Application of New Integral Transform for Ordinary Differential Equation with Unknown Initial Conditions

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Abstract--- *The Shehu transform fundamental properties were showed by Shehu. In this paper, we extended application of Shehu transform to solve ordinary differential where initial conditions known or unknown and formulas of general solutions of some ordinary differential equations are achieved*

Keywords*— solve ordinary, Shehu transform, equations*

I. INTRODUCTION

Ordinary differential equations are absolutely fundamental to modern science and engineering, many problems of physics interesting are described by differential and integral equations with appropriate initial or boundary conditions, these problems are usually formulated as initial value problem or boundary value problem that seem to be mathematically more vigorous and physical realistic in applied and engineering sciences [6]. Many researchers have faced advanced problems in engineering and science using the method of integral transformations, so several new transforms appeared that addressed some of the problems like Laplace, Elzaki, AlTememi, Novel etc [1,2,3,5,12]. Recently a new transform is called Shehu transform [9]. This transform was used to solve many types of ordinary and partial differential equations [4,10,13]. Moreover, it applies to solve fractional in differential equation by Sania Qureshi, Prem Kumar in [11]. After that, they found anthers applications of the Shehu transformation for the problems of growth and decay. These problems are great importance in the grounds of economics, chemistry, physics, biology, social sciences and zoology [7,8]. In our research, we extended on ideas of shehu transform and investigated for solving linear equations whether their initial conditions are known or unknown. The Shehu transformation was shown to be the article dual of the Laplace transform, therefore, we adopted this relationship to find shehu transform for some function. while the Laplace transform is defined by,

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$$F(s) = L\{f(t)\} = \int_0^\infty e^{-st} f(t)d(t)$$

The Shehu transform of the function φ(t) of exponential order is defined over the set of functions,

$$A = \left\{\varphi(t): \exists N, \eta_1, \eta_2 > 0, |\varphi(t)|\ N \exp\left(\frac{|t|}{\eta_i}\right), if\ t \in (-1)^i \times [0,\infty)\right\},$$

$$\mathbb{S}[\varphi(t)] = \psi(\delta,\vartheta) = \int_0^\infty \exp\left(\frac{-\delta t}{\vartheta}\right)\varphi(t)dt \qquad (1)$$

$$= \lim_{\alpha\to\infty}\int_0^\alpha \exp\left(\frac{-\delta t}{\vartheta}\right)\varphi(t)dt; \delta > 0, \vartheta > 0,$$

It converges if the limit of the integral exists.

In the second section, we reviewed some important definitions and evidences that we need in the following of research. In the third section, we deduced the formula of general solutions for linear ordinary equations, when the conditions exist and represent arbitrary constants. Moreover, we conclude the formula of general solution for linear equation of order n without subjected to initial conditions. In the last section, we applied formula that obtained in section three for solving some equations.

Basic definitions and fundamental properties

The inverse Shehu transform is defined by

$$\mathbb{S}^{-1}[\psi(\delta,\vartheta)] = \varphi(t), \qquad for\ t \geq 0$$

which is rewarded

$$\varphi(t) = \mathbb{S}^{-1}[\psi(\delta,\vartheta)] = \frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty}\frac{1}{\vartheta}\exp\left(\frac{\delta t}{\vartheta}\right)\psi(\delta,\vartheta)d\delta \qquad (2)$$

the Shehu transform variables are δ and ϑ , and the real constant is α, the integral in equation (2) is taken along $\delta = \alpha$ in the complex plane $\delta = x + iy$

property 1: Let the functions $\alpha\varphi(t)$ and $\beta\omega(t)$ be in set A, then $\big(\alpha\varphi(t) + \beta\omega(t)\big)\epsilon A$, $\alpha, \beta > 0$ arbitrary constants,

$$\mathbb{S}[\alpha\varphi(t) + \beta\omega(t)] = \alpha\mathbb{S}[\varphi(t)] + \beta\mathbb{S}[\omega(t)] \qquad (3)$$

Proof: By definition of Shehu transform.

