International Journal of Psychosocial Rehabilitation, Vol. 24, Issue 10, 2020
ISSN: 1475-7192

# New Extrapolation Formula (Al-karamy3) to improve the results of numerical integrals

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***ABSTRACT-*** *The main objective of this research is to find a new Extrapolation formula derivative to produce results for numerical integrations. By using the Regardson acceleration as well as utilizing the rest of the excitation lines and the Mcluren series of the Pocket function to improve the accuracy of the results obtained from the Regardson acceleration application.*

***Keywords-*** *Numerical integration 65D30; Regardson Accelerating 65B99; Taylor series 30K05. Newton–Cotes formulas*

## I INTRODUCTION

There are numerical methods for calculating single integrals that are bounded in their integration intervals such as:

1. Trapezoidal Rule
2. Midpoint Rule
3. Simpson's Rule

Which are called "Newton–Cotes formulas".

The new method is to use the first limit of the excitation limits to improve the accuracy of the results resulting from the application of the trapezoidal rule or the integral point base by using the Regardson acceleration Sefi[4],as well as taking advantage of the rest of the spacing and sequence MClureen for the Pocket function to improve the accuracy of the results obtained from the application of Regardson acceleration and our symbol of the way Al-karamy3.

Where the researcher has reached Al karamy in 2017 [2],as well as Alsharify Alsharify[1] in 2018 to the two formats it was better than accelerate Rijardson Sefi[4], as well as accelerate Aitken in improving the results Asthma Al karamy 1 Al karamy [2] and Al karamy 2 Alsharify [1].

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**Received: 27 Feb 2019 | Revised: 20 Mar 2019 | Accepted: 30 Apr 2020** 

In 2019, Mohammad and others presented another version AL- Tamimi Mohammed[3],to improve the results of integrations but the new version was the best.

We assume our following integration:

$I = \int_a^b g(t)dt$ Where $g(t)$ it continues during the period $t \in [a,b]$.

$$I = Q(k) + E(k)$$

$$E(k) = A_1k^2 + A_2k^4 + A_3k^6 + \cdots \qquad \cdots(1)$$

Where $Q(K)$ one of the bases of Newton - Cotes (trapezoidal or middle point)and E(x)

It is a series to be added with the base correction limits $Q(k)$

$k = \frac{b-a}{n}$ and $n$ Is the number of partial periods of the period $[a,b]$

$A_1, A_2, \cdots$ Constants do not depend on their values $k$ Depending on the main limit $(A_1k^2)$ from the series of limits of correction and neglect the rest of our border becomes

$$I - Q(k) \cong A_1k^2 \qquad (2)$$

if it was $Q(k_1)$ and $Q(k_2)$ Two different values of Newton-Coates (trapezoidal or center point) at two different values $k$ they $k_1$, $k_2$ Using equation (2) becomes our

$$I - Q(k_1) \cong A_1k_1^2 \qquad (3)$$

$$I - Q(k_2) \cong A_1k_2^2 \qquad (4)$$

Of the two equations mentioned above we get

$$I = Q(k_2) + \frac{n_1^2\left(Q(k_2) - Q(k_1)\right)}{n_2^2 - n_1^2} \qquad (5)$$

Equation (5) represents the completion of the base Rijardson at two different periods of two numbers of partial .

To take advantage of the rest of the correction limits to find an approximate value for more accurate integration we ignore the first limit and rely on the rest of the limits.

Know from equation (1) that

Received: 27 Feb 2019 | Revised: 20 Mar 2019 | Accepted: 30 Apr 2020 

$$E(k) = A_1k^2 + A_2k^4 + A_3k^6 + \cdots$$

$$E(k) = A_1k^2 + k^3\left(A_2k^1 + A_3k^3 + \cdots\right) \qquad \cdots(6)$$

And sequential McLaurin function $\sin(k)$ is:

$$\sin(k) = B_1k^1 + B_2k^3 + \cdots \qquad \cdots(7)$$

$$E(k) = A_1k^2 + k^3\sin(k) \qquad \cdots(8)$$

After neglecting the first limit we get

$$E(k) \simeq k^3\sin(k) \qquad \cdots(9)$$

And taking the two values $Q(k_1)$ and $Q(k_2)$ We get:

$$E(k_1) \simeq k_1^3\sin(k_1) \rightarrow I - Q(k_1) \simeq k_1^3\sin(k_1) \qquad \cdots(10)$$

$$E(k_2) \simeq k_2^3\sin(k_2) \rightarrow I - Q(k_2) \simeq k_2^3\sin(k_2) \qquad \cdots(11)$$

