

OPTIMAL CONTROL OF ROBOTIC ARM MOVEMENT INVOLVING CORIOLIS, CENTRIFUGAL AND GRAVITATIONAL FORCES

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ABSTRACT--This paper discusses the optimal control solution of the robotic arm model involving coriolis, setrifugal, and gravitational forces. The interaction of these three forces forms the basis of this research. The robot movement model is modifications of the Geise optimal control model which only minimizes time. In this study, the model minimizes time and energy. Given object's coordinates converted into polar coordinates that indicate the final position of the robotic arm. Some tests by adding regularization are carried out to check whether the chosen objective function is correct and produces a stable solution. The optimal solution is determined using a numerical approach through discretizing the dynamic model of the robotic arm as well as the objective of the optimal control by using the Pseudo Spectral Method (PSM). The results show that the model uses less energy than the Geise model does. Robotic arms having arm twice as long can hamper the movement of the robotic arm to reach the destination point. Therefore, it takes longer in time and uses more energy than robotic arms that have normal arm lengths.

Keywords-- Optimal control, polar robot, robot movement, robotic arm, Pseudo Spectral Method

I. INTRODUCTION

As the development of technology increasingly advances, robotic technology is experiencing a very rapid progress. Sophisticated technology has replaced manual equipments that require a lot of manpower to operate, one of which is the use of robots. The development of robotic technology has made the quality of human life even higher. Currently the development of robotics technology has been able to improve the quality and quantity of production of various industries[1].

In general there are two types of robots, namely controlled robots and autonomous robots. Most people assume that a robot is a machine that resembles a human, has a body with a head, arms and legs, but only the structure of the body is different that is made of metal. However, most robots are not shaped like humans, each robot has a different shape depending on the task it does, for example robotic arms in the industrial field. Robotic arm is a mechanical system used in manipulating the movement of lifting, moving, and manipulating workpieces to relieve human work [2]. Robotic arms are made to resemble human arms. The configuration of the robotic arm can be

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divided into 4 parts, namely: polar, cylindrical, Cartesian and arm joints. Robotic arms have other components as their constituents, such as actuators, sensors and controllers themselves. Robotic arms also recognize the degree of freedom that determines the amount of movement in the robot [3].

Research on modeling the robotic arms was first introduced by Monika Mössner-Beigel [4]. Monika modeled the movements of robotic arms by ignoring the coriolis, centrifugal and gravitational forces to minimize time. Later, coriolis, centrifugal and gravity forces were added [5]. The interaction of these three forces forms the basis of this study. This study implement the forces to a polar type robotic arm. In [5], the model minimized the time of movement of the robot and the final position of the robotic arm such as the length of the arm must be removed as well as the large angle of rotation of the robot arm was known as the final value. In this study, the movement of the robot will be modified to minimize time and energy and the final position of the arm can be determined from the coordinates of the given object. The coordinates of this object will be converted into polar coordinates which will indicate the final position of the robotic arm.

Some testing needs to be carried out to check whether the chosen objective function is correct and produces stable solutions. Therefore, a modification of the objective function of the model will be modified by adding regularization. This regulation aims to prevent overfitting so that the resulting solutions are stable or in other words the robot movement is not broken due to coriolis, centrifugal and gravitational forces. Numerical solutions are sought using the Pseudo Spectral Method (PSM).

II. RESEARCH METODOLOGY

The robotic arm modelled in this study is the polar type robotic arm as described in [5]. The physical model of this robotic arm can be simply described as in Figure 1.

