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Structural and Kinematic Analysis of Gear and Lever Differential Mechanisms by Symmetric Movement of Rotation Centers for Driving and Slave Gear Wheels

Sh.P. Alimukhamedov, A. Abdukarimov, H.A. Sharifhodjaeva and K.J. Rustamov

Abstract--- *The article presents a methodology for structural and kinematic analysis of gear-lever differential mechanisms by symmetric movement of the centers of rotation of the driving and driven gears with parallel contours. The degree of mobility of the differential mechanism is determined taking into account excess connections. Expressions are derived for analogues of angular velocities and angular accelerations of links of the gear mechanism, as well as linear velocities and accelerations of the linkage.*

Keywords--- *Mechanism, Gear-lever, Differential, Structure, Kinematics, Degree of Mobility, Speed, Acceleration, An Analog of Speed, Analog of Acceleration.*

I. INTRODUCTION

In recent years, in our country and abroad, much attention has been paid to the theory and practice of the use of gear lever mechanisms. This is due to the fact that with the help of gear-lever mechanisms, various and complex laws of link movement can be implemented.

Figure 1 shows the kinematic diagram of a differential gear mechanism, consisting of a hinge and gear mechanisms. This mechanism allows you to change the center distance AE by moving the link VD along the guides in the horizontal direction.

The degree of mobility of the lever mechanism is W = 1, then the number of excess bonds is determined by the formula. (1)

Where n is the number of movable links;

P4, P5 - the number of kinematic pairs of the fourth and fifth grades.

In our case, n = 5, P5 = 8, P4 = 0, q = 2, therefore, in the linkage mechanism there are two redundant bonds.

The degree of mobility of the gear mechanism at n = 7, P5 = 8, P4 = 3 is equal to W = 2.

Sh.P. Alimukhamedov, Professor, Department of Allied Mechanics, Assistant Professor, The Tashkent Institute of Projection, Building and Maintenance of Automobile Roads, Tashkent, Uzbekistan. E-mail: instmech@academy.uz

A. Abdumalikov, Senior Researcher, Candidate of Engineering Sciences, Head of Laboratory TMM of Institute of Mechanics and Earthquake Engineering, Academy of Sciences of the Republic of Uzbekistan, Tashkent, Uzbekistan. E-mail: aabdusalam54@gmail.com

H.A. Sharifhodjaeva, Assistant Professor, Docent of the Department of Allied Mechanics, The Tashkent Institute of Projection, Building and Maintenance of Automobile Roads, Tashkent, Uzbekistan.

K.J. Rustamov, Assistant Professor, Department of Allied Mechanics, The Tashkent Institute of Projection, Building and Maintenance of Automobile Roads, Tashkent, Uzbekistan. E-mail: k.j.rustamov82@mail.ru

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The degree of mobility of the gear-lever mechanism, taking into account the number of excess bonds q = 2, n = 9, P5 = 12, P4 = 3, is determined by the formula (1), W = 2.

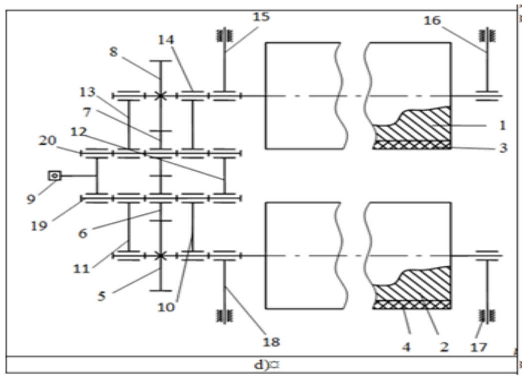


Fig. 1: Gear-lever differential gear (ZRDPM): d) 1,2-working shafts; 3,4-coating; 5,6,7,8 gears; 9-cross 10,11,12,13,14-levers; 15,16,17,18 guides; 19.20-axis.