$$\mathbb{S}[\alpha\varphi(t) + \beta\omega(t)] = \int_0^\infty \exp\left(\frac{-\delta t}{\vartheta}\right)(\alpha\,\varphi(t) + \beta\omega(t))dt$$

$$= \int_0^\infty \exp\left(\frac{-\delta t}{\vartheta}\right)(\alpha\varphi(t))dt + \int_0^\infty \exp\left(\frac{-\delta t}{\vartheta}\right)(\beta\omega(t))dt$$

$$= \alpha\int_0^\infty \exp\left(\frac{-\delta t}{\vartheta}\right)\varphi(t)dt + \beta\int_0^\infty \exp\left(\frac{-\delta t}{\vartheta}\right)\omega(t)dt$$

$$= \alpha\vartheta\int_0^\infty \exp(-\delta t)\,\varphi(\vartheta t)dt + \beta\vartheta\int_0^\infty \exp(-\delta t)\,\omega(\vartheta t)dt$$

$$= \alpha\mathbb{S}[\varphi(t)] + \beta\mathbb{S}[\omega(t)]$$

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Property 2: Change of scale property of shehu transformation, let $\varphi(\beta t)$ be the function in set A,B is an arbitrary constant, then

$$\mathbb{S}[\varphi(\beta t)] = \frac{\vartheta}{\beta}\psi\left(\frac{\delta}{\beta},\vartheta\right) \quad (4)$$

Proof: By definition of Shehu transform, we get

$$\mathbb{S}[\varphi(\beta t)] = \int_0^\infty \exp\left(\frac{-\delta t}{\vartheta}\right)\varphi(\beta t)dt$$

suppose $\eta = \beta t$, then $t = \frac{\eta}{\beta}$ and $dt = \frac{d\eta}{\beta}$, equation(4) yields:

$$\mathbb{S}[\varphi(\beta t)] = \frac{1}{\beta}\int_0^\infty \exp\left(\frac{-\delta\eta}{\vartheta\beta}\right)\varphi(\eta)d\eta$$
$$= \frac{1}{\beta}\int_0^\infty \exp\left(\frac{-\delta t}{\vartheta\beta}\right)\varphi(t)dt$$
$$= \frac{\vartheta}{\beta}\int_0^\infty \exp\left(\frac{-\delta t}{\beta}\right)\varphi(\vartheta t)$$
$$= \frac{\vartheta}{\beta}\psi\left(\frac{\delta}{\beta},\vartheta\right)$$

Lemma (1) : Derivative of Shehu transform

If $\varphi(t), \varphi'(t), \ldots, \varphi^{(n-1)}(t)$ are continuous functions for $t>0$ and of exponential order as $t \to \infty$, also $\varphi^{(n)}(t)$ is continuous functions , it follows that :

$$\mathbb{S}\left[\varphi^{(n)}(t)\ \right] = \frac{\delta^n}{\vartheta^n}\varphi\ \psi(\delta,\vartheta) - \sum_{r=0}^{n-1}\left(\frac{\delta}{\vartheta}\right)^{n-(r+1)}\varphi^{(r)}(0) \quad (5)$$

If n=1,2 and 3 , we get:

$$\mathbb{S}[\varphi'(t)\ ] = \frac{\delta}{\vartheta}\ \psi(\delta,\vartheta) - \varphi(0) \quad (6)$$

$$\mathbb{S}[\varphi''(t)] = \frac{\delta^2}{\vartheta^2}\psi(\delta,\vartheta) - \frac{s}{\vartheta}\varphi(0) - \varphi'(0) \quad (7)$$

$$\mathbb{S}[\varphi'''(t)] = \frac{\delta^3}{\vartheta^3}\psi(\delta,\vartheta) - \frac{\delta^2}{\vartheta^2}\psi(0) - \frac{\delta}{\vartheta}\varphi'(0) - \varphi''(0) \quad (8)$$