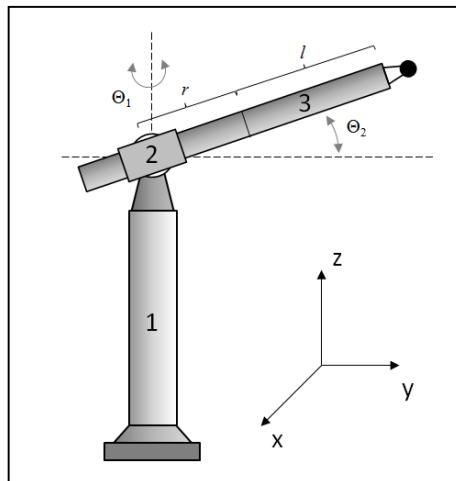


Figure 1: A robotic arm model

The robot model in this study is divided into three parts, namely the base part (part 1), the middle part (part 2) and the arm part (part 3), where r is the arm length of the robotic arm, θ_1 is the horizontal turning angle and θ_2 the vertical turning angle. The approaches of this study are explained as follows:

1. Determine the objective function. The objective function used in this study is to move the robot to reach its final position in the shortest possible time and minimum energy use. The optimal solution in this study uses the objective function as follows:

$$J = \int_0^{t_f} (1 + u_r^2 + u_{\theta_1}^2 + u_{\theta_2}^2) dt \quad (1)$$

It is also necessary to test the optimal control model with the objective function using regularization as follows:

- The robotic arm model with an objective function uses the regularization on u_{θ_1} .

$$J = \int_0^{t_f} (1 + u_r^2 + u_{\theta_1}^2 + u_{\theta_2}^2 + \dot{u}_{\theta_1}^2) dt \quad (2)$$

- The robotic arm model with an objective function uses the regularization on u_{θ_2} .

$$J = \int_0^{t_f} (1 + u_r^2 + u_{\theta_1}^2 + u_{\theta_2}^2 + \dot{u}_{\theta_2}^2) dt \quad (3)$$

- The robotic arm model with an objective function uses the regularization on u_{θ_1} and u_{θ_2} .

$$J = \int_0^{t_f} (1 + u_r^2 + u_{\theta_1}^2 + u_{\theta_2}^2 + \dot{u}_{\theta_1}^2 + \dot{u}_{\theta_2}^2) dt \quad (4)$$

2. Determine the constraint function. The constraint equations in this study is the robot arm movement model with state variables r, θ_1, θ_2 and control variables $u_r, u_{\theta_1}, u_{\theta_2}$ in equations (6), (7), and (8), where $-800 \leq u_r \leq 800, -1400 \leq u_{\theta_1} \leq 1400$, and $-300 \leq u_{\theta_2} \leq 300$ [5].

Based on Figure 1, by using Newton's Law 2 the state equation reads:

$$\ddot{r} = (u_r + F_{z,r \leftarrow \theta_1} + F_{z,r \leftarrow \theta_2} + F_g) / m_{LB} \quad (5)$$

where $m_{LB} = m_l + m_3$, m_l is the mass of the object raised by the arm. In this study $m_l = 0$ because there is no object raised by the robot arm, while $m_3 = 40$ which is the mass of the arm in the robot arm.

$$F_{z,r \leftarrow \theta_1} = (m_{LB} + m_3 l) \dot{\theta}_1^2 \cos^2 \theta_2 \quad (6)$$

$F_{z,r \leftarrow \theta_1}$ is the centrifugal force that affects r which is caused by horizontal motion θ_1 , where m_3 is the robot's part 3 mass and m_{LB} is an object's mass added with part 3 mass.

$$F_{z,r \leftarrow \theta_2} = (m_{LB} r + m_3 l) \dot{\theta}_2^2 \quad (7)$$

$F_{z,r \leftarrow \theta_2}$ is centrifugal force which affects r due to vertical movement θ_2 .

$$F_g = -m_{LB} g \sin \theta_2 \quad (8)$$

F_g is gravitational force affected by linear movements.

$$\ddot{\theta}_1 = (u_{\theta_1} + N_{c,\theta_1 \leftarrow r} + N_{c,\theta_1 \leftarrow \theta_2}) / I(\theta_2, r) \quad (9)$$

where,

$$N_{c,\theta_1 \leftarrow r} = -2(m_3 r + m_l l) \dot{r} \dot{\theta}_1 \cos^2 \theta_2 \quad (10)$$