II. METHODOLOGY

To determine the analogues in the angular velocities of the driven wheel z_8 of the differential mechanism composed of wheels 7 and 9, carrier 1, using the reverse movement method, we write the gear ratio i_{98} (Fig. 2)

$$i_{98}^{(1)} = \frac{\omega_9 - \omega_1}{\omega_8 - \omega_1} = -\frac{r_8}{r_9} \quad (2)$$

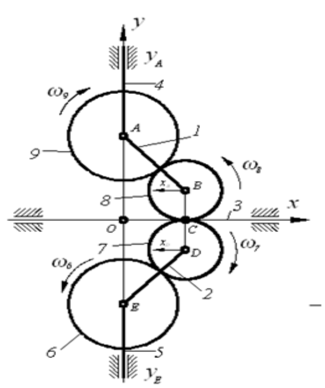
Where the angular speed 8 of the driven wheel is equal to

$$\omega_8 = (1 + \frac{r_9}{r_8})\omega_1 - \frac{r_9}{r_8}\omega_9 \quad (3)$$

Where ω_1 is the angular speed of the connecting rod 1;

ω_9-angular speed of the leading link 6;

r_8, r_9 are the radii of the pitch circles of the gears 8 and 9, respectively.



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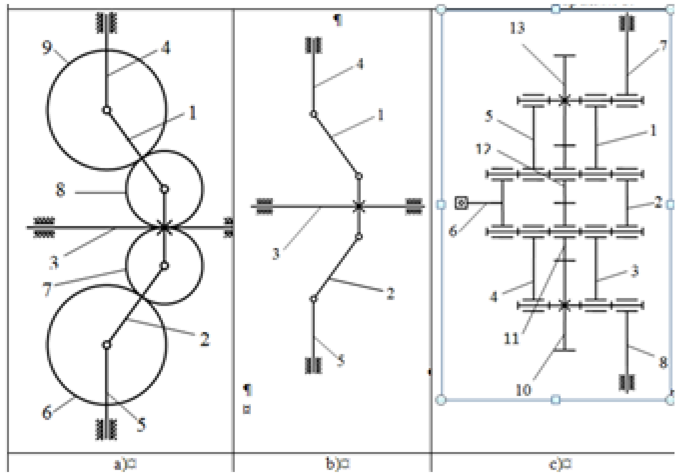

Fig. 2: The design scheme for the determined analogues of angular velocities differential mechanism

III. LITERATURE SURVEY

Structural and kinematic analysis of linkages made a significant contribution to the fetus [3].

Theories of differential gear mechanisms are devoted to the work [1] of the problem of structural and kinematic analysis of gear-lever differential mechanisms solved in [1], [2], [3].

However, in solving the problem of kinematic analysis of gear-lever differential mechanisms with parallel contours with the symmetric movement of the centers of rotation of the driving and driven gears there is no single approach. For a particular such mechanism, one has to look for an individual approach to solve the problem [4], [5], [6].

IV. EXPERIMENT AND DISCUSSION

Dividing the right and left parts (3) by the angular velocity of the reduction link 9, we obtain an expression for the analog of the angular velocity of the wheel z_8.

$$\frac{\omega_8}{\omega_9} = (1+\frac{z_9}{z_8})\frac{\omega_1}{\omega_9} - \frac{r_9}{r_8} \qquad (4)$$

Where $\frac{\omega_1}{\omega_9}$ is the analog of the angular velocity of link 1 of four OAVSD links.

Introducing the four-link as a vector outline

$$y_A + l_1 + r_8 = \overline{X}_B \qquad (5)$$

We find the projections of the vector equation (5) on the x and y coordinate axes.

$$\begin{cases} x_B = l_1 \cos\varphi_1 \\ y_A = l_1 \sin\varphi_1 + r_8 \end{cases} \qquad (6)$$

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We differentiate along the generalized coordinate φ9 (6) and, taking into account that $\frac{d^2\varphi_1}{d\varphi_9} = \frac{\omega_1}{\omega_9}$, $\frac{dx_B}{d\varphi_9} = \frac{\upsilon_B}{\omega_9}$, $\frac{dy_A}{d\varphi_9} = \frac{\upsilon_B}{\omega_9}$, we get

$$\begin{cases} \frac{V_B}{\omega_9} = -\frac{\omega_1}{\omega_9} l_1 \sin\varphi_1 \\ \frac{V_A}{\omega_9} = \frac{\omega_1}{\omega_9} l_1 \cos\varphi_1 \end{cases} \qquad (7)$$

where ω_1 is the angular velocity of the link 1.