Proof: We let equation (5) is true for $n=r$, we get

$$\mathbb{S}\left[\varphi^{(r)}(t)\right] = \frac{\delta}{\vartheta}\mathbb{S}[\varphi^{(r)}(t)] - \varphi^{(r)}(0)$$

$$= \frac{\delta}{\vartheta}\left[\frac{\delta^r}{\vartheta^r}\mathbb{S}[\varphi(t)] - \sum_{i=0}^{r-1}\left(\frac{\delta}{\vartheta}\right)^{r-(i+1)}\varphi^{(i)}(0)\right] - \varphi^{(r)}(0)$$

$$= \left(\frac{\delta}{\vartheta}\right)^{r+1}\mathbb{S}[\varphi(t)] - \sum_{i=0}^{r}\left(\frac{\delta}{\vartheta}\right)^{r-i}\varphi^{(i)}(0),$$

By induction, the last equation implies that equation (5) holds for $n=r+1$ and the proof is complete.

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Table: the following table showed list of Shehu transform special function such as:

S.No	$\varphi(t)$	$\mathbb{S}[\varphi(t)]$
1	1	$\frac{\delta}{\vartheta}$
2	T	$\frac{\delta^2}{\vartheta^2}$
3	$exp\alpha(t)$	$\frac{\vartheta}{\delta-\alpha\vartheta}$
4	$sin(\alpha t)$	$\frac{\alpha\vartheta^2}{\delta^2+\alpha^2\vartheta^2}$
5	$cos(\alpha t)$	$\frac{\vartheta\delta}{\delta^2+\alpha^2\vartheta^2}$
6	$t\ exp\alpha(t)$	$\frac{\vartheta^2}{(\delta-\alpha\vartheta)^2}$
7	$\frac{\exp(\beta t)\sin(\alpha t)}{\alpha}$	$\frac{\vartheta^2}{(\delta-\beta\vartheta)^2+\alpha^2\vartheta^2}$
8	$\frac{\exp(\alpha t)}{\beta-\alpha}$	$\frac{\vartheta^2}{(\delta-\alpha\vartheta)(\delta-\beta\vartheta)}$

The formulas of General solutions of linear differential equations

Formula 1:

The Linear differential equation of order one has the form

$$\frac{d\varphi(t)}{dt}+k\varphi(t)=f(t);\ \varphi(t)=\alpha_0\ , \tag{9}$$

where *K* is constant and *f(t)* is integrable function, we take Shehu transform to both sides

$$\frac{\delta}{\vartheta}\psi(\delta,\vartheta)-\psi(0)+k\psi(\delta,\vartheta)=\mathbb{S}[f(t)]$$

Then, we get the desired result

$$\psi(\delta,\vartheta)=\left[\frac{(\mathbb{S}[f(t)]+\alpha_0)\vartheta}{\delta+\vartheta k}\right]$$

The inverse transform given:

$$\varphi(t)=\mathbb{S}^{-1}\psi(\delta,\vartheta)=\mathbb{S}^{-1}\left[\frac{(\mathbb{S}[f(t)]+\alpha_0)\vartheta}{\delta+\vartheta k}\right], \tag{10}$$

which represent the general solution of equation (9)

Formula 2:

Consider linear equation of order two

$$\frac{d^2\varphi(t)}{dt^2}+\beta_1\frac{d\varphi(t)}{dt}+\beta_2\varphi(t)=f(t)\ , \tag{11}$$

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with initial condition $\varphi(0) = \alpha_1$, $\varphi'(0) = \alpha_2$,where β_1 ,β_2 are constants , f(t) is an integrable function .

