanced masses can be shifted in phase. Differential equations of small vibrations of the foundation under the action of unbalanced forces developed by machines have the form

$$\begin{aligned} &M\ddot{x} + \tilde{c}_x x - \tilde{c}_{x\varphi}\varphi = (A_1 e^{i\varphi_1} + A_2 e^{i\varphi_2})e^{i\omega t}, \\ &M\ddot{y} = 0, \qquad\qquad (1) \\ &M\ddot{\varphi} - \tilde{c}_{x\varphi}x + (\tilde{c}_{x\varphi}h_1 + \tilde{c}_{\varphi})\varphi = h(A_1 e^{i\varphi_1} + A_2 e^{i\varphi_2})e^{i\omega t}, \end{aligned}$$

where

Received: 27 Feb 2019 | Revised: 20 Mar 2019 | Accepted: 30 Apr 2020

$$\tilde{c}_x \ f(t) = c_{x0}\left[f(t) - \int\limits_{-\infty}^{t} R_x(t-\tau) f(\tau) d\tau \right];$$

$$\tilde{c}_{x\varphi} \ f(t) = c_{x\varphi 0}\left[f(t) - \int\limits_{-\infty}^{t} R_{x\varphi}(t-\tau) f(\tau) d\tau \right],$$

$$\tilde{c}_{\varphi} \ f(t) = c_{\varphi 0}\left[f(t) - \int\limits_{-\infty}^{t} R_{\varphi}(t-\tau) f(\tau) d\tau \right].$$

Here $R_x(t-\tau)$ $R_x(t-\tau), R_{x\varphi}(t-\tau), R_{\varphi}(t-\tau)$ - relaxation kernels respectively

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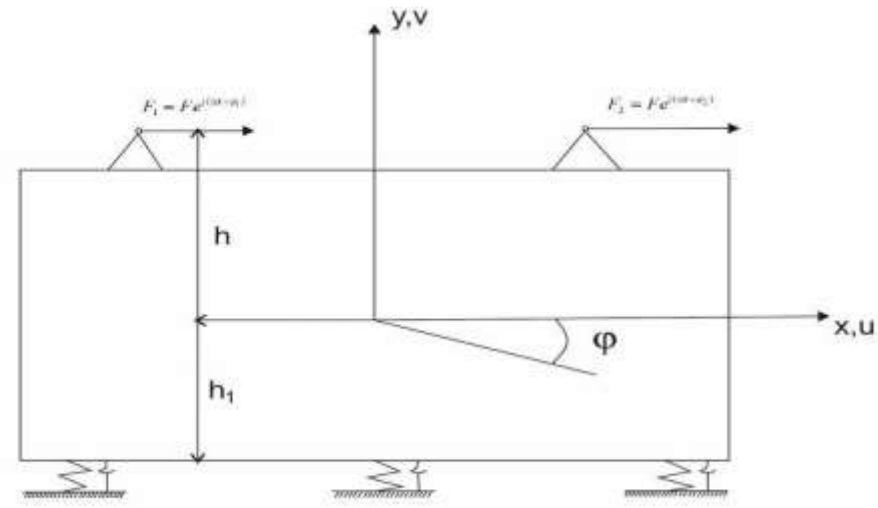


Figure 1: Design scheme

Here$x$and$y$ - coordinates of the center of gravity O_1 foundation with machines in a fixed coordinate systemxOy,coinciding in the position of static equilibrium with the axis systemuO_1v, rigidly connected to the foundation; φ is the angle of rotation of the foundation, measured along the clock;$M$and$I$ - accordingly, the mass and central moment of inertia of the foundation with the machines; $\tilde{c}_x$ and $\tilde{c}_y$ - operator coefficients characterizing the rigidity of the base; $\tilde{c}_{x\varphi} = \tilde{c}_x h_1$ -operator coefficient characterizing the relationship of oscillations in coordinates x and φ, h_1 - distance from the bottom edge of the foundation to the center of gravity O_1 .