The angular velocity ω_1 is determined by the formula.

$$\omega_1 = \frac{|V_A - V_B|}{r_9 + r_8}, \qquad (8)$$

Substituting the values of VB and VA from (7) into (8), we obtain:

$$\omega_1 = (\sin\varphi_1 + \cos\varphi_1)\omega_9, \qquad (9)$$

From expression (9) we find an analog of the angular velocity of link 1 (Fig. 3).

$$\frac{\omega_1}{\omega_9} = \sin\varphi_1 - \cos\varphi_1, \qquad (10)$$

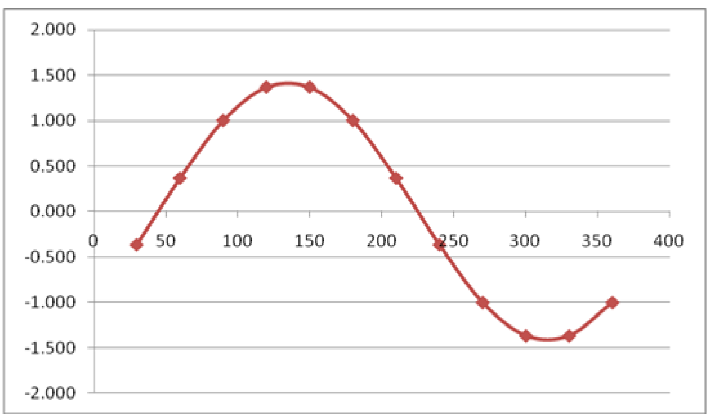

Fig. 3: The curve of analogues of the angular velocity of the link 1

Substituting the value $\frac{\omega_1}{\omega_9}$ in (4), we obtain an analog of the angular velocity of the driven link 8.

$$\frac{\omega_8}{\omega_9} = (1 + \frac{r_9}{r_8})(\sin\varphi_1 - \cos\varphi_1) - \frac{r_9}{r_8} \qquad (11)$$

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Differentiating expression (3) with respect to time, we obtain

$$\frac{d\omega_8}{dt} = (1+\frac{r_9}{r_8})\frac{d\omega_1}{dt} - \frac{r_9}{r_8}\frac{d\omega_9}{dt} \tag{12}$$

Marking through $\varepsilon_8 = \frac{d\omega_8}{dt}$, $\varepsilon_1 = \frac{d\omega_1}{dt}$ and $\varepsilon_9 = \frac{d\omega_9}{dt}$ we get

$$\varepsilon_8 = (1+\frac{r_9}{r_8})\varepsilon_1 - \frac{r_9}{r_8}\varepsilon_9 \tag{13}$$

Dividing both sides of equation ω_9 (13) by, we obtain the following expression for the analog of the angular acceleration of the wheel Z_8,

$$\frac{\varepsilon_8}{\omega_9{}^2} = (1+\frac{r_9}{r_8})\frac{\varepsilon_1}{\omega_9{}^2} - \frac{r_9}{r_8}\frac{\varepsilon_9}{\omega_9{}^2} \tag{14}$$

where $\frac{\varepsilon_1}{\omega_9{}^2}$ and $\frac{\varepsilon_9}{\omega_9{}^2}$ -analogs of angular accelerations of links 1 and 9

V. EXPERIMENTAL RESULTS

To determine the analogs of angular accelerations $\frac{\varepsilon_1}{\omega_9{}^2}$ and $\frac{\varepsilon_9}{\omega_9{}^2}$ twice, we differentiate equations (6).