$N_{c,\theta_1 \leftarrow r}$ is coriolis force on θ_1 affected by linear movements r .

$$N_{c,\theta_1 \leftarrow \theta_2} = -[I_2^{\theta_1 \theta_2} - (I_3^{\theta_1 \theta_2} + m_3 r^2 + m_l (r + l)^2)] \times \dot{\theta}_1 \dot{\theta}_2 \sin(2\theta_2) \quad (11)$$

$N_{c,\theta_1 \leftarrow \theta_2}$ is coriolis force on θ_1 affected by vertical movements θ_2 .

$$I(\theta_2, r) = I_3^r + I_2^{\theta_1 \theta_2} \sin^2 \theta_2 + (I_2^{\theta_1 \theta_2} + m_3 r^2 + m_l (r + l)^2) \cos^2 \theta_2 \quad (12)$$

$$\ddot{\theta}_2 = (u_{\theta_2} + N_{c,\theta_2 \leftarrow r} + N_{z,\theta_2 \leftarrow \theta_1} + N_g) / I(r) \quad (13)$$

where,

$$N_{c,\theta_2 \leftarrow r} = -2(m_{LB} r + m_3 l) \dot{r} \dot{\theta}_2 \quad (14)$$

$N_{c,\Theta_2 \leftarrow r}$ is coriolis force on Θ_2 affected by linear movements r .

$$N_{z,\Theta_2 \leftarrow \Theta_1} = -[I_3^{\Theta_1\Theta_2} - (I_2^{\Theta_1\Theta_2} + m_3r^2 + m_L(r+l)^2)] \times \dot{\Theta}_2 \sin\Theta_1 \cos\Theta_2 \quad (15)$$

$N_{z,\Theta_2 \leftarrow \Theta_1}$ is centrifugal force on Θ_2 affected by horizontal movements Θ_1 .

$$N_g = -(m_{LB}r + m_3l)g \cos\Theta_2 \quad (16)$$

N_g is gravitational force affected by circular motion.

$$I(r) = I_1^{\Theta_1\Theta_2} + m_3r^2 + m_L(r+l)^2 \quad (17)$$

and applying the following parameter values [5]:

$$g = 10 \quad I_1^{\Theta_1\Theta_2} = 18,5$$

$$I_3^{\Theta_1\Theta_2} = 18,5 \quad I_2^{\Theta_1\Theta_2} = 0,12$$

$$I_1^r = 0 \quad l = 0,75$$

3. Determine the initial value and final value of the robotic arm model. The robotic arm move from the initial position $(r(0), \Theta_1(0), \Theta_2(0))$ to reach an object at coordinates (x, y, z) . Object coordinates (x, y, z) which are the final positions of the robotic arm are converted into polar coordinates $(r(t_f), \Theta_1(t_f), \Theta_2(t_f))$ by using equations (22), (23), and (24). The initial value and the final value are as follows:

- Initial value (starting point):

$$r(0) = 0 \quad (18)$$

$$\theta_1(0) = 0 \quad (19)$$

$$\theta_2(0) = -1,2 \text{ rad} \quad (20)$$

$$\dot{r}(0) = \dot{\theta}_1(0) = \dot{\theta}_2(0) = 0 \quad (21)$$

- Final value (final point):

The target coordinate is $(x, y, z) = (0,5, 0,5, 0,5)$. The poin is converted into polar coordinate $(r(t_f), \Theta_1(t_f), \Theta_2(t_f))$ by using the method presented in [6].