Solution of a system of integro-differential equations (1) corresponding to forced frequency oscillations ω, has the form

$$\begin{pmatrix} x(t) \\ y(t) \\ \varphi(t) \end{pmatrix} = \begin{pmatrix} A \\ Y_0 \\ \Phi_0 \end{pmatrix} e^{i\omega t}, \qquad (2)$$

Where A, Y_0, Φ_0 - the amplitude is mixed (complex value), $\omega = \omega_R + i\omega_I$.It is assumed that the movement of the system occurs far from resonances, that is, the frequencies of the forced vibrations of the body are sufficiently different from the frequencies of the main vibrations.Substituting (2) into (1), then we obtain the following expression of the displacement amplitude

$$\Phi_0=\left[\frac{\bar{c}_{x\varphi}h+\bar{c}_{\varphi}-\omega^2}{\bar{c}_{x\varphi}}-\frac{\bar{c}_{x\varphi}}{\bar{c}_x-\omega^2}\right]^{-1}\left[\frac{\bar{c}_{x\varphi}+\bar{c}_x h-\omega^2 h}{\bar{c}_{x\varphi}(\bar{c}_x-\omega^2)}\right](A_1e^{i\varphi_1}+A_2e^{i\varphi_2}) \ ,$$

$$A=(A_1e^{i\varphi_1}+A_2e^{i\varphi_2})\left(\left[\frac{\bar{c}_{x\varphi}h+\bar{c}_{\varphi}-\omega^2}{\bar{c}_{x\varphi}}\left[\frac{\bar{c}_{x\varphi}h+\bar{c}_{\varphi}-\omega^2}{\bar{c}_{x\varphi}}-\frac{\bar{c}_{x\varphi}}{\bar{c}_x-\omega^2}\right]^{-1}\left[\frac{\bar{c}_{x\varphi}+\bar{c}_x h-\omega^2 h}{\bar{c}_{x\varphi}(\bar{c}_x-\omega^2)}\right]\right]-\frac{h}{\bar{c}_{x\varphi}}\right) \quad (3)$$

$Y_0=0$.

Here A_1 and A_2 - the amplitude of the displacement of external vibrational loads, a φ_1 and φ_2 relative phase shift of the disturbing forces of the vibrators; h - distance from the center of gravity of the foundation O_1 to the line of action of the disturbing forces of the vibrators. After simple transformations of body movements under the influence of external loads (2) and (3) takes the following form:

$$x=\frac{F}{M\omega^2}\frac{\lambda_{x\varphi}^2+\frac{hh_1}{\eta^2}\lambda_x^2-1}{(\lambda_x^2-1)(\lambda_{x\varphi}^2-1)-\lambda_x^4h_1^2/\eta^2}e^{i\omega t},$$
$$y=0, \quad (4)$$
$$x=\frac{Fh}{I\omega^2}\frac{\lambda_{x\varphi}^2+\frac{h_1}{h}\lambda_x^2-1}{(\lambda_x^2-1)(\lambda_{x\varphi}^2-1)-\lambda_x^4h_1/\eta^2}e^{i\omega t},$$

where indicated

$$\lambda_x^2=\frac{1}{\omega^2}\frac{c_{x0}}{M}(1-\Gamma_x^c-i\Gamma_x^s),$$
$$\lambda_{x\varphi}^2=\frac{1}{\omega^2}\frac{c_{x\varphi0}(1-\Gamma_{x\varphi}^c-i\Gamma_{x\varphi}^s)h_1+c_{\varphi0}(1-\Gamma_{\varphi}^c-i\Gamma_{\varphi}^s)}{I}.$$

Here

$$\Gamma_x{}^C(\omega_R)=\int_0^\infty R_x(\tau)\cos\omega_R\tau\,d\tau,\ \Gamma_x{}^S(\omega_R)=\int_0^\infty R_x(\tau)\sin\omega_R\tau\,d\tau,$$
$$\Gamma_{x\varphi}{}^C(\omega_R)=\int_0^\infty R_{x\varphi}(\tau)\cos\omega_R\tau\,d\tau,\ \Gamma_{x\varphi}{}^S(\omega_R)=\int_0^\infty R_{x\varphi}(\tau)\sin\omega_R\tau\,d\tau.$$

To establish the nature of stable synchronous movements of machines, the so-called integral criterion for the stability of synchronous movements, proposed in [11, 12], is used. According to this criterion, a stable synchronous movement of the same or almost the same vibrators corresponds to such a combination of the rotation phases of the machine shafts, for which the average value of the

