

Solving Neutrosophic Travelling Salesman Problem in Triangular Fuzzy Number Using Ones Assignment Method

S. Subasri and K. Selvakumari*

Abstract--- Travelling salesman problem is a well-known studied problem and intensely used in computational mathematics and management studies. In this paper we discuss a Neutrosophic triangular fuzzy numbers which are more realistic in nature. Neutrosophic triangular fuzzy numbers can be converted to a Neutrosophic fuzzy numbers by using a Pascal's graded mean approach and then the problem is solved by ones assignment method. Neutrosophic crisp travelling salesman problem also solved by using a score function and finally find the optimality. Numerical examples are included for a fruitful way.

Keywords--- Neutrosophic Fuzzy Number, Triangular Neutrosophic Fuzzy Number, Fuzzy Travelling Salesman Algorithm, Graded Mean Approach, Score Function.

I. INTRODUCTION

The travelling salesman problem is the well-known problem and one of the most intensely studied problem in the field of computational mathematics. A huge number of techniques were constructed to solve the problem. The problem ascends from the characteristic situation of a salesman who wants to travel a city only once in a map and return to his home town. In such a way that the length of the journey is too short among all other possible journey. Fuzzy sets were proposed by Prof. Zadeh in 1965, to handle data and information feature of non-statistical ambiguity. One of the main applications of fuzzy arithmetic is accommodated the parameters and is represented by a fuzzy number.

The Neutrosophic set was first proposed by F. Smarandache in 1995. The evolution of Neutrosophic components are truth, indeterminacy and falsity membership values respectively and it is non-standard unit interval. Here, we generate two methods to solve the Neutrosophic travelling salesman problem. The Neutrosophic triangular fuzzy number using a graded mean approach. (i.e.,) all the Neutrosophic triangular fuzzy numbers can be converted to a Neutrosophic fuzzy number. Neutrosophic crisp travelling salesman problem also discussed in this paper.

II. PRELIMINARIES

2.1 Definition [10]

A **fuzzy set** characterized by a membership function mapping element of a domain, space, or the universe of discourse X to the unit interval $[0,1]$ i.e. $A = \{(x, \mu_A(x)) / x \in X\}$. Here $\mu_A(x): X \rightarrow [0,1]$ is a mapping called the

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degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging [0,1].

2.2 Definition [11]

A fuzzy set A of real line R with membership function $\mu_A(x): R \rightarrow [0,1]$ is called **fuzzy number** if

- (i) A is normal and convexity.
- (ii) A must be bounded.
- (iii) $\mu_A(x)$ is piecewise continuous.

2.3 Definition [11]

A fuzzy number $A = (a_0, a_1, a_2)$ is said to be **triangular fuzzy number** if its membership function is given by

$$\mu_A(x) = \begin{cases} \frac{x-a_0}{a_1-a_0}, & a_0 \leq x \leq a_1 \\ 1, & x = a_1 \\ \frac{a_2-x}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ 0, & \text{otherwise} \end{cases}$$

Where $a_0 \leq a_1 \leq a_2$ are real numbers.

2.4 Definition [8]

Let X be a universe. A **Neutrosophic set** A over X is defined by

$A^N = \{x: T_{a^N}(x), I_{a^N}(x), F_{a^N}(x) : x \in X\}$ where $T_{a^N}, I_{a^N}, F_{a^N}: X \rightarrow]0^-, 3^+[$ are called the truth, indeterminacy and falsity membership function of the element $x \in X$ to the set A^N with $0^- \leq T_{a^N}(x) + I_{a^N}(x) + F_{a^N}(x) \leq 3^+$

2.5 Definition [8]

Let X be the finite universe of discourse and $F^N[0,1]$ denoted by the set of all triangular fuzzy numbers on [0,1]. A **Neutrosophic triangular fuzzy set** A in X is represented by $A^N = \{x: T_{a^N}(x), I_{a^N}(x), F_{a^N}(x) / x \in X\}$