After take Shehu transform to both sides, we get:

$$\left[\frac{\delta^2}{\vartheta^2}\psi(\delta,\vartheta) - \frac{\delta}{\vartheta}\varphi(0) - \varphi'(0)\right] + \beta_1\left[\frac{\delta}{\vartheta}\psi(\delta,\vartheta) - \varphi(0)\right] + \beta_2\psi(\delta,\vartheta) = \mathbb{S}[f(t)]$$

$$\psi(\delta,\vartheta)\left[\frac{\delta^2+\beta_1\vartheta\delta+\beta_2\vartheta^2}{\vartheta^2}\right] = \frac{\delta}{\vartheta}\alpha_1 + \beta_1\alpha_1 + \alpha_2 + \mathbb{S}[f(t)]$$

By taking the inverse Shehu transform of $\psi(\delta,\vartheta)$ we obtained the general solution of (11), such as:

$$\varphi(t) = \mathbb{S}^{-1}\psi(\delta,\vartheta) = \mathbb{S}^{-1}\left[\frac{\alpha_1\vartheta\delta+(\beta_1\alpha_1+\alpha_2)\vartheta^2+\mathbb{S}[f(t)]\vartheta^2}{\delta^2+\beta_1\vartheta\delta+\beta_2\vartheta^2}\right], \tag{12}$$

which represent the general solution of equation (11)

Formula3:

The linear equation of order three has the form

$$\frac{d^3\varphi(t)}{dt^3} + \beta_1\frac{d^2\varphi(t)}{dt^2} + \beta_2\frac{d\varphi(t)}{dt} + \beta_3\varphi(t) = f(t), \tag{13}$$

with initial condition $\varphi(0) = \alpha_1$, $\varphi'(0) = \alpha_2$,$\varphi''(0) = \alpha_3$ where β_1 ,β_2 and β_3 are constants , f(t) is an integrable function ,by similar way in the formula 2, we get:

$$\left[\frac{\delta^3}{\vartheta^3}\psi(\delta,\vartheta) - \frac{\delta^2}{\vartheta^2}\varphi(0) - \frac{\delta}{\vartheta}\varphi'(0) - \varphi''(0)\right] + \beta_1\left[\frac{\delta^2}{\vartheta^2}\psi(\delta,\vartheta) - \frac{\delta}{\vartheta}\psi(0) - \varphi'(0)\right] +$$
$$\beta_2\left[\frac{\delta}{\vartheta}\psi(\delta,\vartheta) - \varphi(0)\right] + \beta_3\psi(\delta,\vartheta) = \mathbb{S}[f(t)] ,$$

$$\varphi(t) = \mathbb{S}^{-1}\left[\frac{\alpha_1\vartheta\delta^2+(\alpha_2+\beta_1\alpha_1)\vartheta^2\delta+(\alpha_3+\beta_1\alpha_2+\beta_2\alpha_2)\vartheta^3+\mathbb{S}[f(t)]\vartheta^3}{\delta^3+\beta_1\vartheta\delta^2+\beta_2\vartheta^2\delta+\beta_3\vartheta^3}\right], \tag{14}$$

where $\varphi(t)$ represent the general solution of equation (1.13).

Formula 4:

In the general, consider the n-th order linear equation,

$$\beta_1\psi^{(k)}(t) + \beta_2\psi^{(k-1)}(t) + \beta_3\psi^{(k-2)}(t) + \ldots + \beta_{k+1}\psi(t) = f(t), \tag{15}$$

with initial condition $\psi(0) = \alpha_1$, $\psi'(0) = \alpha_2$,…, $\psi^{(k-1)}(0) = \alpha_n$, where β_1 ,β_2, … ,β_{k+1} , are constants , f(t) is an integrable function.

We take Shehu transform to both sides:

$$\mathbb{S}[\beta_1\psi^{(k)}(t)] + \mathbb{S}[\beta_2\psi^{(k-1)}(t)] + \mathbb{S}[\beta_3\psi^{(k-2)}(t)] + \ldots + \mathbb{S}[\beta_{k+1}\psi(t)] = \mathbb{S}[f(t)]$$