Lagrange function corresponding to the bodies over the period which machines are installed, has a minimum [13]

$$\Delta=(\omega/(2\pi))\int_0^{2\pi/\omega}[L]dt=(\omega/(2\pi))\int_0^{2\pi/\omega}[T-\Pi]dt= \tag{5}$$
$$=\ (\omega/(2\pi))\int_0^{2\pi/\omega}\left\{\left[\frac{1}{2}M\,\dot{x}^2+\frac{1}{2}I\dot{\varphi}^2\right]-\left[\frac{1}{2}\widetilde{c}_x(x-h_1\varphi)^2+\frac{1}{2}\widetilde{c}_\varphi\varphi^2\right]\right\}dt$$

If we use (4) and (5) we obtain the following expression of the Lagrange function

$$\Delta=-\frac{F^2}{2M\omega^2}\frac{1}{((\lambda_x^2-1)(\lambda_{x\varphi}^2-1)-\lambda_x^4\eta^2\xi^2)^2}((\lambda_{x\varphi}^2+\lambda_x^2\eta^2\xi-1)(\lambda_x^2-2\lambda_x^2\eta^2\xi\frac{\lambda_x^2+\lambda_x^2\xi-1}{\lambda_{x\varphi}^2+\lambda_x^2\eta^2\xi-1}-1)+ \tag{6}$$
$$+\eta^2(\lambda_x^2+\lambda_x^2\xi-1)(\lambda_{x\varphi}^2+\lambda_x^2\eta^2\xi-1))cos\alpha+C,$$

Here Δ average for the period $2\pi/\omega$ the value of the Lagrange function $L=T-\Pi$ (T - kinetic, Π-potential energy);

$$\eta^2=\frac{h_1^2}{\rho},\zeta=\frac{h_1}{h};$$

$\rho=\sqrt{\dfrac{I}{M}}$ - base inertia radius with machines;C - constant independent of.

When the condition is met

$$\left[\left(\lambda_\varphi^2-1+\lambda_x^2\eta^2\zeta\right)^2\left(\lambda_x^2-1-2\lambda_x^2\eta^2\zeta\frac{\lambda_x^2-1+\lambda_x^2\zeta}{\lambda_\varphi^2-1+\lambda_x^2\eta^2\zeta}\right)+\right. \tag{4}$$
$$\left.+\eta^2\left(\lambda_x^2-1+\lambda_x^2\zeta\right)\left(\lambda_\varphi^2-1\right)\right]<0$$

Function Δ at the point $\alpha=\pi$ has a minimum, and at the point $\alpha=0$ - the maximum, that is, in accordance with the integral criterion, the antiphase motion is stable, and the in-phase motion is unstable [14,15,16].

III. RESULTS OF CALCULATIONS AND DISCUSSION

In the calculations, the three-parameter Rizhanitsen – Koltunov core was used as the relaxation core $R(t)=\frac{Ae^{-\beta t}}{t^{1-\alpha}}$. Here A,α,β - parameters of the relaxation core [17], which takes the following value $A=0{,}048;\quad \beta=0{,}05;\quad \alpha=0{,}1$.As follows from Fig. 2, and also from equality (2), with the antiphase motion of the vibrators ($\alpha=\pi$), there are no base vibrations.

Thus, the fulfillment of inequality (4) is a condition for the presence of a favorable situation when the machines exhibit a tendency toward mutual balancing.On drawing. 3, 4, 5 areas of stability and instability of antiphase motion are depicted for various parameter values η^2 and ξ. Areas of stability are shaded.For comparison on drawing.2 the same areas are shown for various parameter values η^2 when (see above), when horizontal and rotational vibrations are not interconnected, that is, the coefficient $\widetilde{c}_{x\varphi}$ taken equal to zero.The areas of stability are shown in the figure. 2 by shading the boundaries within the stability regions.As follows from picture. 2, at $\lambda_x^2 < 1$ and $\lambda_{x\varphi}^2 < 1$, that is, when the frequencies of natural vibrations of the foundation are less than the frequencies of forced vibrations (in the after resonance region), the antiphase motion is always stable.Thus, the case of “soft” setting of machines always leads to a favorable situation.At $\lambda_x^2 > 1$ and $\lambda_{x\varphi}^2 > 1$ (in the pre-resonance region) the antiphase motion of the vibrators is unstable [18,19,20].

In the case under consideration, the stability regions are substantially changed, but the main regularity of the previous case is preserved here.

