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FUZZY PORTFOLIO OPTIMIZATION USING QUADRATIC PROGRAMMING

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Abstract--- This paper presents the arrangement of fuzzy portfolio advancement utilizing Lagarange multiplier technique. The definition of nonlinear improvement model for portfolio advancement issue and its answer are given. As information for portfolio improvement the recorded returns of benefits and estimations of expected returns are taken. At the point when portfolio improvement issues are portrayed with vulnerability, adaptability in issue imperatives, the fuzzy sets are appropriate portrayal for demonstrating of this kind of enhancement issue. The detailing of fluffy portfolio advancement issue and its answer are given. The α level method is utilized to characterize vitality work for fuzzy portfolio and find ideal fuzzy estimations of the protections.

Keywords--- *Portfolio optimization, quadratic programming, fuzzy optimization, fuzzy portfolio selection.*

I. INTRODUCTION

In portfolio streamlining the normal return and the hazard estimated by the change are the two fundamental parts of a portfolio. The finding ideal estimations of these boundaries, for example, exceptional yield and least hazard are fundamental issue of portfolio streamlining. Sadly resource having exceptional yields typically have high hazard. it's far the point of the portfolio director to discover a portfolio that reinforces ex-pected return below given chance level or a portfolio that limits danger beneath giv-en carry degree back. planning the right association of benefits requires pre-despatched day, amazing and solid numerical gadgets and initiatives.

The numerical definition of portfolio investigation originally by Prof. Harry Markowitz [1]. In this model the degree of hazard limited for given degree of return. This model is called mean-variance model of Markowitz. In light of this model other portfolio choice models were proposed. Some of them are direct programming models [2,3]. The greater part of portfolio advancement models depend on likelihood hypothesis. The likelihood hypothesis is one of principle devices for breaking down vulnerability in fund. Be that as it may, some of vulnerability factors vary from the arbitrary ones. Consequently likelihood hypothesis can't portray vulnerability totally. At the point when portfolio streamlining issues are portrayed by some vulnerability, adaptability in issue imperatives, the fuzzy sets are reasonable portrayal for demonstrating this sort of enhancement issue.

This paper the utilization of quadratic programming for deterministic and fluffy portfolio streamlining have been thought of.

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2. Nonliner Programming Model for Optimization

Nonlinear programming can be seen as enhancement issue. One of class of nonlinear writing computer programs is quadratic programming. to consider the accompanying quadratic programming issue with uniformity and imbalance imperatives

$$f(x) = \sum_{j}^{n} c_j x_j + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} x_i \sigma_{ij} x_j \rightarrow \min \qquad (1)$$

$$\textit{subject to}$$

$$\sum_{j=1}^{n} a_{ij} x_j = b_i, \quad \text{for i} = 1,2,\dots,\text{p}$$

$$\sum_{j=1}^{n} a_{qj} x_j \geq b_k, \quad \text{for q} = p+1, \text{p}+2,\dots,\text{m} \qquad (2)$$

$$\text{x}_1 \geq 0, \text{x}_2 \geq 0,\dots, x_n \geq 0$$

problem isto restrict the target work beneath arrangement of limitations. here limitations are remoted into two subset s. the main subset incorporates of p correspondence impera-tives, and the following subset has m-p disparity obstacles. To tackle this problem the Lagarange multiplier method is applied. From the outset step the affordable power paintings is deliberate. for the reason that fairness and disparity units are disjoint, the power work is deliberate as composite equation [4]. vitality work is constructed with the intention that it punishes every infringement of the balance and imbalance imperatives.

$$E(x) = E_1(x) + E_2(x) + E_3(x) \qquad (3)$$

$$\textit{where,}$$

$$E_1(x) = c^T x + \frac{1}{2}\text{x}^{\text{T}}\sigma\text{x} \qquad (4)$$

$$E_2(x) = \frac{K_1}{2}(A_p x - b_p)^T (A_p x - b_p) + \lambda_p^T (A_p x - b_p) \qquad (5)$$

$$E_3(x) = \frac{K_2}{2}\sum_{i=p+1}^{m}\Phi[r_1(x)] \qquad (6)$$

right here K1, K2, α are fine coefficients. here E1(x) is electricity feature to be mini-mized. E2(x) and E3(x) are Lagarange multipliers that penalizes each violation of the equality and inequality constraints, correspondingly. making use of gradient method the gradient of power feature with recognize to x is calculated