Equations (10) and (11) are obtained:

$$\frac{I - Q(k_1)}{I - Q(k_2)} \simeq \frac{k_1^3\sin(k_1)}{k_2^3\sin(k_2)} \qquad \cdots(12)$$

We conclude that the value of integration is:

$$I \simeq \frac{Q(k_1)k_2^3\sin(k_2) - Q(k_2)k_1^3\sin(k_1)}{k_2^3\sin(k_2) - k_1^3\sin(k_1)} \qquad \cdots(13)$$

Mohammed [3]

We would like to point out when using the Simpson Rule to find the value of integration the accompanying correction boundary series will be as follows:

$$E(k) = A_1k^4 + A_2k^6 + A_3k^8 + \cdots$$

From this we conclude that to apply the Al karamy method we use the following rules (14) and (15):

$$I = Q(k_2) + \frac{n_1^4\left(Q(k_2) - Q(k_1)\right)}{n_2^4 - n_1^4} \qquad (14)$$

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ISSN: 1475-7192

$$I \simeq \frac{Q(k_1)k_2^5 \sin(k_2) - Q(k_2)k_1^5 \sin(k_1)}{k_2^5 \sin(k_2) - k_1^5 \sin(k_1)} \quad \cdots(15)$$

## II Examples and results

We applied the new method to several integrals and used the results of the trapezoidal base and the midpoint base for the application of the new method and obtained good results below in Tables 1 and 2 and compared the results obtained from the application of the new method with the results of the completion of Regardson Acceleration of Triangle AL- Tamimi Mohammed[3].

(Tr) symbols refer to the trapezoidal base, indicating (Me) to the base of the middle point, (Ri )refers to the acceleration of Regardson Sefi[4], (Tim) refers to the acceleration of the triangular ALTamimi Mohammed[3] ,( Ka )refers to the Al karamy method.

| ت | Function and its analytical value | n | Tr | Ri | Tim | Ka |
|---|---|---|---|---|---|---|
| 1 | $\int_1^2 (x^2+1)/x^3 dx$ 1.06814718056 | 95 | 1.0681801 3 Decimal places | 1.068147181686 9 Decimal places | 1.068147181686 9 Decimal places | 1.06814718056 11 Decimal places |
| 2 | $\int_3^4 \sqrt{x^2-x}dx$ 2.9576301324760 | 38 | 2.9576295 4 Decimal places | 2.9576301324647 10 Decimal places | 2.9576301324648 10 Decimal places | 2.9576301324760 13 Decimal places |
| 3 | $\int_2^3 \ln(x^2+1)dx$ 1.972673563322 | 25 | 1.9726468 3 Decimal places | 1.972673563352 9 Decimal places | 1.972673563355 9 Decimal places | 1.972673563322 12 Decimal places |
| 4 | $\int_0^1 e^x \cos(x)dx$ 1.37802461355 | 57 | 1.3779779 3 Decimal places | 1.37802461279 8 Decimal places | 1.37802461279 8 Decimal places | 1.37802461355 11 Decimal places |
| 5 | $\int_1^2 \frac{e^x}{x}dx$ Its analytical value is unknown | 78 | 3.0591418 5 4 Decimal places | 3.059116539890 9 Decimal places | 3.059116539890 9 Decimal places | 3.05911653965 11 Decimal places |

Table (1) shows some functions, analytical values, and numerical values using the trapezoidal rule and the methods of propagation (Ri, Tim, Ka)

ISSN: 1475-7192

| ت | Function and its analytical value | n | Me | Ri | Tim | Ka |
|---|---|---|---|---|---|---|
| 1 | $\int_1^2 (x^2+1)/x^3 dx$ 1.06814718056 | 95 | 1.0681307 4 Decimal places | 1.06814717957 8 Decimal places | 1.06814717957 8 Decimal places | 1.06814718056 11 Decimal places |
| 2 | $\int_3^4 \sqrt{x^2 - x} dx$ 2.957630132476 | 29 | 2.957630641 5 Decimal places | 2.957630132506 9 Decimal places | 2.957630132506 9 Decimal places | 2.957630132476 12 Decimal places |
| 3 | $\int_2^3 \ln(x^2+1) dx$ 1.972673563322 | 31 | 1.97268223 4 Decimal places | 1.9726735633111 10 Decimal places | 1.9726735633106 10 Decimal places | 1.972673563322 12 Decimal places |
| 4 | $\int_0^1 e^x \cos(x) dx$ 1.37802461355 | 71 | 1.37803964 4 Decimal places | 1.378024613819 9 Decimal places | 1.378024613818 9 Decimal places | 1.37802461355 11 Decimal places |
| 5 | $\int_1^2 \frac{e^x}{x} dx$ Its analytical value is unknown | 55 | 3.05909199 4 Decimal places | 3.05911653877 8 Decimal places | 3.05911653877 8 Decimal places | 3.05911653965 11 Decimal places |

