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Comparison of Some Methods for Estimating Poisson Regression Model Parameters Using the Genetic Algorithm

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Abstract--*The Poisson regression model is one of the log- linear most important models and this model is suitable for analyzing data in the form of a Counting data Or rates This model deals with the effects of the response variable that are rare in This paper, the procedure for the maximum likelihood estimation of the regression coefficient apply for Poisson regression and the Bayesian Poisson regression using some of Artificial intelligence algorithms*

Key words--*Artificial Intelligence, Genetic algorithms, Poisson Equation, maximum likelihood estimation, Bayesian Poisson regression*

I. INTRODUCTION

Data in the form of counts to be a important area in the practice of statistics data is appear in many application areas, from medicine, Social and natural sciences to econometrics, finance and industry The statistical method for analyzing count data type (nonnegative integer values (0,1,2,.) in the response or dependent variable is the Poisson regression model(Rashwan & Kamel, 2011) The aim of this paper is comparison between the methods to estimated parameters of Poisson regression model and Bayesian Poisson regression with genetic algorithm of this paper is organized as follows. Section (2) describes the discrete Poisson regression model Section (3) describes Bayesian parameter estimation for a discrete Poisson regression model Section (4) 4- Genetic Algorithms (GA) Section (5) presents an extensive simulation study, finally, we draw some conclusions in Section (6).

Discrete Poisson distribution

Poisson Distribution is a discrete probability distribution which can be used to express the probability of a given number of events occurring in fixed interval, Let Yi be the random variable takes nonnegative values, i=1, 2,… , n where n is the number of observation . If Yi is a count variable and follows a Poisson distribution with the parameter λ>0 with the probability density function: (Huang, Jiang, Ding & Zhou, 2019; Månsson & Shukur, 2011; Rashwan & Kamel, 2011)

$$P_r(Y_i = y_i) = \frac{\exp(-\lambda_i)\lambda_i^{y}}{y_i!} \quad , y_{i=0,1,2,.} \quad \ldots(1)$$

Not that the mean and variance of a Poisson random variable are same and related as

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$$E(Y_i) = Var(Y_i) = \lambda \qquad \cdots(2)$$

Where λ_i =exp($X_i^{`}\beta$), X_i *is the* i^{th} row of covariate matrix , and

$$\beta = (\beta_0, \beta_1, ..., \beta_n) \text{ are vector of coefficient is } (p+1)x1$$

The express the Poisson regression model as

$$y_i = \mathrm{E}(y_i) + \varepsilon_i \; i = 1,2,.n \;\; where \quad \varepsilon_i \quad , are\ disturbance\ terms$$

Maximum likelihood estimation of parameters

We use the method of maximum likelihood estimation to estimate the parameters of the Poisson regression model.

the likelihood function of $y_1, y_2, .., y_n$ is

$$L(y, \lambda) = \prod_{i=1}^{n} P_i(y_i) \qquad ...(3)$$

$$= \prod_{i=1}^{n} \frac{\exp(-\lambda_i)\lambda_i^{y_i}}{y_i!} \qquad ...(4)$$

$$\ln l(y, \lambda) = \sum_{i=1}^{n} y_i \ln(\lambda_i) - \sum_{i=1}^{n} \lambda_i - \sum_{i=1}^{n} \ln(y_i!) \qquad ...(5)$$

$$\ln l(y, \lambda) = \sum_{i=1}^{n} y_i \ln(\exp(X_i^{`}\beta) - \sum_{i=1}^{n} \exp(X_i^{`}\beta) - \sum_{i=1}^{n} \ln(y_i\,!) \quad ...(6)$$

The maximum likelihood estimation of the Poisson regression model is obtained by taking the first derivative by solving the following equation

$$\frac{\partial \ln l(y, \lambda)}{\partial \beta} = \sum_{i=1}^{n} (y_i - \exp(X_i^{`}\beta)) X_i = 0 \qquad ...(7)$$

The equation (7) is nonlinear in B the solution equaling zero is found using the following iterative weighted least square (IWLS) algorithm

$$\beta_{ML} = (X^{`}W^{\wedge}X)^{-1}(X^{`}W^{\wedge}Z^{\wedge}) \qquad ...(8)$$

$$where$$

$$W^{\wedge} = diag[M_i^{\wedge}\;] \qquad ...(9)$$

$$Z_i^{\wedge} = \log(M_i^{\wedge}) + \frac{Y_i - M_i^{\wedge}}{M_i^{\wedge}} \qquad ...(10)$$

Bayesian Analysis regression

Bayesian modeling has increasingly been used in diverse data analysis areas, such as regression and classification. This is one of two approaches to statistic. In Bayesian the interest the posterior distribution $p(\lambda / y)$ which is a product of likelihood function and prior distribution $p(\lambda)$ We started Bayesian count data modeling from the following expression. (Chan & Vasconcelos, 2009),(Jun, 2018).