$$r(t_f) = \sqrt{x^2 + y^2 + z^2} - l = 0,1160 \quad (22)$$

$$\theta_1(t_f) = \sin^{-1}\left(\frac{y}{\sqrt{x^2+y^2}}\right) = 0,7854 \quad (23)$$

$$\theta_2(t_f) = \sin^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) = 0,6155 \quad (24)$$

$$\dot{r}(t_f) = \dot{\theta}_1(t_f) = \dot{\theta}_2(t_f) = 0 \quad (25)$$

4. Determine the solutions by implementing Pseudo Spectral Method (PSM).

5. Compare the solutions of the optimal control model (model with objective fuction without regularization, with regularization, and model in [5])

III. RESULTS AND DISCUSSION

1.1 Solution of Optimal Control Model of the Robotic Arms

The optimal control solution of the robotic arm model with $(x, y, z) = (0.5, 0.5, 0.5)$ is shown in Figure 2. In Figure 2 it can be seen that the control variable u_r rises from $-696,2093$ to $42,2958$ in $0,7272$ seconds, and goes to $-103,1428$, u_{θ_1} decreases from $75,6080$ to $-106,4904$, while u_{θ_2} increases from 300 to $180,7904$. The minimum time needed to move the robotic arm from the initial position to the position of the object is $0,9923$ seconds.

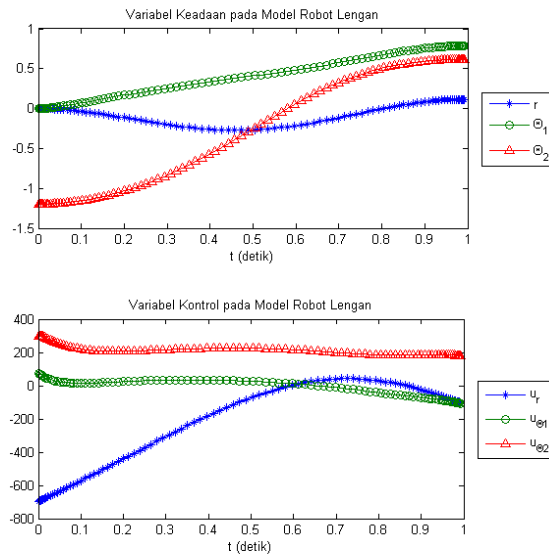


Figure 2: Plot of state and control variables

The optimal control solution of the robotic arm model with the objective function using regularization at u_{θ_1} is shown in Figure 3. In Figure 3 it can be seen that the control variable u_r rises from $-677,0511$ to $66,6193$ in $0,8207$ seconds and goes to $-83,9764$, while u_{θ_1} decreases from $35,2393$ to $-33,5919$, and u_{θ_2} decreases from $233,8513$ to $188,6965$ in $0,1565$ seconds and then increases to $195,2571$. The minimum time needed to move the robotic arm from the initial position to the position of the object is $1,0993$ seconds.

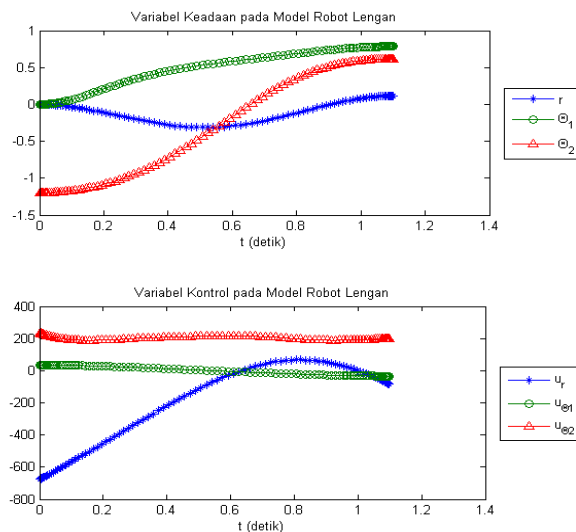


Figure 3: Plot of state and control variables with objective function using regularization at u_{θ_1} .