On the picture. 3, 4, 5 dashed lines plot the resonance curves, where the frequency of the forced oscillations coincides with one of the frequencies of the main oscillations. The resonance curves coincide with the boundaries of the regions of stability and instability, which is also characteristic of the previous case.

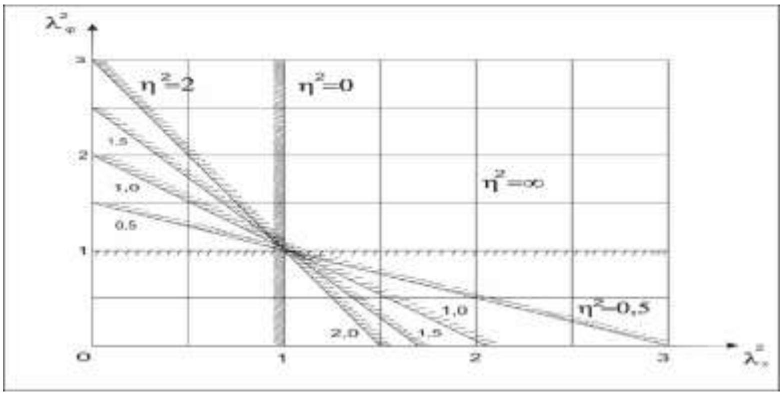
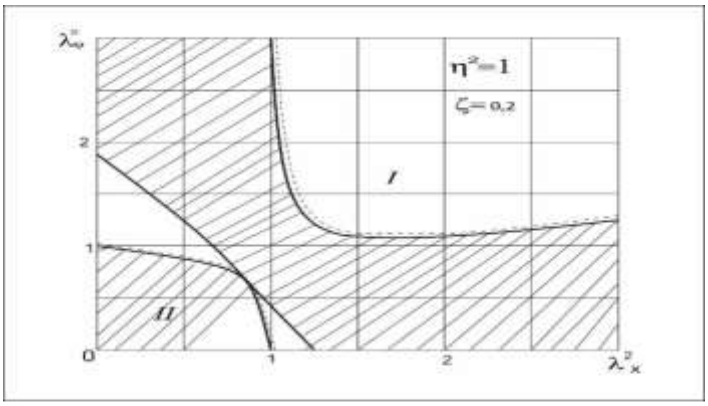


Figure 2: The real parts of the frequency of the natural vibrations of the foundation are less than the frequencies of the forced vibrations (in the after resonance region) ($\lambda_x^2 < 1$ and $\lambda_{x\varphi}^2 < 1$).

In the resonance region, indicated byI, as before, the ant phase motion is unstable, and in the after resonance region, indicated by a number II, antiphase motion is stable, that is, the “soft” setting of machines still leads to mutual balancing of their dynamic effects.



Received: 27 Feb 2019 | Revised: 20 Mar 2019 | Accepted: 30 Apr 2020

Figure 3: The frequency of the forced vibrations matches the real parts with one of the frequencies of the main vibrations.

Due to inaccuracies in manufacturing and assembly, partial speeds of machines ω_1 and ω_2 may differ from one another. Stable stationary values of phase shifts in this case will differ from π, so that the effects of machine imbalances are not completely, but only partially mutually compensated [21,22].

The new values of the phase shifts are determined from equation [23]

$$\sin\alpha = \frac{q_\omega}{p_\omega} \quad (5)$$

Marked here

$q_\omega = \dfrac{\Delta\omega}{2\omega_0} = \dfrac{\omega_1 - \omega_2}{2\omega_0}$ -Relative dispersion of partial speeds of vibrators; ω_0 -nominal value of the angular velocity of the vibrators;

$p_\omega = \dfrac{W_0}{\omega_0(z_L + z_R)}$ -maximum value of relative deviation of partial angular velocities q_w, in which self-synchronization of machines is still possible.

