

ALGORITHMS FOR SUSTAINABLE RECOVERY OF INPUT INFLUENCE ON THE BASIS OF DYNAMIC FILTRATION METHODS

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Abstract--- *The problems of constructing algorithms for the sustainable recovery of unknown signals in dynamic control systems are considered. Algorithms for recovering input actions based on dynamic filtering methods and solving incorrectly posed problems are presented. The regularized Cholesky factorization method for symmetric matrices is used as a regular procedure. The above algorithms make it possible to stabilize the matrix inversion procedure when assessing the state of stochastic objects and thereby improve the accuracy of determining the true state vector estimate under perturbation of the object and observer parameters.*

Keywords--- *sustainable recovery, input impact, dynamic filtering method, regularization.*

I. Introduction

The task of restoring the initial state and input of a dynamic system from the results of measuring the output belongs to the class of inverse problems of the dynamics of controlled systems [1]. Since the indicated problem is incorrectly posed, methods developed in the corresponding theory should be applied for its solution [2–9].

Consider a linear dynamic system with the observation:

$$x_{k+1} = A_k x_k + B_k w_k, \quad x(k_0) = x^0, \quad (1)$$

$$y_k = C_k x_k + D_k w_k, \quad (2)$$

where $x \in R^n$, $w \in R^p$, $y \in R^m$; $x = x_k$ – state of the system; x^0 – initial state of the system; $w_k \in L_2^p$ – input unmeasured disturbing effect on the system; $y_k \in L_2^m$ – system output; A_k, B_k, C_k, D_k – matrices of corresponding dimensions.

Let

$$\Theta = R^n \times L_2^p, \quad Y = L_2^m.$$

We transform the space Θ into a Hilbert space by defining the scalar product

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$$\langle \theta_1, \theta_2 \rangle_{\Theta} = \langle x_1^0, x_2^0 \rangle_{R^n} + \langle w_1, w_2 \rangle_{L_2^2}$$

on it.

Relations (1), (2) determine the linear operator $F : \Theta \rightarrow Y$, which each pair $\theta = (x_0, w) \in \Theta$, i.e. system input, associates function $y \in Y$ at the system output. Let y^* be some output of system (1), (2). We denote by Θ^* the nonempty set of all inputs $\theta \in \Theta$ such that

$$F\theta = y^*. \quad (3)$$

II. Formulation of the problem

Consider the variational problem

$$\Omega(\theta) \rightarrow \min, \theta \in \Theta^*,$$

where $\Omega : \Theta \rightarrow R$ is a non-negative, lower semicontinuous and strictly uniformly convex functional.

Consider the following task: on output y^* , restore Ω - a normal input compatible with this output. Suppose that the output y^* is not known to us, and only the function $y_{\delta} \in Y$ (the result of measuring the output) is known such that

$$\|y_{\delta} - y^*\|_Y \leq \delta,$$

where δ is a known non-negative parameter characterizing the accuracy of the measurements. In this case, it is impossible to accurately restore the set Θ^* and especially the element $\theta^* \in \Theta^*$. Therefore, we pose the problem of the approximate restoration of the element $\theta^* = (x_*^0, w^*)$ according to the results of inaccurate measurements of the output y^* under the assumption that the matrices A, B, C, D and functional Ω are known exactly. Using function y_{δ} and parameter $\delta > 0$, it is necessary to find a pair of $\theta_{\delta} = (x_{\delta}^0, w_{\delta}(\cdot)) \in \Theta$ such that

$$\|\theta_{\delta} - \theta^*\|_{\Theta} \rightarrow 0$$

at $\delta \rightarrow 0$.

III. Decision

To solve equation (3), you can also use the concept of dynamic filtering. To dynamite equation (3), we write it in the form:

$$\theta_{k+1} = \theta_k + w_k, \quad \theta(0) = \theta_0, \quad (4)$$

$$y_{k+1}^* = F_{k+1} \theta_{k+1} + v_{k+1} \quad (k = 0, 1, \dots),$$

where θ_k - state vector of the system, y_k^* - measurement vector, w_k and v_k - Gaussian white noises with zero mathematical expectation and intensities of Q_k , R_k , θ_0 - Gaussian random vectors with known characteristics of $M(\theta_0)$ and $M(\theta_0\theta_0^T) = P_0$.

We will assume that w_k and v_k are not correlated with θ_0 , but

$$M[w_k v_j^T] = S_k \delta_{kj}, \quad S_k \neq 0,$$

where δ_{kj} is the Kronecker symbol. Matrix R_k is positive definite.

It is also assumed that

$$w_k = v_k + w_k^0,$$

where

$$M[w_k^0 v_j^T] = 0,$$

$$\text{at } \forall k, j, M[w_k^0 w_j^{0T}] = Q_k^0 \delta_{kj}.$$

In accordance with [10], we have

$$\hat{\theta}_{k+1|k} = \hat{\theta}_{k|k} + M[w_k | y_k]. \quad (5)$$

It can be shown that in the case under consideration, condition

$$M[w_k^0 y_j^{*T}] = \begin{cases} 0, & \text{если } j < k, \\ S_k, & \text{если } j = k, \end{cases} \quad (6)$$

is satisfied.