The optimal control solution of the robotic arm model with the objective function using regularization at u_{θ_2} is shown in Figure 4. In Figure 4 it can be seen that the control variable u_r rises from -691.0989 to 54.3275 in 0,7615 seconds and falls to -82.8104, u_{θ_1} decreases from 57,5424 to -100,0755, while u_{θ_2} decreases from 211,4075 to 195,4073. The minimum time needed to move the robotic arm from the initial position to the position of the object is 1,0591 seconds.

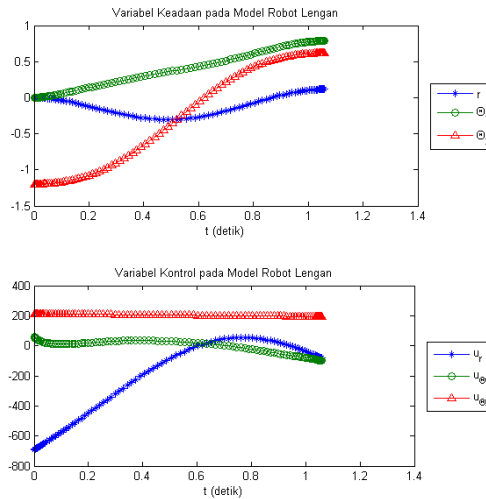


Figure 4: Plot of state and control variables with objective function using regularization at u_{θ_2} .

The optimal control solution of the robotic arm model with the objective function using regularization at u_{θ_1} and u_{θ_2} is shown in Figure 5. In Figure 5 it can be seen that the control variable u_r increases from -682,7574 to 75,2944 in 0,8137 seconds and falls to -78,8037, while u_{θ_1} falls from 34,6423 to -33,1487, and u_{θ_2} increases from 199,4691 to 199,6703 in 0,4431 seconds and then decreases to 197,4952. The minimum time needed to move the robotic arm from the initial position to the position of the object is 1,1103 seconds.

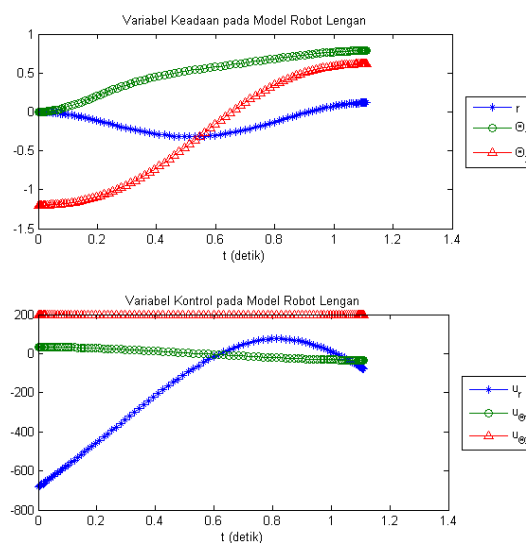


Figure 5: Plot of state and control variables with objective function using regularization at u_{θ_1} and u_{θ_2} .

1.2 Comparison of the Optimal Control Solutions of the Robotic Arm Models

From the four objective functions in equations (1), (2), (3), and (4) that were tested in this study, the results of the comparison contained in Figure 2 to Figure 5 and Table 1. In Table 1, it can be seen that the lowest total energy u_r is 208,9434 in the objective function case without using regularization, while the highest total energy u_r is 230,2721 in the case of the objective function using regularization at u_{θ_1} and u_{θ_2} . Furthermore, the lowest total energy u_{θ_1} is 22,8168 in the case of the objective function with regularization in u_{θ_1} and u_{θ_2} , the highest total energy u_{θ_1} is 32,9519 in the case of the objective function without using regularization. Then the lowest total energy u_{θ_2} is 210,0957 in the case of the objective function without using regularization, while the highest total energy u_{θ_2} is 222,0099 in the case of the goal function with the regularization at u_{θ_1} . The shortest time required is 0,9923 seconds in the objective function without using regularization, while the longest time is 1,1103 seconds in the objective function with regularization at u_{θ_1} and u_{θ_2} . From the four cases of the objective functions, it can be concluded that the objective function without regularization is better because it gives the shortest time and the lowest amount of energy u_r .