$$\begin{cases} \frac{d^2 x_B}{d\varphi_9^2} = l_1 \cos\varphi_1 (\frac{d\varphi_1}{d\varphi_9})^2 + l_1 \sin\varphi_1 \frac{d^2\varphi_1}{d\varphi_9^2} \\ \frac{d^2 x_A}{d\varphi_9^2} = -l_1 \sin\varphi_1 (\frac{d\varphi_1}{d\varphi_9})^2 + l_1 \cos\varphi_1 \frac{d^2\varphi_1}{d\varphi_9^2} \end{cases} \tag{15}$$

Given that $\frac{d^2 x_B}{d\varphi_9^2} = \frac{a_B}{\omega_9^2}$, $\frac{d^2 x_A}{d\varphi_9^2} = \frac{a_A}{\omega_9^2}$,

$\frac{d^2\varphi_1}{d\varphi_9^2} = \frac{\varepsilon_1}{\omega_9^2}$ and $\frac{d\ \varphi_1}{d\varphi_9} = \frac{\omega_1}{\omega_9}$, we get

$$\begin{cases} \frac{a_B}{\omega_9^2} = l_1 \cos\varphi_1 (\frac{\omega_1}{\omega_9})^2 + l_1 \sin\varphi_1 \frac{\varepsilon_1}{\varphi_9^2} \\ \frac{a_A}{\omega_9^2} = -l_1 \sin\varphi_1 (\frac{\omega_1}{\omega_9})^2 + l_1 \cos\varphi_1 \frac{\varepsilon_1}{\varphi_9^2} \end{cases} \tag{16}$$

From the first equation (16) we determine the analog of the angular acceleration of link 1

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$$\frac{\varepsilon_1}{\omega_9^2} = \frac{a_B}{\omega_9^2}\frac{1}{l_1 \sin\varphi_1} - \frac{\cos\varphi_1}{\sin\varphi_1}\left(\frac{\omega_1}{\omega_9}\right)^2 \quad (17)$$

Substituting (17) into (14) we obtain an analog of the angular acceleration of the gear

$$\frac{\varepsilon_8}{\omega_9^2} = \left(1+\frac{r_9}{r_8}\right)\left[\frac{a_B}{\omega_9^2} = \frac{1}{l_1 \cos\varphi_1} - tg\varphi_1\left(\frac{\omega_1}{\omega_9}\right)^2\right] \quad (18)$$

The data on this formula are given in table I.

Table I: An analogue of the values of the angular acceleration of the point B and A of the gear depending on the φ angle

φ / cm	30^0	60^0	90^0	120^0	150^0	180^0	210^0	240^0	270^0	300^0	330^0	360^0
$\frac{a_B}{\omega_9^2}$	12,44	19,307	21	17,067	8,56	-2,24	-12,44	-19,31	-21	-17,07	-8,56	2,24
$\frac{a_A}{\omega_9^2}$	17,067	8,56	-2,24	-12,44	-19,31	-21	-17,07	-8,56	2,2399	12,44	19,306	21

The true values of the linear acceleration AB and AA are determined by the method of plans based on vector equations

$$\begin{cases} \bar{a}_A = \bar{a}_B + \bar{a}^n_{A/B} + a^\tau_{A/B} \\ \bar{a}_A = \bar{a}_{A/y-y} \end{cases} \quad (19)$$

$$\begin{cases} \bar{a}_B = \bar{a}_A + \bar{a}^n_{B/A} + \bar{a}^\tau_{B/A} \\ \bar{a}_B = \bar{a}_{B/x-x} \end{cases} \quad (20)$$

Similarly to the above method, we will conduct a kinematic analysis for the differential mechanism composed of gears 6 and 7 and carrier 2, the gear ratio

$$i_{67} = \frac{\omega_6-\omega_2}{\omega_7-\omega_2} = -\frac{r_7}{r_6} \quad (21)$$

$$\omega_7 = \left(1+\frac{r_6}{r_7}\right)\omega_2 - \frac{r_7}{r_6}\omega_6 \quad (22)$$

where are ω_2 the conditions of the connecting rod speed 2;

- $\omega_6$conditions link speed 6;

-$r_6$radius of the motor circumference of the gear 6;

- $r_7$the radius of the motor circumference of the gear 7;