$$x(k+1) = x(k) + \mu\frac{\partial E}{\partial x} \qquad (7)$$

$$\lambda_p(k+1) = \lambda_p(k) + \gamma\frac{\partial E}{\partial \lambda_p}$$

Then

$$\frac{\partial E}{\partial x} = \Big(c + \sigma x + A_p^T K_1 r_p + A_p^T \lambda_p + + K_2 \sum_{i=p+1}^{m} \psi[r_i(x)] \begin{bmatrix} a_{i1} \\ a_{i2} \\ \cdots \\ a_{in} \end{bmatrix} \Big) \quad (8)$$

Here

$$r_p = A_p x - b_p \quad (9)$$

$$\psi(v) = \frac{d\Phi(v)}{dv} \quad (10)$$

$$\lambda_p(k+1) = \lambda_p(k) + \gamma[r_p(k) - \alpha\, \lambda_p(k)] \quad (11)$$

where μ and γ are learning rates. Ψ(v) is differentiable piecewise function.

Using formula (8) - (11) the optimal values of unknown parameters x satisfying equality and inequality constraints are calculated.

3. Portfolio Modeling

In this segment the upper depicted procedure is applied for taking care of portfolio streamlining issue.

The information for the model is chronicled returns and anticipated returns in future. Expect that a portfolio chief needs to designate his advantages among n unsafe protections dependent on ongoing recorded information or the enterprise's money related report and he need to limit hazard levels under some given portfolio return. Let xj are extent of the complete speculation gave to the unsafe security j, j=1,2,… ,n. Accept that the information is gotten for the dangerous security j at period T. Acquired information are pace of return of dangerous security j at period t, where t=1,2,… ,T.

In Markowitz model the change or standard deviation is acknowledged as proportion of hazard and this model is detailed as

$$z = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} x_i x_j \rightarrow \min \quad (12)$$

Here σ_{ij} is covariance matrix and is calculated as

$\sigma_{ij} = \frac{1}{T} \sum_{t=1}^{T} (r_{it} - R_i)(r_{jt} - R_j)$. Here r_{it} is return rate invested in security j over period t. R_j is average return (or expected return) of portfolio for j-th stock in time T. x_j is portfolio allocation of security. The following inequality and equality constraints are given

$$\sum_{j=1}^{n} R_j x_j \geq b$$

$$\sum_{j=1}^{n} x_j = d \qquad (13)$$

$$x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$$

Here b is least return requested by portfolio chief. d is all out spending plan put resources into portfolio

The issue is to discover such ideal weight estimations of xj under disparity and uniformity conditions (13), by utilizing they in target work (12) the estimation of portfolio hazard would be least.

The above depicted inclination strategy is applied to take care of issues (12) and (13). As a model the chronicled estimations of return rates for six stocks from Istanbul Stock Exchange Market are taken. The estimations of profits of six stocks for a year are given in table 1.

The estimations of expected returns for six stocks are assessed as (0.1252, 0.0862, 0.0793, 0.0521, 0.0637, 0.042), correspondingly. Utilizing inclination strategy the ideal estimations of each stock are resolved. In figure1 the consequence of recreation is given. The bends portray learning aftereffects of the directions of weight

Table 1. Historical values of return rates

	Stocks					
	X1	X2	X3	X4	X5	X6
	0.0667	0.3200	0.1100	0.1560	0.1600	0.0500
	0.2121	0.1300	0.1030	0.4500	0.3300	0.1700
	0.1647	0.1530	0.0860	0.4000	0.2700	0.3800
Times	0.0000	-0.1900	-0.0790	-0.2200	-0.1580	-0.3600
	0.4167	0.1560	0.0000	0.0000	0.0330	-0.1970
	0.2632	0.1300	0.0350	-0.0960	-0.0980	-0.1400
	-0.0306	0.0100	0.3100	0.0000	0.0100	0.0760
	-0.0392	-0.0780	-0.0540	-0.1560	-0.1140	0.0560
	0.4366	0.6200	0.5900	0.2070	0.4070	0.0870
	0.1094	-0.0300	0.1200	0.0820	-0.0690	0.3530
	-0.4386	-0.2100	-0.2700	-0.2500	-0.0417	-0.0730
	0.3412	0.0240	0.0010	0.0520	0.0345	0.1000