By using lemma1 and property 1, we get:

$$\beta_1[(\frac{\delta^k}{\vartheta^k}\psi(\delta,\vartheta)-\frac{\delta^{k-1}}{\vartheta^{k-1}}\varphi(0)-\frac{\delta^{k-2}}{\vartheta^{k-2}}\varphi'(0)+\ldots.+\varphi^{(k-1)}(0)]+\beta_2[\Big(\frac{\delta^{k-1}}{\vartheta^{k-1}}\psi(\delta,\vartheta)-\frac{\delta^{k-2}}{\vartheta^{k-2}}\varphi(0)-\frac{\delta^{k-3}}{\vartheta^{k-3}}\varphi'(0)+\ldots.+\varphi^{(k-2)}(0)\Big]+\ldots+$$

$$\beta_{k+1}\mathbb{S}[\psi(\delta,\vartheta)]=\mathbb{S}[f(t)]$$

After substitution initial conditions and simple calculation, we have:

$$\varphi(t)=\mathbb{S}^{-1}\left[\frac{\mathbb{S}[f(t)]-[\beta_1\alpha_1\frac{\delta^{k-1}}{\vartheta^{k-1}}+(\beta_1\alpha_1+\beta_2\alpha_1)\frac{\delta^{k-2}}{\vartheta^{k-2}}+(\beta_1\alpha_3+\beta_2\alpha_2+\beta_3\alpha_1)\frac{\delta^{k-3}}{\vartheta^{k-3}}+\ldots+(\beta_1\alpha_k+\beta_2\alpha_{k-1}+\cdots+\beta_1\alpha_k)}{\beta_1\frac{\delta^k}{\vartheta^k}+\beta_2\frac{\delta^{k-1}}{\vartheta^{k-1}}+\beta_3\frac{\delta^{k-3}}{\vartheta^{k-3}}\ldots+\beta_k}\right]$$

where $\varphi(t)$ represent the general solution of equation (15).

Formula5:

The differential equation for n-th order is:

$$\beta_1\psi^{(k)}(t)+\beta_2\psi^{(k-1)}(t)+\beta_3\psi^{(k-2)}(t)+\ \ldots+\beta_k\psi(t)=f(t), \qquad (16)$$

with unknown condition ,where $\beta_1, \beta_2\ldots\beta_{k+1}$, are constant and $f(t)$ is integrable function after take Shehu transform to both sides ,we have:

$$\mathbb{S}[\beta_1\psi^{(k)}(t)]+\mathbb{S}[\beta_2\psi^{(k-1)}(t)]+\mathbb{S}[\beta_3\psi^{(k-2)}(t)]+\ldots+\mathbb{S}[\beta_{k+1}\psi(t)]=\mathbb{S}[f(t)]$$

By using lemma1 and property 1, we get:

$$\beta_1[(\frac{\delta^k}{\vartheta^k}\psi(\delta,\vartheta)-\frac{\delta^{k-1}}{\vartheta^{k-1}}\varphi(0)-\frac{\delta^{k-2}}{\vartheta^{k-2}}\varphi'(0)+\ldots.+\varphi^{(k)}(0)]+\beta_2[\frac{\delta^{k-1}}{\vartheta^{k-1}}\psi(\delta,\vartheta)-\frac{\delta^{k-2}}{\vartheta^{k-2}}\varphi(0)-\frac{\delta^{k-3}}{\vartheta^{k-3}}\varphi'(0)+\ldots.+\varphi^{(k-1)}(0)]+\ldots+\beta_{k+1}\mathbb{S}[\psi(\delta,\vartheta)]=\mathbb{S}[f(t)]$$

$$\psi(\delta,\vartheta)=\frac{\mathbb{S}[f(t)]-[\beta_1\varphi(0)\frac{\delta^{k-1}}{\vartheta^{k-1}}+\left(\beta_1\varphi'(0)+\beta_2\varphi(0)\right)\frac{\delta^{k-2}}{\vartheta^{k-2}}+\left(\beta_1\varphi''(0)+\beta_2\varphi'(0)+\beta_3\varphi(0)\right)\frac{\delta^{k-3}}{\vartheta^{k-3}}+\ldots+\left(\beta_1\varphi^{(k)}(0)+\beta_2\varphi^{(k-1)}(0)+\cdots+\beta_1}{\beta_1\frac{\delta^k}{\vartheta^k}+\beta_2\frac{\delta^{k-1}}{\vartheta^{k-1}}+\beta_3\frac{\delta^{k-3}}{\vartheta^{k-3}}\ldots+\beta_k}$$