Dividing the right and left parts (22) by the angular velocity of link 6, we obtain an analog of the angular velocity of the gear r_7.

$$\frac{\omega_7}{\omega_6} = \left(1 + \frac{r_6}{r_7}\right)\frac{\omega_2}{\omega_6} - \frac{r_6}{r_7} \quad (23)$$

Where $\frac{\omega_2}{\omega_6}$ an analog of the angular velocity of link 2 of the four-link OEDSO. $\frac{\omega_2}{\omega_6}$

Introducing the four-link as a vector outline

$$\overline{\mathrm{y}}_{\mathrm{E}} + \bar{l}_2 + r_7 = X_D$$

We find the projections of the vector equation (21) on the *x* and *y* axis

$$\begin{cases} \mathrm{X}_D = l_2 cos\varphi_2 \\ Y_E = l_2 sin\varphi_2 + r_7 \end{cases} \quad (25)$$

We differentiate (28) along the generalized coordinate and taking into account that

$$\frac{d\varphi_2}{d\varphi_6} = \frac{\omega_2}{\omega_6}, \qquad \frac{dX_D}{d\varphi_6} = \frac{v_D}{\omega_6}, \qquad \frac{dY_E}{d\varphi_6} = \frac{v_E}{\omega_6},$$

we get

$$\begin{cases} \frac{v_D}{\omega_6} = -\frac{\omega_2}{\omega_6} l_2 sin\varphi_2 \\ \frac{v_E}{\omega_6} = -\frac{\omega_2}{\omega_6} l_2 sin\varphi_2 \end{cases} \quad (26)$$

The angular velocity ω_2 is determined by the formula

$$\omega_2 = \frac{|v_E - v_D|}{r_6 + r_7} \quad (27)$$

Substituting the values of v_D and v_E from (26) into (27) we obtain

$$\omega_2 = (sin\varphi_2 + cos\varphi_2)\omega_6; \ (28)$$

From expression (28) we obtain an analog of the angular velocity of link 2.

$$\frac{\omega_2}{\omega_6} = \sin\varphi_2 + \cos\varphi_2; \quad (29)$$

Substituting the value (29) into (23) we obtain an analog of the angular velocity of link 7.

$$\frac{\omega_7}{\omega_6} = \left(1 + \frac{r_6}{r_7}\right)(sinl_2 + cos\varphi_2) - \frac{r_6}{r_7}; \ (30)$$

We differentiate expression (22) with respect to time, we obtain

$$\frac{d\omega_7}{dt} = \left(1 + \frac{r_6}{r_7}\right)\frac{d\omega_7}{dt} - \frac{r_6}{r_7}\frac{d\omega_7}{dt}; \quad (31)$$

Marking through $\varepsilon_2 \frac{d\omega_7}{dt}$, $\varepsilon_6 \frac{d\omega_7}{dt}$ and $\varepsilon_7 \frac{d\omega_7}{dt}$, we get

$$\varepsilon_7 = \left(1 + \frac{r_6}{r_7}\right)\varepsilon_2 - \frac{r_6}{r_7}\varepsilon_6; \quad (32)$$

Having spilled both sides of ω_6^2equation (32) on, we write the following expression of the analogue of the carbohydrate acceleration of the gear r_7

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$$\frac{\varepsilon_7}{\omega_6^2} = \left(1 + \frac{r_6}{r_7}\right)\frac{\varepsilon_7}{\omega_6^2} - \frac{r_6}{r_7}\frac{\varepsilon_7}{\omega_6^2}, \qquad (33)$$

where $\frac{\varepsilon_2}{\omega_6^2}$and $\frac{\varepsilon_6}{\omega_6^2}$ are the analogues of the angular accelerations of links 2 and 6.

