International Journal of Psychosocial Rehabilitation, Vol. 24, Issue 10, 2020
ISSN: 1475-7192

MATLAB Programming for Simulation of Discrete Dynamic Model

[1]Adel A. Abed AL Wahab, [2]Nihad Mahmoud Nasir, [3]Alaa Hussien Lafta

ABSTRACT- *Here, we considered a discrete-time dynamic model of prey-predator populations' involved ratio-dependent functional response with the goal of disclosing a simulation method using MATLAB. Mathematical model was implemented in MATLAB function that allows simulating the effect of ratio-dependent functional response. The graphical interface shown in this paper is performed using the MATLAB Program.*

Keywords- *MATLAB Programming , Discrete Dynamic Model , prey-predator model*

I INTRODUCTION

A Modern mathematical population dynamics started with the famous Lotka-Volterra prey-predator model. Since that time many mathematical models, which describe the population dynamics, have been proposed and analyzed. These models can be taken many forms depending on the time scale and space structure of the problem [1]. Some of these forms are represented by discrete-time dynamical models. In fact, many researchers showed that the discrete- time prey-predator models may be have more sets of patterns than continuous-time models have, see [2-6] and the references there in. In particular Danca et al [3] "demonstrated the existence of the chaotic dynamics in a simple discrete-time prey-predator model with Holling type - I functional response". Agiza et al [2] "proposed and analyzed a discrete-time prey-predator model with Holling type- II functional response, and they observed that the proposed model has a complex dynamic".

In all the above mentioned studies the functional response (which is known as prey-dependent) depends entirely on the prey species with absence of predator one. Although these prey and predatory prey models use their brutality, which suffers from two paradoxical problems: the fertilization paradox and biological control.

.

Later, Ariditi and Ginzburg [7] proposed and studied a new form of continuous time prey-predator model, in which the functional response depends on both the prey as well as predator species, and they called "ratio-

[1] *Computer science department, College of education for pure science, University of Diyala, Diyala-Iraq*
[2] *Computer science department, College of education for pure science, University of Diyala, Diyala-Iraq*
[3] *Computer science department, College of education for pure science, University of Diyala, Diyala-Iraq*

Received: 27 Feb 2019 | Revised: 20 Mar 2019 | Accepted: 30 Apr 2020

dependent prey-predator model" and they obtained the ratio-dependent model can solve the paradoxes problems.

Therefore, the discrete-time prey-predator model given by Agiza et al [2] is modified so that it involve the ratio-dependent type of functional response instead of Holling type-II of functional response.

Numerically, with all above literatures the authors used many types of programs for instance EXCEL sheets at Microsoft office as well as MATLAB program to simulate their Mathematical model to confirm the analytical results.

II MATHMATICAL MODEL

One of the possible ways to understand the complex dynamical behavior between two interacting species is the use of the discrete-time model formulation. In the present work we study the dynamics of prey-predator model with ratio-dependent functional response that may be describes by the following two difference equations:

$$\begin{aligned} P_{t+1} &= aP_t(1-P_t) - N_t \tfrac{bP_t}{N_t+eP_t} \\ N_{t+1} &= N_t \tfrac{dP_t}{N_t+eP_t} \end{aligned} \qquad (1)$$

Where xn and yn represents the numbers of the prey and predator population at the iteration n where t=0, 1,.... The terms and represent the functional and numerical response, respectively.

The positive parameters a and e stand for intrinsic growth rate of prey species and the limitation of the growth velocity of the predator species with increase in number of prey while the positive parameters b and d denoted to maximum attack rate and conversion rate of predator, respectively.

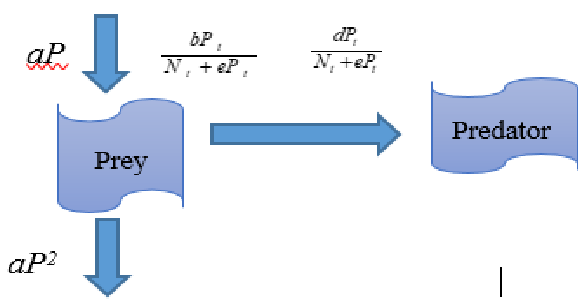

FIGURE 1. The diagram of model (1).