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$$p(\lambda / y) = \frac{p(y/\lambda)\, p(\lambda)}{p(y)} \qquad ...(11)$$

where

$$p(y) = \int p(y/\lambda)\, p(\lambda) d(\lambda)$$

Bayesian Poisson regression

Suppose (y_i), $i = 1,2,3...n$ is a random sample from Poisson distribution with mean λ and that the prior distribution of λ is Gamma distribution with parameter α >0 and β >0, the Gamma distribution is conjugate prior for Poisson parameter (Chan & Vasconcelos, 2009),(Joyce, 2014),(Jun, 2018).

$$g(\lambda / y) \propto (\prod_{i=1}^{n} e^{-\lambda} \lambda^{y_i}) \quad \frac{\alpha^{\beta}}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\lambda\beta} \qquad ...(12)$$

$$\propto e^{-\lambda(\beta-n)} \ \lambda^{\alpha+n\,\bar{y}\ -1}$$

Is Gamma distribution with parameter posterior distribution of the λ

$$\tilde{\alpha} = \alpha + n\,\bar{y}$$

$$\tilde{\beta} = (\beta + n)$$

The mean of the prior Gamma distribution $E(\lambda) = \dfrac{\alpha}{\beta}$

The posterior mean

$$\tilde{\lambda} = \frac{\tilde{\alpha}}{\tilde{\beta}} = \frac{\alpha + n\,\bar{y}}{\beta + n} \qquad ...(13)$$

$$= \frac{\beta}{\beta + n} E(\lambda) + \frac{n}{\beta + n}\,\bar{y}$$

The Poisson Gamma model is an example for a common result in Bayesian statistic, namely that the posterior mean is a weighted average of prior mean and sample mean

When the conjugate prior exists for the parameter β in the Poisson regression model where the likelihood is proportional to

$$L(\beta / y, x) \propto \prod_{i=1}^{n} \exp[-\exp(x_i^{`}\ \beta)]\, [\exp(x_i^{`}\ \beta)]^{y_i} \qquad ...(14)$$

This expression is not the kernel of any known parametric distribution, there are two Solutions one is an approximation methods is necessary, other is the evaluation of the exact posterior distribution using simulation methods.

Genetic Algorithms

In order to obtain optimal solutions to mathematical problems, genetic algorithms were used as one of the most frequently used iterative methods for making the right decisions Genetic algorithms (GA) may contain a chromosome, a gene, set of population, fitness, fitness function, breeding, mutation and selection. Genetic

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algorithms (GA) begin with a set of solutions represented by chromosomes, called population 1 . Solutions from one population are taken and used to form a new population, which is motivated by the possibility that the new population will be better than the old one. Further, solutions are selected according to their fitness to form new solutions, that is, offspring's. The above process is repeated until some condition is satisfied. Algorithmically, the basic genetic algorithm (GA) as below (Huang, Jiang, Ding & Zhou, 2019; Jebari, Madiafi & Moujahid, 2014; Misra & Sebastian, 2013):

Generate random population of chromosomes, then Evaluate the fitness of each chromosome in the population. Create a new population by repeating following steps until the new population is complete. Select two parent chromosomes from a population according to their fitness. Better the fitness, the bigger chance to be selected to be the parent. , cross over the parents to form, new offspring, that is, children. If no crossover was performed, offspring is the exact copy of parents. with a mutation probability. Place new off spring in the new population. Replace Use new generated population for a further run of the algorithm. Test If the end condition is satisfied, stop, and return the best solution in current population.

II. EXPERIMENTAL RESULTS

In this section, we perform a simulation study where we show the effectiveness of the Poisson and Bayesian Poisson estimation procedure for Genetic Algorithms (GA) ,by use the compared between different methods ,we calculate the Mean Squared Error(MSE) using the following equation :

$$MSE = \frac{\sum_{i=1}^{R} mse_i}{R} = \frac{1}{R}\sum_{i=}^{R} \frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{N} \quad ...(15)$$

III. RESULTS

Simulation results are presented for the method of (Poisson and Bayesian Poisson) regression using the genetic algorithm where the research program for MATLAB was written. The simulation results are obtained according to the following tables.

Table 1: Mean Squares Error (MSE) for each methods when p=5

Sample size / **methods**	**N=20**	**N=50**	**N=100**	**N=150**
ML	0.6705	0.2701	0.0372	0.0262
Bayes	0.0889	0.0312	0.0055	0.0041
GA-ML	0.0425	0.0325	0.000049	0.0998
GA-Bayes	0.0012	0.000054	0.000017	0.000012
Best	GA-Bayes	GA-Bayes	GA-Bayes	GA-Bayes

From Table No. 1, we note that the method GA-ML is better than ML, as well as GA-Bayes better method than Bayes and for all sizes. And the best for all is GA-Bayes, note that when the sample size increases, the mean squared error decreases.