Table 1: Total energy of each control variable and optimal time of the four objective functions

No.	Objective functions	u_r	u_{θ_1}	u_{θ_2}	t_{opt}
1	Without regularization	208,9434	32,9519	210,0957	0,9923
2	With regularization at u_{θ_1}	227,5346	22,9171	222,0099	1,0993
3	With regularization at u_{θ_2}	216,8839	32,7731	215,3515	1,0591
4	With regularization at u_{θ_1} and u_{θ_2}	230,2721	22,8168	220,8768	1,1103

1.3 Comparison with Previous Research

To observe the performance of the optimal control model in this study in comparison with the previous study, especially in [5], it should be noted that the objective function in the Geise optimal control model [5] is to minimize the time as in equation (26). The obtained minimum time was 0,38 seconds.

$$J = t_f \tag{26}$$

Meanwhile this study uses an optimal control model with an objective function to minimize time and energy as shown in equation (1) and it is obtained that the optimum time is 0,8817 seconds. For $(r(t_f), \theta_1(t_f), \theta_2(t_f)) = (0, 0, -1,4)$, the results obtained in [5] and this study can be seen in Table 2, Figure 6 and Figure 7.

Table 2: Comparison of the results of the Geise model [5] and the optimal control model of in this study.

Variables	Robotic armmodel with minimizing time(Geiseet.al, 2004 [5])	Robotic armmodel with minimizing time and energy
Maximum r	0,1	$-5,2492 \times 10^{-8}$
Minimum r	0	-0,2259
Maximum θ_1	0,6	$8,4621 \times 10^{-27}$
Minimum θ_1	0	$-4,1621 \times 10^{-27}$
Maximum θ_2	0,4	0,4
Minimum θ_2	-1,2	-1,2
Maximum \dot{r}	1,1	0,7844
Minimum \dot{r}	-1,5	-0,8035
Maximum $\dot{\theta}_1$	15	0
Minimum $\dot{\theta}_1$	-7,5	0
Maximum $\dot{\theta}_2$	4	2,9977
Minimum $\dot{\theta}_2$	0	0,0012
Maximum u_r	800	-19,8436
Minimum u_r	-800	-677,5028
Maximum u_{θ_1}	1400	0
Minimum u_{θ_1}	-1400	0
Maximum u_{θ_2}	300	300
Minimum u_{θ_2}	-300	99,8698

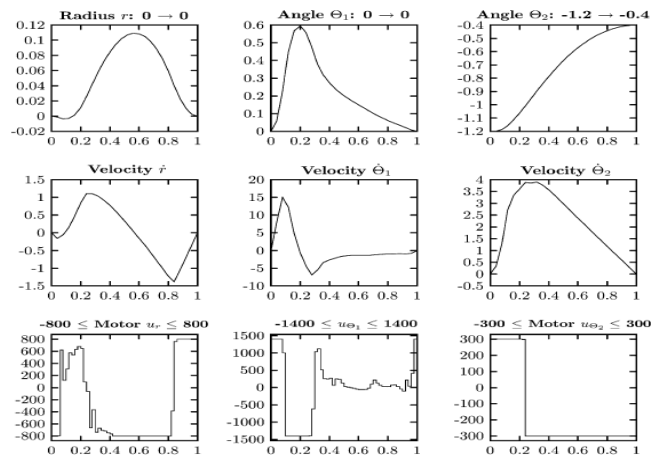


Figure 6: Plot of state variables and control variables of the objective function of the Geise model [5].