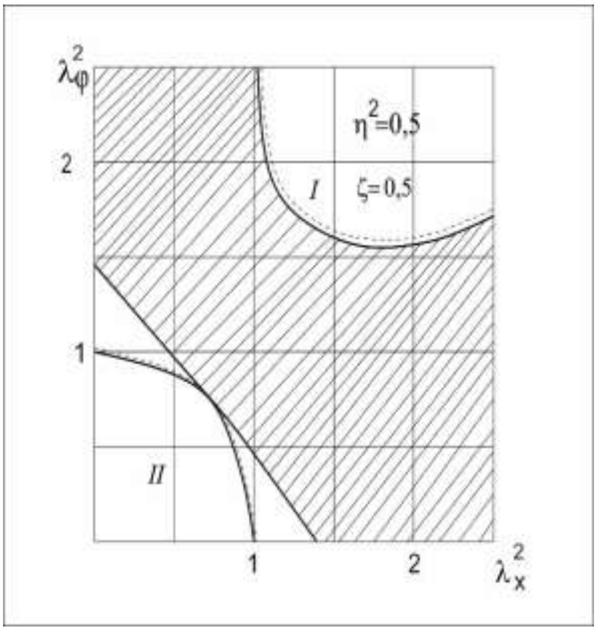


Figure 4: The frequency of the forced vibrations matches the real parts with one of the frequencies of the main vibrations

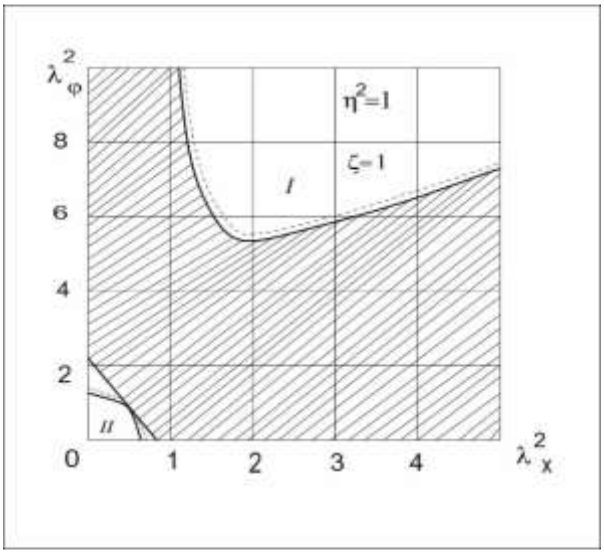


Figure 5 : The frequency of the forced vibrations matches the real parts with one of the frequencies of the main vibrations

W_0- the maximum possible value of vibration moments W_s, reflecting the inverse effect of foundation vibrations on the movement of the rotors of the vibrators. The presence of vibrational moment's leads to equalization of the angular velocities of the machines;$z_L$and$z_R$constants determined by the catalog data of engines.It was shown in [24] that the relations

$$W_s = \frac{\partial \Delta}{\partial \alpha_s} \quad (6)$$

Using (3) and (6), we obtain the following expression:

$$W_0 = \frac{(m\varepsilon)^2 \omega_0^2}{2M} \frac{1}{\left[\left(\lambda_x^2 - 1\right)\left(\lambda_\varphi^2 - 1\right) - \lambda_x^4 \eta^2 \zeta^2\right]^2} \times$$
$$\times \left[\left(\lambda_\varphi^2 - 1 + \lambda_x^2 \eta^2 \zeta\right)^2 \left(\lambda_x^2 - 1 - 2\lambda_x^2 \eta^2 \zeta \frac{\lambda_x^2 - 1 + \lambda_x^2 \zeta}{\lambda_\varphi^2 - 1 + \lambda_x^2 \eta^2 \zeta}\right) + \right. \quad (7)$$
$$\left. + \eta^2 \left(\lambda_x^2 - 1 + \lambda_x^2 \zeta\right)^2 \left(\lambda_\varphi^2 - 1\right)\right]$$

When inequality

$$\frac{|q_\omega|}{p_\omega} < 1 \quad (8)$$

уравнение (5) имеетвещественныерешенияотносительно equation (5) has real solutions with respect to α.

Thus, the fulfillment of inequality (8) is a condition for the onset of self-synchronization. With sufficient maximum possible values of the vibrational moments, self-synchronization can take place even in the case when the partial angular velocities differ significantly from one another.