Figure 1. The trajectories of investment for risky securities

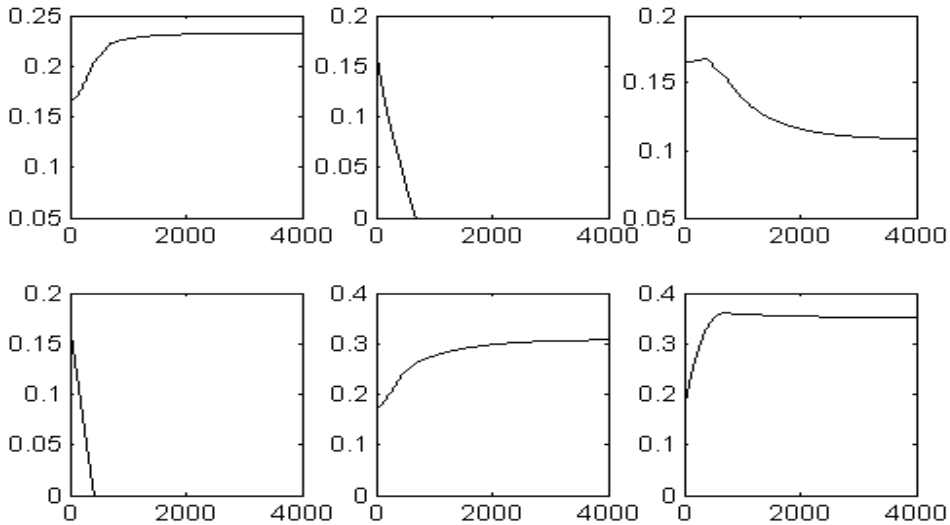

Figure 1. The trajectories of investment for risky securities

values for six stocks. The learning is completed for 4000 emphasess. At the aftereffect of learning the ideal estimation of stocks have been discovered x =(0.2322, 0 0.1085, 0, 0.3074, 0.3519). The reenactment can be completed for various degrees of portfolio return. By changing estimations of portfolio return the diverse venture procedures can be created.

4. Fuzzy Portfolio Optimization

Fuzzy smoothing out is the combination of strategies that arrangement improvement issues with versatile, estimated or questionable prerequisites and goals. All things considered, fuzzy en-hancement can be segregated into two classes: 1) To address defenselessness in the im-peratives and the goals (target limits), 2) To address versatility in the prerequisites and the destinations.

In this work the main sort of fuzzy improvement issue is thought of. As an im-provment model we are considering Markowitz model for portfolio determina-tion. The fuzzyheadway will be definite as

$$\tilde{z} = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i \tilde{\sigma}_{ij} x_j \rightarrow \mathit{fuzzy}\ \text{maximize} \quad (14)$$

under set of fuzzy constraints

.

$$\begin{aligned} &\sum_{j=1}^{n} \tilde{a}_j x_j \geq \tilde{b}, \quad \text{t} = 1,2,\ldots,\text{T} \\ &\sum_{j=1}^{n} x_j = \tilde{d} \qquad\qquad (15) \\ &x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0 \end{aligned}$$

Here $\tilde{\sigma}_{ij}$ coefficients are fuzzy covariance between returns of security i and of security j. $\tilde{a}_j$ are fuzzy values of expected returns of securities. $\tilde{b}$ is fuzzy value of minimum return demanded by portfolio manager. $\tilde{d}$ is fuzzy value of total budget in portfolio.

The issue is to discover such ideal weight estimations of xj under fuzzy imbalances and fairness conditions (15), by utilizing they in target work (14) the estimation of portfolio hazard would be least.

Some fuzzy advancement techniques have been proposed in the writing so as to manage various parts of delicate requirements. Zimmerman in [7] has thought about the fuzzy improvement as a symmetric issue. In this definition, fluzzy sets speak to both the difficult objectives and the adaptable (delicate) imperatives. In this plan the fluffiness emerges in light of meaning of fuzzy expansion and the rough disparity. These are characterized by fuzzy objective and fuzzy requirements. In this plan the fuzzy objectives and the imperatives are totaled to a solitary capacity that is saying ized. This structure can deal with fresh requirements just as fuzzy limitations

In this work the first type of fuzzy optimization for portfolio selection problem is considered. In the work the α level procedure and interval arithmetic are used to solve fuzzy optimization problem and find optimal fuzzy values of the securities.