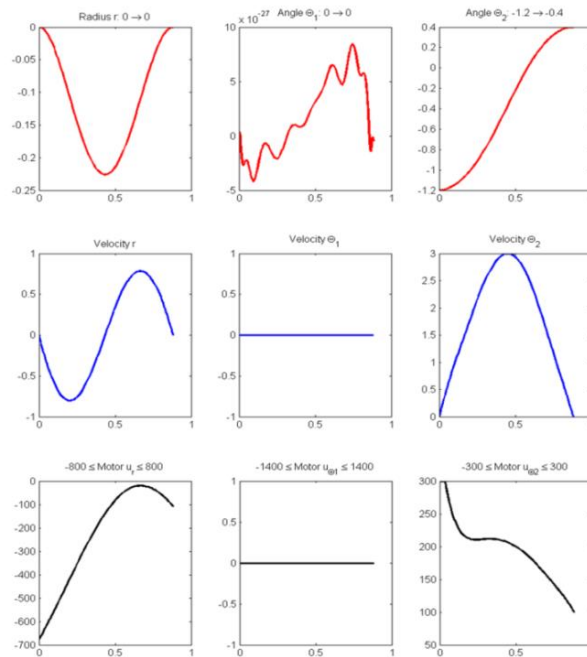


Figure 7: Plot of state and control variables with the objective function of equation (1)

In [5], the maximum value of r reaches 0.1 and the minimum approaches 0. While in this study the maximum value of r is $-5,2492 \times 10^{-8}$ and the minimum is $-0,2259$. For the maximum value of θ_1 in [5], it reaches 0,6 the minimum is 0. Meanwhile in this study, the maximum value of θ_1 is $8,4621 \times 10^{-27}$ and the minimum is $-4,1621 \times 10^{-27}$, in other words it is almost constant at 0. Furthermore, the value of θ_2 based on Geise model [5] and this research is the same which is maximum at 0,4 and minimum at -1,2. The maximum velocity \dot{r} in [5] is 1,1 and the minimum reaches -1,5, while in this study the maximum value of \dot{r} is 0,7844 and the minimum is -0,8035. The maximum angular velocity $\dot{\theta}_1$ in [5] is 15 and the minimum approaches -7,5, while in this study the value of $\dot{\theta}_1$ is constant at 0. Furthermore, the maximum angular velocity $\dot{\theta}_2$ in [5] approaches 4 and its minimum is 0. In this study the maximum angular velocity $\dot{\theta}_2$ is 2,9977 and the minimum is 0,0012.

In terms of energy use in the Geise model [5], the maximum value of the control variable u_r is 800 and the minimum is -800. In this study the maximum value of the control variable u_r is -19,8436 and the minimum is -

677,5028. For the value of control variable u_{θ_1} , in [5] its maximum and minimum are 1400 and -1400 respectively, while in this study the value is constant at 0. Furthermore, the maximum value of control variable u_{θ_2} in [5] is 300 and its minimum is -300. In contrast to the value of the control variable u_{θ_2} in this study, the maximum value is 300 and the minimum is 99,8698. From some of the results which have been described, it can be concluded that although the time required for the model in this study is longer than the Geise model [5] with a time difference of 0.5017 seconds, but the model in this study gives less energy use as shown in Table 2.

IV. CONCLUSION

From the results of the study it can be concluded that the time taken by the robotic arm to reach the object is 0,9923 seconds with a minimum total energy use u_r is 208,9434, total energy u_{θ_1} is 32,9519, and total energy u_{θ_2} is 210,0957. Optimal control model solutions for control variables with stable conditions do not need to add regularization because it can increase the performance time of the robotic arm to reach the destination point. However, the optimal control model for control variables with unstable conditions requires regularization because it can smooth the movement of the robotic arm in reaching the destination point. The time difference between the optimal control model solution and the objective function minimizing time and energy (this study) and the optimal control model solution with the objective function only minimizing the time (model in [5]) is 0,5017 seconds. However, the use of energy in this study is less than the Geise model [5]. Robotic arm with arm twice as long can hamper the movement to reach the destination point so that it takes longer and uses more energy than robotic arm that has normal arm lengths.

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