Where the Neutrosophic triangular fuzzy numbers

$T_{a^N}(x) = (T_{a_1^N}(x), T_{a_2^N}(x), T_{a_3^N}(x)), I_{a^N}(x) = (I_{a_1^N}(x), I_{a_2^N}(x), I_{a_3^N}(x)), F_{a^N}(x) = (F_{a_1^N}(x), F_{a_2^N}(x), F_{a_3^N}(x))$ be the truth, indeterminacy and falsity membership degree of x in A and for every $x \in X$ such that $0^- \leq T_{a^N}(x) + I_{a^N}(x) + F_{a^N}(x) \leq 3^+$

III. NEUTROSOPHIC FUZZY TRAVELLING SALESMAN PROBLEM

Suppose a salesman has to visits n cities. He visits one particular city and return to the home town within a short period of time. The Neutrosophic fuzzy travelling salesman problem in the following matrix may be formulated as

City → ↓	1	2	...	j	...	n
1	∞	$T_{12}^N, I_{12}^N, F_{12}^N$...	$T_{1j}^N, I_{1j}^N, F_{1j}^N$...	$T_{1n}^N, I_{1n}^N, F_{1n}^N$
2	$T_{21}^N, I_{21}^N, F_{21}^N$	∞	...	$T_{2j}^N, I_{2j}^N, F_{2j}^N$...	$T_{2n}^N, I_{2n}^N, F_{2n}^N$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
J	$T_{j1}^N, I_{j1}^N, F_{j1}^N$	$T_{j2}^N, I_{j2}^N, F_{j2}^N$...	∞	...	$T_{jn}^N, I_{jn}^N, F_{jn}^N$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
N	$T_{n1}^N, I_{n1}^N, F_{n1}^N$	$T_{n2}^N, I_{n2}^N, F_{n2}^N$...	$T_{nj}^N, I_{nj}^N, F_{nj}^N$...	∞

$$\begin{pmatrix} (T_{a_{11}}^N, I_{a_{11}}^N, F_{a_{11}}^N)/a_1^N b_1^N & (T_{a_{12}}^N, I_{a_{12}}^N, F_{a_{12}}^N)/a_1^N b_2^N & (T_{a_{13}}^N, I_{a_{13}}^N, F_{a_{13}}^N)/a_1^N b_3^N & \cdots & (T_{a_{1n}}^N, I_{a_{1n}}^N, F_{a_{1n}}^N)/a_1^N b_n^N \\ (T_{a_{21}}^N, I_{a_{21}}^N, F_{a_{21}}^N)/a_2^N b_1^N & (T_{a_{22}}^N, I_{a_{22}}^N, F_{a_{22}}^N)/a_2^N b_2^N & (T_{a_{23}}^N, I_{a_{23}}^N, F_{a_{23}}^N)/a_2^N b_3^N & \cdots & (T_{a_{2n}}^N, I_{a_{2n}}^N, F_{a_{2n}}^N)/a_2^N b_n^N \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ (T_{a_{n1}}^N, I_{a_{n1}}^N, F_{a_{n1}}^N)/a_n^N b_1^N & (T_{a_{n2}}^N, I_{a_{n2}}^N, F_{a_{n2}}^N)/a_n^N b_2^N & (T_{a_{n3}}^N, I_{a_{n3}}^N, F_{a_{n3}}^N)/a_n^N b_3^N & \cdots & (T_{a_{nn}}^N, I_{a_{nn}}^N, F_{a_{nn}}^N)/a_n^N b_n^N \end{pmatrix} \begin{matrix} a_1^N \\ a_2^N \\ \vdots \\ a_n^N \end{matrix}$$

$$b_1^N b_2^N b_3^N \cdots b_n^N$$

Step 4: Obtain the optimum schedule and minimum cost for the travelling salesman problem