The values of expected returns for each stock are taken as fuzzy values that have trapezoid form. Then the fuzzy energy function for optimization is defined. Using α- cuts the derivatives in (7) will be determined for adjusting the values of securities [8, 9].

If a trapezoid form is used for description of fuzzy variables, then any fuzzy number can be described by four parameters $x = (x_1, x_2, x_3, x_4)$. Then problem consists in determining the optimal values of four parameters. Using (7) the updating formulas for these variables can be derived.

$$\begin{aligned} x_1(k+1) &= x_1(k) + \mu \frac{\partial E}{\partial x_1} \\ x_2(k+1) &= x_2(k) + \mu \frac{\partial E}{\partial x_2} \\ x_3(k+1) &= x_3(k) + \mu \frac{\partial E}{\partial x_3} \\ x_4(k+1) &= x_4(k) + \mu \frac{\partial E}{\partial x_4} \end{aligned} \quad (16)$$

Using the α level procedure the values of derivatives in (16) can be determined. α-cut of the fuzzy number is defined as

$$x(\alpha) = [x^L(\alpha), x^R(\alpha)], \quad x^L(\alpha) \le x(\alpha) \le x^R(\alpha) \quad (17)$$

here $x^L(\alpha)$ and $x^R(\alpha)$ are the left and right sides of $x(\alpha)$ correspondingly (figure 2). To find values of unknown parameters the α level procedure is applied to energy function. Then the values of energy function for left and right sides would be determined. In the work the common energy function is determined as

$E(\alpha)=(E^L(\alpha)+E^R(\alpha))/2$ (18)

Using derivatives of energy function for $x^L(\alpha)$ and $x^R(\alpha)$, the derivatives of four parameters in (16) can be determined.

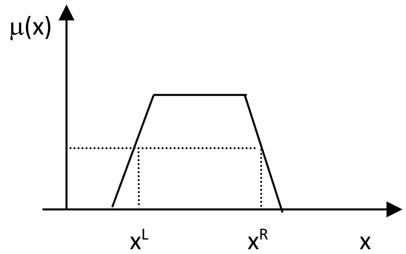

Figure 2

The left and right sides of trapezoid fuzzy variables are defined as

$$\begin{cases} x^L(\alpha) = (1-\alpha)x_1 + \alpha x_2 \\ x^R(\alpha) = \alpha x_3 + (1-\alpha)x_4 \end{cases} \qquad (19)$$

Using formulas (3-6) and (18-19) the values of derivatives $\frac{\partial E}{\partial x^L(\alpha)}$ and $\frac{\partial E}{\partial x^R(\alpha)}$ for the left and right sides are determined. The values of derivatives

$\frac{\partial E}{\partial x_1}, \frac{\partial E}{\partial x_2}, \frac{\partial E}{\partial x_3}$ and $\frac{\partial E}{\partial x_4}$ in (16) are determined through derivatives of left and right sides. After determining the values of derivatives of four parameters of the trapezoid the correction of the values of unknown parameters have been carried out.

The described procedure is used to solve (14) and (15) under fuzzy values of expected returns. As result of simulation the fuzzy values of risky securities are determined.

5. Simulation

At times the normal return of protections can't be assessed precisely and is characterized by fuzzy qualities. Let information is gotten for the unsafe security j at period T. Gotten information are pace of return of hazardous security j at period t, where t=1,2,… ,T. Utilizing these information measurable information it is expected to discover such ideal weight estimations of xj under given degree of portfolio return, the estimation of portfolio hazard will be least.