Now, we can put the above equation as

$$\psi(\delta,\vartheta)=\frac{H(\delta,\vartheta)}{(\beta_1\frac{\delta^k}{\vartheta^k}+\beta_2\frac{\delta^{k-1}}{\vartheta^{k-1}}+\beta_3\frac{\delta^{k-3}}{\vartheta^{k-3}}\ldots+\beta_{k+1})Z(\delta,\vartheta)}\ . \qquad (17)$$

By taking inverse of Shehu transform for both sides of equation (17):

$$\varphi(t)=\mathbb{S}^{-1}\left[\frac{H(\delta,\vartheta)}{(\beta_1\frac{\delta^k}{\vartheta^k}+\beta_2\frac{\delta^{k-1}}{\vartheta^{k-1}}+\beta_3\frac{\delta^{k-3}}{\vartheta^{k-3}}\ldots+\beta_{k+1})Z(\delta,\vartheta)}\right]\ , \qquad (18$$

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where $Z(\delta,\vartheta)$ is polynomial of Shehu transform for the δ and represents denominator of Shehu transform for the function $f(t)$. $H(\delta,\vartheta)$ is also polynomial of δ ,which has degree of smaller than the degree of product of($\beta_1\frac{\delta^k}{\vartheta^k}+\beta_2\frac{\delta^{k-1}}{\vartheta^{k-1}}+\beta_3\frac{\delta^{k-3}}{\vartheta^{k-3}}\ldots.+\beta_{k+1})$ with $Z(\delta,\vartheta)$. It is not necessary to know the terms of $H(\delta,\vartheta)$,but just denoted it by this symbol . Equation (18) is a general solution of the equation (16) can be written of the form

$$\varphi(t)=A_1\,H_1\,(t)+A_2\,H_2\,(t)+\cdots,A_n\,H_n\,(t)\,;k<n\ , \tag{19}$$

where $H_1\,(t),H_1\,(t),\ldots,H_n\,(t)$ are functions of t ,and $A_1\,,A_2\ \ldots A_n$ are constants. We note that the number of constants in equation (19) is greater than the order of equation (1.16) therefore, to eliminate the extra constants, we substitute the general solution in equation (16).

Applications

In this section, some applications are given to demonstrate the effectiveness for solving linear ordinary differential equations.

Example 1:

Consider the first order differential equation:

$$\frac{d\varphi(t)}{dt}+10\varphi(t)=e^{3t}\quad,\quad\varphi(0)=\alpha_o \tag{20}$$

Using equation (10), we obtain:

$$\varphi(t)=\mathbb{S}^{-1}\left[\frac{\left(\frac{\vartheta}{\delta-3\vartheta}+\alpha_o\right)\vartheta}{\delta+10\vartheta}\right]$$

After simplification by using partial fractions with taking inverse of Shehu transform

$$\varphi(t)=\frac{1}{13}e^{3t}+Ae^{-10t}\quad,\quad\text{where }A=\frac{-1}{13}+\alpha_o \tag{21}$$

Example 2:

Consider the second order differential equation:

$$\frac{d^2\varphi(t)}{dt^2}+9\varphi(t)=\sin 2t\,,\qquad\qquad\varphi(0)=\alpha_1\,,\varphi'(0)=\alpha_2 \tag{22}$$

Using equations (1.12), we obtain

$$\varphi(t)=\mathbb{S}^{-1}\left[\frac{\alpha_1\vartheta\delta^3+4\alpha_1\vartheta^3\delta+\alpha_2\vartheta^2\delta^2+4\alpha_2\vartheta^4+\vartheta^3\delta}{(\delta^2+4\vartheta^2)(\delta^2+9\vartheta^2)}\right]$$