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Table 2: Mean Squares Error (MSE) for each methods when p=8

Sample size / Methods	N=20	N=50	N=100	N=150
ML	0.0579	0.0262	0.0132	0.0044
Bayes	0.0331	0.0097	0.0061	0.0018
GA-ML	0.04205	0.00555	0.00134	0.00052
GA-Bayes	0.000449	0.0000333	0.00000529	0.00000514
Best	GA-Bayes	GA-Bayes	GA-Bayes	GA-Bayes

From Table No. 2, we note that the method GA-ML is better than ML, as well as GA-Bayes better method than Bayes and for all sizes. And the best for all is GA-Bayes, note that when the sample size increases, the mean squared error decreases.

Table 3: Mean Squares Error (MSE) for each methods when p=10

Sample size / Methods	N=20	N=50	N=100	N=150
ML	0.0943	0.0161	0.0148	0.0099
Bayes	0.0474	0.0066	0.0065	0.0037
GA-ML	0.0320	0.0254	0.0215	0.01547
GA-Bayes	0.000759	0.000101	0.0000185	0.0000017
Best	GA-Bayes	GA-Bayes	GA-Bayes	GA-Bayes

From Table No. 3, we note that the method GA-ML is better than When N=20 and ML is best in N=50,100,150, GA-Bayes better method than Bayes and for all sizes. And the Bayes is best than ML for all size and the best for all is GA-Bayes, note that when the sample size increases, the mean squared error decreases

Table 4: Mean Squares Error (MSE) for each methods when p=12

Sample size / Methods	N=20	N=50	N=100	N=150
ML	0.1262	0.0176	0.0142	0.0109
Bayes	0.0530	0.0074	0.0062	0.004
GA-ML	0.0323	0.01444	0.01151	0.0020
GA-Bayes	0.000451	0.000165	0.0000095	0.0000013
Best	GA-Bayes	GA-Bayes	GA-Bayes	GA-Bayes

From Table No. 4, we note that the method GA-ML is better than ML, as well as GA-Bayes better method than Bayes and for all sizes. And the Bayes is best than ML for all size And the best for all is GA-Bayes, note that when the sample size increases, the mean squared error decreases.

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Table 5: Mean Squares Error (MSE) for each methods when p=15

Sample size / Methods	N=20	N=50	N=100	N=150
ML	0.2048	0.0265	0.0157	0.0082
Bayes	0.083	0.0107	0.0061	0.0031
GA-ML	0.0471	0.02995	0.021	0.014
GA-Bayes	0.00033	0.0001119	0.0000131	0.0000092
Best	GA-Bayes	GA-Bayes	GA-Bayes	GA-Bayes

From Table No. 5, we note that the method GA-ML is better than When N=20 and ML is best in N=50,100,150, GA-Bayes better method than Bayes and for all sizes. And the Bayes is best than ML for all size and the best for all is GA-Bayes, note that when the sample size increases, the mean squared error decreases

Recommendations

Were compared between the methods by calculate the mean squared error (MSE) using Monte Carlo simulations with taking samples of different dimensions and sizes. We recommend using artificial intelligence algorithms because they have the advantage of having internal iterations and extracting several solutions for them and choosing the best solution from them

REFERENCES

1. Chan, A. B., & Vasconcelos, N. (2009, September). Bayesian poisson regression for crowd counting. In *2009 IEEE 12th international conference on computer vision* (pp. 545-551). IEEE.
2. Huang, H., Jiang, M., Ding, Z., & Zhou, M. (2019). Forecasting emergency calls with a poisson neural network-based assemble model. *IEEE Access*, *7*, 18061-18069.
3. Jebari, K., Madiafi, M., & Moujahid, A. E. (2014). Solving poisson equation by genetic algorithms. *arXiv preprint arXiv:1401.0523*.
4. Joyce, D. (2014). MATH 217--Probability and Statistics.
5. Jun, S. (2018). Bayesian Count Data Modeling for Finding Technological Sustainability. *Sustainability*, *10*(9), 3220.
6. Månsson, K., & Shukur, G. (2011). A Poisson ridge regression estimator. *Economic Modelling*, *28*(4), 1475-1481.
7. Misra, A. K., & Sebastian, J. (2013). Portfolio optimization of commercial banks-an application of genetic algorithm. *European Journal of Business and Management*, *5*(6), 120-129.
8. Rashwan, N. A., & Kamel, M. M. (2011). Using generalized Poisson log linear regression models in analyzing two-way contingency tables. *Applied Mathematical Sciences*, *5*(5), 213-222.

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