The above portrayed angle technique is applied to tackle issues (14) and (15). As info information the recorded estimations of return rates for six stocks are taken. The estimations of return paces of the six stocks for a year are given in table 2

The fuzzy estimations of expected returns of protections are acknowledged as

R1=(0.0197, 0.0306, 0.0313, 0.0327, 0.0366)
R2=(0.0207, 0.0316, 0.0323, 0.0337, 0.0376)
R3=(0.0217, 0.0326, 0.0333, 0.0347, 0.0386)

Here R1, R2, R3 are left, middle and right parts of fuzzy values of expected return, correspondingly. Because of triangular fuzzy numbers are special cases of trapezoid fuzzy numbers, in the work membership functions of fuzzy values of parameters are accepted in triangular form.

The fuzzy value of minimum return demanded by portfolio manager is b=(0.029 0.03 0.031). Using these input data the fuzzy optimization model (14) and (15) for portfolio is formulated. To find optimal fuzzy values of weight coefficients, the training of parameters of the model has been performed. During training using left and right sides the values of four parameters of risky securities that have trapezoid form are corrected.

Table 2. Return of securities over 12 month

0.053	0.046	-0.030	-0.018	0.043	0.047	0.055	0.036	-0.039	-0.043	0.046	0.052
0.032	0.055	-0.036	0.052	0.047	0.034	0.063	0.048	0.025	0.040	0.036	-0.017
0.064	0.056	0.048	0.007	0.053	0.036	0.017	0.047	-0.059	0.047	0.040	0.032
0.038	0.062	-0.037	0.050	0.065	-0.043	0.062	0.034	0.035	0.056	0.057	0.025
0.049	0.067	-0.039	0.051	0.049	0.037	0.055	0.025	0.052	0.020	0.045	0.040

The optimal fuzzy values of weight coefficients at the end of training are determined as:

x1 =(0, 0.1983, 0.3838, 0.0874, 0.3093);
x2=(0.0001,0.1998, 0.3871, 0.0885, 0.3145);
x3 =(0.0018,0.2014, 0.3904, 0.0897, 0.3196);

Here x1, x2, x3 are left, middle and right parts, correspondingly. The obtained results from the simulation demonstrate the efficiency of presented approach.

6. Conclusion

Utilizing quadratic programming model and Lagarange strategy the arrangement proce-dure of deterministic and fuzzy portfolio streamlining issues has been introduced. Utilizing input information taken from Istanbul Stock Exchange Market, the deterministic port-folio advancement model and its answer are given. Likewise the plan of fuzzy advancement is given. Utilizing □ level methodology and span number juggling the discovering operation timal fuzzy estimations of protections of portfolio is done. Utilized strategy permits finding ideal estimations of protections. At the point when number of protections expanded because of neighborhood little mama issue we are utilizing hereditary calculation for portfolio improvement issue.

References

[1] Harry Markowitz. "Portfolio Selection". The Journal of Finance, vol.VII, No.1, March 1952.

[2] Konno, H. and H. Yamazaki (1991): "Mean-Absolute Deviation Portfolio Optimization Model and Its Applications to Tokyo Stock Market", Management Science, 37, p. 519-531.1991

[3] R. Mansini and M.G. Speranza, Heuristic algorithms for the portfolio selection problem with minimum transaction lots, European Journal of Operational Research 114. p. 219–233, 1999.

[4] A.Cichocki and R.Unbehauen. Neural Networks for Optimization and Signal Processing, John Wiley & Sons, Inc., New York, NY, USA

[5] Frank S. Budnick. "Applied Mathematics for Business, Economics, and the Social Sciences' " McGraw-Hill International Edition, USA, 1993.

[6] K.K.Lai, S.Y.Wang, J.P.Xu, S.S.Zhu and Y.Fang, "A class of Linear Interval Programming Problems and Its Application to Portfolio Selection". IEEE Transactions on Fuzzy Systems, Vol.10, No.6,pp.698-703, December, 2002.

[7] H. J. Zimmermann. Description and optimization of fuzzy systems. International Journal of General Systems, 2:209–215, 1976.

[8] H.Ishibuchi, "A fuzzy neural network", in "Fuzzy Modelling: Paradigms and Practice", ed. Witold Pedryz, Kluwer Academic Publisher, Boston, 1996.

[9] Rahib Abiyev, Mustafa Menekay. Linear programming models for portfolio selection. Second International conference on Softcomputing and Computing with Words in System Analysis, Decision and Control. ICSCCW 2003.Dedicated to Prof. Zadeh. Antalya, Turkey, September 9-11, 2003, pp.226-231.