VI. PROCEDURE FOR THE NEUTROSOPHIC CRISP TRAVELLING SALESMAN PROBLEM

Step 1: Construct the cost matrix of Neutrosophi fuzzy travelling salesman problem, $A^N = (C_{ij})_{m \times n}$

Step 2: Determine the evaluation Neutrosophic matrix as $E^N(A) = [(T^N)^l, (T^N)^u]$

$$\text{Where, } [(T^N)^l, (T^N)^u] = \begin{bmatrix} \min\left(\left(\frac{T^N + I^N}{2}\right), \left(\frac{1 - F^N + I^N}{2}\right)\right), \\ \max\left(\left(\frac{T^N + I^N}{2}\right), \left(\frac{1 - F^N + I^N}{2}\right)\right) \end{bmatrix}$$

Step 3: Compute the score function $S^N(A)$ of an alternative $S^N(A) = 2[(T^N)^u - (T^N)^l]$

$$\text{i.e., } S^N(A) = 2 \left[\begin{matrix} \max\left(\left(\frac{T^N + I^N}{2}\right), \left(\frac{1 - F^N + I^N}{2}\right)\right) - \\ \min\left(\left(\frac{T^N + I^N}{2}\right), \left(\frac{1 - F^N + I^N}{2}\right)\right) \end{matrix} \right], \text{ where } 0 \leq S^N(A) \leq 1.$$

Step 4: Take the score function matrix as initial input data for Neutrosophic crisp travelling salesman problem and solve it by ones assignment method.

VII. NUMERICAL EXAMPLE

Method: 1-Solving Neutrosophic fuzzy travelling salesman problem

Let us consider the following travelling salesman problem. A company has four territories. The working salesman has to distribute the products from one territory to other. The cost of the travel from territories A,B,C,D to A,B,C,D is given below.

	A	B	C	D
A	∞	$[(1,2,3); (3,4,5); (4,5,6)]$	$[(0,1,2); (1,2,3); (2,3,4)]$	$[(0,1,6); (0,3,6); (2,8,10)]$
B	$[(0,1,2); (0,2,4); (2,3,4)]$	∞	$[(0,1,6); (2,4,6);]$	$[(1,2,7); (0,3,10); (2,8,10)]$
C	$[(0,1,6); (0,2,8); (2,3,8)]$	$[(0,4,8); (3,4,9); (2,8,10)]$	∞	$[(0,1,2); (0,2,4); (3,4,5)]$
D	$[(0,2,4); (2,3,4); (4,6,8)]$	$[(0,2,4); (1,2,7); (2,4,6)]$	$[(0,1,2); (1,2,3); (2,3,4)]$	∞

Solution

Step 1: To convert the given problem into Neutrosophic number by using graded mean approach for triangular fuzzy number

$P^T(a_{ij})$	MEMBERSHIP	INDETERMINACY	NON-MEMBERSHIP
$P^T(a_{12})$	2	4	5
$P^T(a_{13})$	1	2	3
$P^T(a_{14})$	2	3	7
$P^T(a_{21})$	1	2	3
$P^T(a_{23})$	2	4	5
$P^T(a_{24})$	3	4	7
$P^T(a_{31})$	2	3	4
$P^T(a_{32})$	4	5	7
$P^T(a_{34})$	1	2	4
$P^T(a_{41})$	2	3	6
$P^T(a_{42})$	2	3	4
$P^T(a_{43})$	1	2	3

Step 2: The Neutrosophic fuzzy travelling salesman problem is given by

$$\begin{matrix} & A & B & C & D \\
 \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \left(\begin{matrix} \infty & (2,4,5) & (1,2,3) & (2,3,7) \\
 (1,2,3) & \infty & (2,4,5) & (3,4,7) \\
 (2,3,4) & (4,5,7) & \infty & (1,2,4) \\
 (2,3,6) & (2,3,4) & (1,2,3) & \infty \end{matrix} \right)
 \end{matrix}$$