Knowing the law of distribution of a random variable q_ω, we can calculate the probability of inequality (8), i.e., the probability of the presence of self-synchronization. It is natural to assume that the quantity q_{ω} distributed according to normal law with an average value of zero. Standard deviation σ_{ω}, in view of the lack of accurate data, one can take equal to a third of the maximum possible deviation of the partial velocities.

$$\sigma_{\omega} = \frac{1}{3}|q_{\omega}|_{\max} \quad (9)$$

Magnitude $|q_{\omega}|_{\max}$ usually not difficult to evaluate in each case [25]. Under the assumptions made, the probability of fulfilling the self-synchronization condition will be

$$P_{\omega} = P_{\omega}(|q_{\omega}| < p_{\omega}) = P_{\omega}(|q_{\omega}| < 3\sigma_{\omega}\chi_{\omega}) = \Phi(3\chi_{\omega}) \quad (10)$$

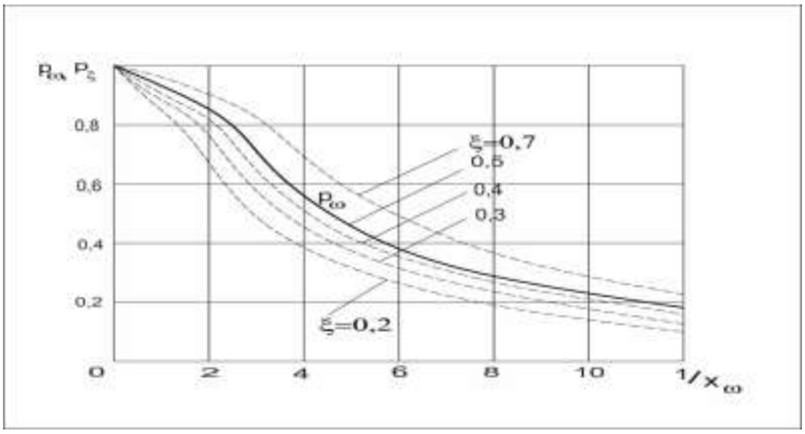


Figure 6: Dependencies P_{ω} from $1/\chi_{\omega}$ at various ξ

Here Φ (x) —Gaussian probability integral;

$\chi_{\omega} = \frac{P_{\omega}}{|q_{\omega}|_{\max}} = \frac{P_{\omega}}{3\sigma_{\omega}}$ -safety factor for self-synchronization.

Dependency graph P_{ω} from $1/\chi_{\omega}$ presented in fig. 6 solid line. From the consideration of the graph it can be seen that with $1/\chi_{\omega} \le 2$, i.e., when $\chi_{\omega} \ge 1/2$ probability of self-synchronization $P_{\omega} \ge 0.9$.

FunctionCharts $P_{\xi} = \Phi(6\xi\chi_{\omega})$ are presented in fig. 6 dashed lines.

Further probabilistic calculation leads to a relation linking the probability of not exceeding the maximum amplitude of the basement oscillations of a certain level with the value of the self-synchronization safety factor χ_{ω}

$$P_{\xi} = P_{\xi}\left(\frac{A_{\max}}{A^{*}} \le \xi\right) = P_{\xi}\left(\frac{1}{2}\left|\frac{1}{2}\frac{\Delta\omega}{\omega_0}\frac{1}{P_{\omega}}\right| \le \xi\right) = $$
$$= P_{\xi}\left(\left|\frac{1}{2}\frac{\Delta\omega}{\omega_0}\right| \le 2p_{\omega}\xi\right) = \Phi\left(\frac{2p_{\omega}\xi}{\sigma_{\omega}}\right) = \Phi(6\xi\chi_{\omega}) \quad (11)$$

where $\Phi(x)$ -Gaussian probability integral; A_{max} -maximum amplitude of foundation vibrations in the presence of self-synchronization; A^*-maximum amplitude of foundation vibrations in the absence of self-synchronization; ξ-set share from A^*.

From their consideration, in particular, it follows that for $1/\chi_\omega = 1/2$, i.e., when $\chi_\omega = 2$ probability of not exceeding $A_{max}$the values 0,2$A^*$is 0.9, in other words, the amplitude of the foundation vibrations decreases by five times compared with the amplitude of oscillations in the presence of self-synchronization.

IV. CONCLUSIONS

1.A mathematical formulation and methods for solving the problem of dynamic stability of a viscoelastic mechanical system with a finite number of degrees of freedom have been developed. Comparison of numerical values of calculations with known results.

2**.**Taking into account the viscous properties of the material has a noticeable effect on the areas of dynamic stability. Viscous properties play a stabilizing factor for parametric vibrations of mechanical systems. Geometrically, the sizes of the regions of dynamic instability corresponding to the main parametric resonances are reduced and shifted above the abscissa axis.

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