By partial fractions, we obtain

$$\varphi(t)=\ \mathbb{S}^{-1}\left[\frac{1}{5}\,\frac{\vartheta\delta}{\delta^2+4\vartheta^2}+\ \frac{\alpha_1\vartheta\delta}{\delta^2+9\vartheta^2}-\frac{1}{5}\,\frac{\alpha_1\vartheta\delta}{\delta^2+9\vartheta^2}+\frac{\alpha_2\vartheta^2}{\delta^2+9\vartheta^2}\right]$$

The general solution obtained by using inveres of Shehu transform,

$$\varphi(t)=\frac{1}{5}cos2t+\alpha_1cos3t-\frac{1}{5}cos3t+\alpha_1sin3t \tag{23}$$

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Example 3:

Solve the equation:

$$\frac{d^3\varphi(t)}{dt^3} - 4\frac{d^2\varphi(t)}{dt^2} - 5\frac{d\varphi(t)}{dt} = 0\,, \quad \varphi(0) = \alpha_1\,, \varphi'(0) = \alpha_2\,, \varphi''(0) = \alpha_3 \quad (24)$$

By similar way in previous examples, we get:

$$\varphi(t) = \mathbb{S}^{-1}\left[\frac{\alpha_1\vartheta\delta^2 + (\alpha_2 + 4\alpha_1)\vartheta^2\delta + (\alpha_3 + 4\alpha_2 + 5\alpha_2)\vartheta^3}{\delta(\delta+5\vartheta)(\delta-\vartheta)}\right].$$

$$\varphi(t) = \mathbb{S}^{-1}\left[\frac{-1}{5}(\alpha_3 - \alpha_2)\frac{\vartheta}{\delta} + (\frac{1}{6}\alpha_1 + \frac{1}{30}\alpha_3 - \frac{1}{5}\alpha_2)\frac{\vartheta}{\delta+5\vartheta} + \frac{1}{6}(5\alpha_1 + \alpha_2)\frac{\vartheta}{\delta-\vartheta}\right]$$

$$\varphi(t) = \frac{-1}{5}(\alpha_3 - \alpha_2) + (\frac{1}{6}\alpha_1 + \frac{1}{30}\alpha_3 - \frac{1}{5}\alpha_2)e^{-5t} + \frac{1}{6}(5\alpha_1 + \alpha_2)e^{t}$$

$$\varphi(t) = \frac{-1}{5}A + Be^{-5t} + \frac{1}{6}Ce^{t}\,; \quad (25)$$

$$A = \alpha_3 - \alpha_2,\ B = \frac{1}{6}\alpha_1 + \frac{1}{30}\alpha_3 - \frac{1}{5}\alpha_2\ \ ,\mathrm{C} = 5\alpha_1 + \alpha_2$$

Which represent the general solution of original equation.

Example 4:

Consider differential equation of order three

$$\frac{d^3\varphi(t)}{dt^3} + \frac{d^2\varphi(t)}{dt^2} - \frac{d\varphi(t)}{dt} - \varphi(t) = e^{t}\ , \quad (26)$$

without subjected to any initial conditions. By using equations (17)

$$\varphi(t) = \mathbb{S}^{-1}\left[\frac{H(\delta,\vartheta)}{(\delta^3 + \vartheta\delta^2 - \vartheta^2\delta - \vartheta^3)Z(\delta,\vartheta)}\right].$$

After simplification by using partial frictions, we have:

$$\varphi(t) = \mathbb{S}^{-1}\left[\frac{A}{\delta-\vartheta} + \frac{B}{(\delta-\vartheta)^2} + \frac{C}{\delta+\vartheta} + \frac{D}{(\delta+\vartheta)^2}\right]$$

$$\varphi(t) = Ae^{t} + Bte^{t} + Ce^{-t} + De^{-t}t \quad (27)$$

which represent the general solution of original equation and A,B,C,D are constants, whereas equation (27) contains four constants, but the equation (26) of order three .Therefore, we substitute the general solution in equation (26) and obtain the value of $B = \frac{1}{4}$, equation (27) become

$$\varphi(t) = Ae^{t} + \frac{1}{4}te^{t} + Ce^{-t} + De^{-t}t$$

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