Step 3: Find out the minimum Neutrosophic element of each row in the distance matrix (say a_i^N) and put it on the right side of the matrix,

$$\begin{matrix} & A & B & C & D \\
 \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \left(\begin{matrix} \infty & (2,4,5) & (1,2,3) & (2,3,7) \\
 (1,2,3) & \infty & (2,4,5) & (3,4,7) \\
 (2,3,4) & (4,5,7) & \infty & (1,2,4) \\
 (2,3,6) & (2,3,4) & (1,2,3) & \infty \end{matrix} \right) \begin{matrix} (1,2,3) \\
 (1,2,3) \\
 (1,2,4) \\
 (1,2,3) \end{matrix}
 \end{matrix}$$

Then divide each element of i^{th} row of the matrix by a_i^N . Each row creates some ones and the matrix will reduce to the following

$$\begin{matrix} & A & B & C & D \\
 \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \left(\begin{matrix} \infty & (2,2,1.6) & 1^N & (2,1.5,2.3) \\
 1^N & \infty & (2,2,1.6) & (3,2,2.3) \\
 (2,1.5,1) & (4,2.5,1.7) & \infty & 1^N \\
 (2,1.5,2) & (2,1.5,1.3) & 1^N & \infty \end{matrix} \right) \begin{matrix} (1,2,3) \\
 (1,2,3) \\
 (1,2,4) \\
 (1,2,3) \end{matrix}
 \end{matrix}$$

Step 4: Now find the minimum element of each column in distance matrix (say b_j^N) and divide each element of j^{th} column of the matrix by b_j^N . This will create some ones to each row and column.

$$\begin{matrix} & A & B & C & D \\
 \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \left(\begin{matrix} \infty & (1,1.3,1.2) & 1^N & (2,1.5,2.3) \\
 1^N & \infty & (2,2,1.6) & (3,2,2.3) \\
 (2,1.5,1) & (2,1.6,1.3) & \infty & 1^N \\
 (2,1.5,2) & 1^N & 1^N & \infty \end{matrix} \right) \begin{matrix} (1,2,3) \\
 (1,2,3) \\
 (1,2,4) \\
 (1,2,3) \end{matrix}
 \end{matrix}$$

Step 5: The optimum schedule is given by

$$\begin{array}{c}
 \\
 \\
 \\
 \\
 \end{array}
 \begin{array}{cccc}
 & A & B & C & D \\
 A & \left(\begin{array}{cccc}
 \infty & (1,1.3,1.2) & [1^N] & (2,1.5,2.3) \\
 [1^N] & \infty & (2,2,1.6) & (3,2,2.3) \\
 (2,1.5,1) & (2,1.6,1.3) & \infty & [1^N] \\
 (2,1.5,2) & [1^N] & 1^N & \infty
 \end{array} \right) & \begin{array}{l}
 (1,2,3) \\
 (1,2,3) \\
 (1,2,4) \\
 (1,2,3)
 \end{array}
 \end{array}$$

Step 6: Territory A assigns to Territory C Distance (1,2,3)

Territory B assigns to Territory A Distance (1,2,3)

Territory C assigns to Territory D Distance (1,2,4)

Territory D assigns to Territory B Distance (2,3,4)

So, the root conditions are satisfied and is given by

$$A \rightarrow C; B \rightarrow A; C \rightarrow D; D \rightarrow B$$

$$A \rightarrow C \rightarrow D \rightarrow B \rightarrow A$$

The Neutrosophic fuzzy optimal solution = (1,2,3)+(1,2,4)+(2,3,4)+(1,2,3)=(5,9,14)

Method 2: Solving Neutrosophic crisp travelling salesman problem

In Previous example, the Neutrosophic fuzzy travelling salesman problem is

$$\begin{array}{c}
 \\
 \\
 \\
 \\
 \end{array}
 \begin{array}{cccc}
 & A & B & C & D \\
 A & \left(\begin{array}{cccc}
 \infty & (2,4,5) & (1,2,3) & (2,3,7) \\
 (1,2,3) & \infty & (2,4,5) & (3,4,7) \\
 (2,3,4) & (4,5,7) & \infty & (1,2,4) \\
 (2,3,6) & (2,3,4) & (1,2,3) & \infty
 \end{array} \right)
 \end{array}$$