Table (2) shows some analytic functions, values, and numerical values using the base point and the cryptographic (methods (Ri, Tim, Ka)

First integration $\int_1^2 (x^2+1)/x^3 dx$ Integrity is a fractional function and its analytical value is1.06814718056 Close to eleven decimal correct note when using two databases (Tr, Me) on the integration we got(3, 4) correct decimal places respectively when the number of partial periods for both rules after the application of the new method (Ka) on the results we obtained eleven correct decimal places, whereas in the application of the acceleration of Regardson Sefi[4], and acceleration al-Tamimi Mohammed[3],on the results of the rules (Tr, Me) the accuracy of their results is (9, 8) correct decimal places respectively, The same number of partial periods.

For the second integration $\int_3^4 \sqrt{x^2 - x} dx$ It has a radical complementarity and its analytical value is 2.9576301324760 rounded to thirteen decimal places correct without its results in the second row of the two tables (1, 2) each note that the value of integration is correct to four decimal places only when the number of

Received: 27 Feb 2019 | Revised: 20 Mar 2019 | Accepted: 30 Apr 2020 

partial periods Using the Tr rule and correct for only five decimal places using Rule Me when the number of partial periods When the Al karamy method was applied, the accuracy increased to (13, 12) decimal places respectively, while the accuracy of the results of the acceleration of Regardson Sefi[4], or acceleration of ALTamimi Mohammed[3],did not exceed ten decimal places correct when applied to the results of Tr (Tr) ) Were the correct decimal places (8) only at the same two periods.

While the Al karamy method is applied with the two rules (Tr, Me) on the third integration $\int_2^3 \ln(x^2+1)dx$
Which is complemented by the natural logarithm function and its analytical value is 1.972673563322 we obtained twelve decimal places Correct when the number of partial periods Respectively, while the value was correct to (3, 4) decimal places only when Tr (Me) was applied without any acceleration with them. When using Regardson acceleration Sefi[4], or acceleration of Tamimi Mohammed[3], with Tr (Me), we obtained (9, 10) Decimal places are correct, respectively, at the same time.

While in the fourth integration $\int_0^1 e^x \cos(x)dx$ Which integrate the function of an exponential trigonometric and value analytical 1.37802461355 close to eleven decimal correct note when applying the rules (Tr, Me) we got (3, 4) decimal places are correct, respectively, and when applying the method of Karmi with them was correct value for one of ten decimal when While applying the Regardson acceleration Sefi[4], on the results of Tr (Me), we obtained (8, 9) correct decimal places respectively, and we obtained the same correct decimal points when applying the triangulation ofAL Tamimi Mohammed[3],triangles in the same two periods.

And when applying the fifth integration $\int_1^2 \frac{e^x}{x}dx$ whose value of analysis is unknown. We note that the values resulting from the application of the new method Ka are closer to the results of the rules (Tr, Me) to a certain amount and their reliability at the value of 3.05911653965. They are valid for eleven decimal places. Hence we conclude that the value of integration is 3.05911653965. Regardson Sefi[4], or the use of trigeminal triangulation on the results of the rules (Tr, Me) are correct to (9, 8) decimal places only respectively when the number of partial periods It is worth mentioning that when using Tr (Me), the value was valid for only four decimal places at the same time of the two periods.

## III Discussion and conclusion

The results obtained from the application of the Al karamy method ka on the integrations we conducted and some of them in the tables of this research note that the application of the method of Al karamy on numerical integrations increased the accuracy of the results of these integrals significantly where the amount of increase in the accuracy of the results between (7) (9) correct decimal places. When applying Tr (Me), the accuracy of the results ranged from (3) to (5) correct decimal places only when the number of partial periods used ranged from to me after using theAL Karmi method on these results, the accuracy ranges from (11) to (13) correct decimal places at the same partial intervals. After comparing the results obtained from the application of the Al karamy method (Ka) with the results obtained from the application of acceleration Regardson Sefi[4], or the application of triangular acceleration ALTamimi Mohammed[3],we noticed the superiority of the Al karamy method on the acceleration of the nose in terms of accuracy and the number of partial periods used as the accuracy of the results three partial levels most often And at the same number of partial periods. It should be noted that the method of Al karamy enabled us to predict and know the value of some integrations unknown analytical value and in a few partial periods by approaching and then proved the numerical value to a certain amount as is evident in the fifth integration.

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