THE FIXED POINTS AND THEIR STABILITY

In this section, the existence and local stability conditions of all possible fixed points are discussed, and the following results are obtained:

The axial fixed point $p_1 = (\frac{a-1}{a}, 0)$ exists if and only if a>1

The positive fixed point $p_2 = (P^*, N^*)$, where

$$P^* = \frac{d(a-1) - b(d-e)}{ad}$$

$$N^* = x^*(d-e)$$

Which exists under the following conditions:

$$d(a-1) > b(d-e) > 0$$

Lemma 1 [5]:

Let

$F(\lambda) = \lambda_2 - Tr\lambda + Det$ (2)

"Suppose that $F(1) > 0$, λ_1 and λ_2 are the two roots of equation (2), Then:

(i) $|\lambda_1| < 1$and $|\lambda_2| < 1$ if and only if $F(-1) > 0$ and $Det < 1$.

(ii) $|\lambda_1| < 1$and $|\lambda_2| > 1$ (or $|\lambda_1| > 1$and $|\lambda_2| < 1$) if and only if $F(-1) < 0$.

(iii) $|\lambda_1| > 1$and $|\lambda_2| > 1$ if and only if $F(-1) > 0$ and $Det > 1$.

(iv) $\lambda_1 = -1$ and $\lambda_2 = 1$ if and only if $F(-1) = 0$ and $Tr = 0, 2$.

(v) λ_1 and $\lambda_2$are complex and $|\lambda_1| = |\lambda_2|$ if and only if $Tr^2 - 4Det < 0$ and $Det = 1$".

Let λ_1 and λ_2 the two roots of the equation (2) which are called the eigenvalues, then we have got that:

The axial fixed point $p_1 = (\frac{a-1}{a}, 0)$ is stable if and only if 1<a<3 and $d < e$. And unstable otherwise while the positive fixed point $p_2 = (x^*, y^*)$ is stable if and only if H2<a<H1 and unstable otherwise, Where $H_1 = \frac{3d^2+bd^2+2bde+3e(d-be)}{d(d+e)}$, and $H_2 = \frac{2bde-2be^2+2de-d^2}{de}$. For more details and proofs see[10]

III NUMERICAL SIMULATIONS

In this section, the global dynamical behavior of model (1) is investigated numerically. The objectives of such study are: first confirm our analytical results and second investigate the existence of complex dynamics (such as chaos) in model (1). The asymptotic behavior of the orbit of model (1) is studied for different sets of parameter values and for different sets of initial conditions. In order to detect about the types of attracting sets exist in model (1).

Example 1:

If we fixed our parameters as b=2, e=1.25and d=2while a as equal to 2 and use the following code:

Received: 27 Feb 2019 | Revised: 20 Mar 2019 | Accepted: 30 Apr 2020

ISSN: 1475-7192

```
a=2;
b=2;
e=1.25;
d=2;
N=500;
x=zeros (1, N);
y=zeros (1, N);
x (1)=0.5;
y (1)=0.5;
For n=1: N
   x(n+1)=a*x(n)*(1-x(n))-(b*x(n)*y(n)/(y(n)+e*x(n)));
   y(n+1)= (d*x(n)*y(n)/(y(n)+e*x(n)));
end
axis([0 1 0 1])
plot(x(1:N),y(1:N),'.','MarkerSize',6);
fsize=15;
set (gca,'XTick',0:0.2:1,'FontSize',fsize)
set (gca,'YTick',0:0.2:2,'FontSize',fsize)
xlabel('\itx','FontSize',fsize)
ylabel ('\ity','FontSize',fsize)
```

Accordingly, we can get the following figure:

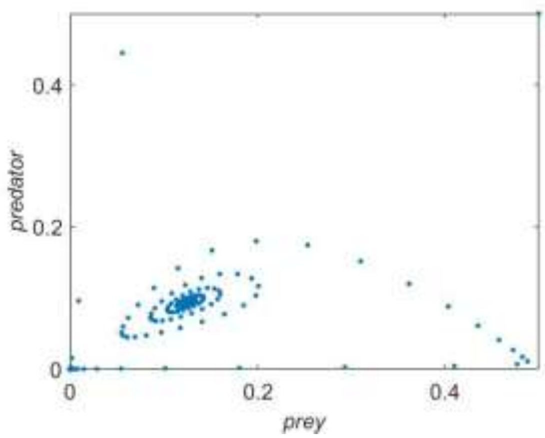

FIGURE 2. The phase portrait of the model (1) when a=2

Example 2:

In this example we fixed our parameters as b=2, e=1.25and d=2while a as equal to 4.85 and use the following code:

```
a=4.85;
b=2;
e=1.25;
d=2;
N=500;
x=zeros (1, N);
y=zeros (1, N);
x (1) =0.5;
y (1)=0.5;
for n=1:N
   x(n+1)=a*x(n)*(1- x(n))-(b*x(n)*y(n)/(y(n)+e*x(n)));
   y(n+1)= (d*x(n)*y(n)/(y(n)+e*x(n)));
end
axis([0 1 0 1])
plot(x(1:N),y(1:N),'.','MarkerSize',6);
fsize=15;
set(gca,'XTick',0:0.2:1,'FontSize',fsize)
set(gca,'YTick',0:0.2:2,'FontSize',fsize)
xlabel('\itx','FontSize',fsize)
ylabel('\ity','FontSize',fsize)
```

According to the above MATLAB code we can get:

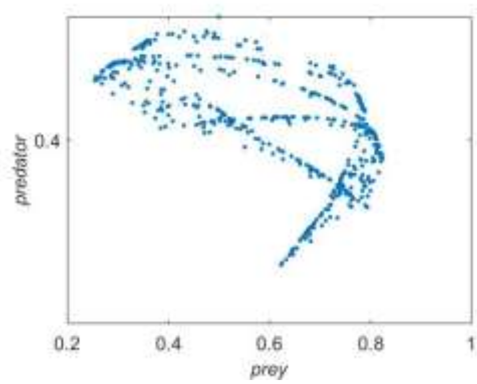

FIGURE 3. The phase portrait of the model (1) when a=4.85

IV CONCLUSIONS

The discrete-time prey-predator with ratio-dependent model is proposed and analyzed. Analytically, the model (1) has a unique positive fixed point. However, model (1) has complex dynamics. Numerically, the simulation result agree with the analytically one. Moreover, model (1) display much interesting dynamic behavior, including stable state, and chaotic set. These results shown numerically by using MATLAB.

Received: 27 Feb 2019 | Revised: 20 Mar 2019 | Accepted: 30 Apr 2020

REFERENCES

1. Y. Kuang, “Basic properties of the mathematical population,” Biomath., vol. 17,pp. 129-142, 2002.
2. H. Agiza, E. Elabbasy, H. Metwally and A. Elsadany, “Chaotic dynamic of discrete prey-predator with holling type II,” Nonlinear Anal.: Real World Appl., vol. 10, pp. 116-129, 2009.
3. M. Danca, S. Codreanu and B. Bako, “Detailed analysis of a nonlinear prey-predator model,” J. Biol. Phys. Vol. 23 pp.11-20 1997.
4. Z. Jing and J. Yang, “Bifurcation analysis of non-linear prey-predator model,” Chaos Solitons Fractals, vol. 27, pp. 259-277 2006.
5. X. Liu and D.Xiao, “Complex dynamic behavior of a discrete-time prey-predator model,” Chaos Solitons Fractals, vol. 32, pp. 80-94, 2007.
6. D. Sumers, C. Justian and H. Brian, “Chaos periodically force discrete-time ecomodel model,” Chaos Solitons Fractals, vol. 11, pp. 2331-2341, 2000.
7. R. Arditi and L. Ginzburg, “Coupling in predator-prey dynamics: ratio-dependent,” J. Theo. Biol., vol. 139, pp. 311-326, 1989.
8. S. Elaydi, Discrete chaos, 2nd ed.,Springer-Verlag publishers, , 2008.
9. J. Murray, Mathematical biology, 3rd ed., New York: Springer-Verlag, 2005.
10. Raid K.Naji and Alaa H. Lafta, On the dynamics of discrete-time prey-predator system with ratio-dependent functional response, Iraqi Journal of Science 54 (1): 157-164,2013.