The evaluation matrix is as follows:

$$\begin{array}{c}
 \\
 \\
 \\
 \\
 \end{array}
 \begin{array}{cccc}
 & A & B & C & D \\
 A & \left(\begin{array}{cccc}
 \infty & (0,3) & (0,1.5) & (-2,2.5) \\
 (0,1.5) & \infty & (0,3) & (-1,3.5) \\
 (0,2.5) & (1,5) & \infty & (-0.5,1.5) \\
 (-1,2.5) & (0,2.5) & (0,1.5) & \infty
 \end{array} \right)
 \end{array}$$

The score function of the above matrix is

$$\begin{array}{c}
 \\
 \\
 \\
 \\
 \\
 \\
 \\
 \\
 \end{array}
 \begin{array}{cccc}
 & A & B & C & D \\
 A & \left(\begin{array}{cccc}
 \infty & 6 & 3 & 9 \\
 3 & \infty & 6 & 9 \\
 5 & 10 & \infty & 4 \\
 7 & 5 & 3 & \infty
 \end{array} \right) & \begin{array}{l}
 3 \\
 3 \\
 4 \\
 3
 \end{array}
 \end{array}$$

$$\begin{array}{c}
 \\
 \\
 \\
 \\
 \\
 \\
 \\
 \\
 \end{array}
 \begin{array}{cccc}
 & A & B & C & D \\
 A & \left(\begin{array}{cccc}
 \infty & 2 & 1^N & 3 \\
 1^N & \infty & 2 & 3 \\
 1.2 & 2.5 & \infty & 1^N \\
 2.3 & 1.6 & 1^N & \infty
 \end{array} \right) & \begin{array}{l}
 3 \\
 3 \\
 4 \\
 3
 \end{array}
 \end{array}$$

$$\begin{matrix}
 & A & B & C & D \\
 A & \left(\begin{matrix} \infty & 3.2 & [1^N] & 3 \\ [1^N] & \infty & 2 & 3 \\ 1.2 & 1.5 & \infty & [1^N] \\ 2.3 & [1^N] & 1^N & \infty \end{matrix} \right) & \begin{matrix} 3 \\ 3 \\ 4 \\ 3 \end{matrix}
 \end{matrix}$$

The resultant matrix is

$$\begin{matrix}
 & A & B & C & D \\
 A & \left(\begin{matrix} \infty & 3.2 & [1^N] & 3 \\ [1^N] & \infty & 2 & 3 \\ 1.2 & 1.5 & \infty & [1^N] \\ 2.3 & [1^N] & 1^N & \infty \end{matrix} \right) & \begin{matrix} 3 \\ 3 \\ 4 \\ 3 \end{matrix}
 \end{matrix}$$

Territory A assigns to Territory C Distance 3

Territory B assigns to Territory A Distance 3

Territory C assigns to Territory D Distance 4

Territory D assigns to Territory B Distance 5

Thus, the crisp optimum solution is

$$\begin{aligned}
 & \mathbf{A \rightarrow C ; B \rightarrow A ; C \rightarrow D ; D \rightarrow B} \\
 & \mathbf{A \rightarrow C \rightarrow D \rightarrow B \rightarrow A = 3 + 4 + 5 + 3 = 15}
 \end{aligned}$$

VIII. CONCLUSION

In this paper, we proposed a new methodology to find the optimal solution for the Neutrosophic fuzzy travelling salesman problem by using the Pascal’s graded mean approach and also compared with crisp travelling salesman problem. This method is very useful for the future study. In future, we extend this method in generalized Neutrosophicfuzzy number.

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