We find analogues of linear accelerations of points D and E of link 2, differentiating equations (26) twice

$$\begin{cases} \frac{d^2X_D}{d\varphi_6^2} = -l_2 cos\varphi_2 \left(\frac{d\varphi_2}{d\varphi_6}\right)^2 - l_2 sin\varphi_2 \frac{d^2\varphi_2}{d\varphi_6^2}; \\ \frac{d^2\mathrm{y_E}}{d\varphi_6^2} = -l_2 sin\varphi_2 \left(\frac{d\varphi_2}{d\varphi_6}\right)^2 + l_2 sin\varphi_2 \frac{d^2\varphi_2}{d\varphi_6^2} \end{cases} (31)$$

Taking into account that

$$\frac{d^2X_D}{d\varphi_6^2} = \frac{\mathrm{a}_D}{\omega_6^2}; \frac{d^2\mathrm{y_E}}{d\varphi_6^2} = \frac{\mathrm{a_E}}{\omega_6^2}; \frac{d^2\varphi_2}{d\varphi_6^2} = \frac{\varepsilon_2}{\omega_6^2} \text{ and } \frac{d^2\varphi_2}{d\varphi_6^2} = \frac{\omega_2}{\omega_6}, \text{ we get}$$

$$\begin{cases} \frac{\mathrm{a}_D}{\omega_6^2} = -l_2 cos\varphi_2 \left(\frac{\omega_2}{\omega_6}\right)^2 - l_2 sin\varphi_2 \frac{\varepsilon_2}{\omega_6^2}; \\ \frac{\mathrm{a_E}}{\omega_6^2} = -l_2 sin\varphi_2 \left(\frac{\omega_2}{\omega_6}\right)^2 + l_2 cos\varphi_2 \frac{\varepsilon_2}{\omega_6^2} \end{cases} (35)$$

The data on this formula are given in table II.

Table II: Position of point D and E in φ angle dependent

φ / cm	30^0	60^0	90^0	120^0	150^0	180^0	210^0	240^0	270^0	300^0	330^0	360^0
$\frac{\mathrm{a}_D}{\omega_6^2}$	-12,44	-19,31	-21	-17,07	-8,56	2,2399	12,44	19,307	21	17,067	8,5602	-2,24
$\frac{\mathrm{a_E}}{\omega_6^2}$	17,067	8,56	-2,24	-12,44	-19,31	-21	-17,07	-8,56	2,2399	12,44	19,306	21

From the first equation (35) we determine the analog of the angular acceleration of link 2.

$$\frac{\varepsilon_2}{\omega_6^2} = -\frac{\mathrm{a}_D}{\omega_6^2}\frac{1}{l_2 sin\, l_2} - \frac{cos\,\varphi_2}{sin\,\varphi_2}\left(\frac{\omega_2}{\omega_6}\right)^2 \ (36)$$

Substituting (36) into (33) we obtain an analog of the angular acceleration of the gear r_7

$$\frac{\varepsilon_7}{\omega_6^2} = -\left(1 + \frac{r_6}{r_7}\right)\left[-\frac{\mathrm{a}_D}{\omega_6^2}\frac{1}{l_2 sin\, l_2} - tg\varphi_2\left(\frac{\omega_2}{\omega_6}\right)^2\right] \qquad (37)$$

The values of linear and define vector equations

$$\begin{cases} \bar{\mathrm{a}}_E = \bar{\mathrm{a}}_D + \bar{\mathrm{a}}_{E/D} + \bar{\mathrm{a}}_{\frac{E}{D}} \\ \bar{\mathrm{a}}_E = \bar{\mathrm{a}}_{\frac{E}{y}-y} \end{cases} (38)$$

$$\begin{cases} \bar{\mathrm{a}}_D = \bar{\mathrm{a}}_E + \bar{\mathrm{a}}_{D/E} + \bar{\mathrm{a}}_{\frac{E}{D}} \\ \bar{\mathrm{a}}_D = \bar{\mathrm{a}}_{\frac{D}{x}-x} \end{cases} (39)$$

VI. CONCLUSION

Thus, on the basis of the proposed methodology, analytical and vector equations, structural and kinematic analysis of, for example, the gear-lever differential mechanism was carried out. The dependences of the analogues of the angular velocities of link 1 on the angle of rotation of link 9 are obtained, it can be seen from the graph that the

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change in the analog of the angular velocity of link 1 is periodic.

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