Based on the properties of conditional mathematical expectations [11], as well as relation (6), we obtain

$$M[w_k | y_k] = W_k [y_k^* - F_k \hat{\theta}_{k|k-1}], \quad (7)$$

where

$$W_k = S_k [F_k P_{k|k-1} F_k^T + R_k]^{-1}, \quad (8)$$

$$P_{k,j} - \text{correlation matrix of the estimation error } \varepsilon_{k|j} = \theta_k - \hat{\theta}_{k|j}.$$

Substituting (7) into (5), we find

$$\hat{\theta}_{k+1|k} = \hat{\theta}_{k|k} + W_k [y_k^* - F_k \hat{\theta}_{k|k-1}]. \quad (9)$$

Based on the representations of [10, 12], we express $\hat{\theta}_{k+1|k+1}$ in terms of $\hat{\theta}_{k+1|k}$:

$$\hat{\theta}_{k+1|k+1} = \hat{\theta}_{k+1|k} + K_{k+1} \tilde{y}_{k+1|k}^*,$$

$$K_{k+1} = P_{k+1|k} F_{k+1}^T P_{k+1}^{-1}, \quad (10)$$

$$\mathbf{P}_{k+1} = F_{k+1} \mathbf{P}_{k+1|k} F_{k+1}^T + \mathbf{R}_{k+1}, \quad (11)$$

$$\tilde{y}_{k+1|k}^* = F_{k+1} \tilde{\theta}_{k+1|k} + v_{k+1}, \quad \tilde{\theta}_{k+1|k} = \theta_{k+1} - \hat{\theta}_{k+1|k},$$

$$y_{k+1}^* - F_{k+1} \hat{\theta}_{k+1|k} = [F_{k+1} \mathbf{P}_{k+1|k} F_{k+1}^T \mathbf{R}_{k+1}^{-1} + I][y_{k+1}^* - F_{k+1} \hat{\theta}_{k+1|k+1}],$$

where $G_\alpha(\mathbf{P}_{k+1})$ – generating system of functions for the regularization method, α – regularization parameter.

Then

$$\hat{\theta}_{k+1|k} = \hat{\theta}_{k|k} + D_k [y_k^* - F_k \hat{\theta}_{k|k}], \quad (12)$$

where

$$D_k = S_k \mathbf{R}_k^{-1}.$$

A matrix \mathbf{P}_{k+1} of the form (11), the pseudoinverse of which \mathbf{P}_{k+1}^+ is used in (10), is a symmetric ill-conditioned sign-indefinite matrix. In order to stabilize the desired solution and give greater numerical stability to the pseudo inversion procedure in (10), it is necessary to use regular methods [13-19]. When implementing (10), we will use the regularized Cholesky factorization method for symmetric matrices [20].

Based on a symmetric matrix \mathbf{P}_{k+1} of order n with $\rho_{k+1,ij}$ elements, a sequence of matrices is constructed:

$$\mathbf{P}_{k+1}^{(r)} = \begin{bmatrix} \mathbf{P}_{k+1,1}^{(r)} & \vdots & \mathbf{P}_{k+1,2}^{(r)} \\ -\mathbf{P}_{k+1,1}^{(r)} & \vdots & \mathbf{P}_{k+1,3}^{(r)} \\ 0 & \vdots & \mathbf{P}_{k+1,3}^{(r)} \end{bmatrix}, \quad r = 0, 1, \dots, \quad (13)$$

where $\mathbf{P}_{k+1,1}^{(r)}$ – an upper triangular matrix of size $k \times k$, $\mathbf{P}_{k+1,2}^{(r)}$ – rectangular matrix, $\mathbf{P}_{k+1,3}^{(r)}$ – symmetric matrix of order $n - k$, 0 – zero matrix.

For this, the leading element is determined in cell $\mathbf{P}_{k+1,3}^{(r)}$ by comparing its maximum elements standing on the diagonal and outside the diagonal:

$$|\rho_{k+1,\zeta\zeta}^{(r)}| = \max_{k < i \leq n} |\rho_{k+1,ii}^{(r)}|, \quad |\rho_{k+1,\varpi}^{(r)}| = \max_{k < i \leq n, i < j \leq n} |\rho_{k+1,ij}^{(r)}|.$$

If $|\rho_{k+1,\zeta\zeta}^{(r)}| \geq |\rho_{k+1,\varpi}^{(r)}|$ and $|\rho_{k+1,\zeta\zeta}^{(r)}| > \varepsilon$, then the matrix $\mathbf{P}_{k+1}^{(r)}$ swaps the ζ -th row and column with $(k+1)$ -th row and column, respectively.

After permutations, the matrix $\mathbf{P}_{k+1}^{(r+1)}$ is determined, which differs from that obtained after permutations $\mathbf{P}_{k+1}^{(r)}$ only by the elements of the cell

$$\mathbf{P}_{k+1,3}^{(r)} = \begin{bmatrix} \rho_{k+1,k+1}^{(r)} & \vdots & d^{(r)} \\ (d^{(r)})^T & \vdots & W_k \end{bmatrix}, \quad (14)$$

which takes the form

$$\left[\begin{array}{c|c} |\rho_{k+1,k+1}^{(r)}|^{1/2} & \alpha^{(r)} \\ \hline 0 & \hat{\mathbf{P}}_{k+1,3}^{(r+1)} \end{array} \right], \quad (15)$$

where $\alpha^{(r)} = |\rho_{k+1,k+1}^{(r)}|^{-1/2} d^{(r)} \text{sign} \rho_{k+1,k+1}^{(r)}$, $\mathbf{P}_{k+1,3}^{(r+1)} = \mathbf{W}_3^{(r)} - (\rho_{k+1,k+1}^{(r)})^{-1} (d^{(r)})^T d^{(r)}$. Then, the transition to the next factorization step is made.

If $|\rho_{\zeta\zeta}^{(r)}| < |\rho_{\tau\tau}^{(r)}|$ and $|\rho_{\tau\tau}^{(r)}| > \varepsilon$, then the orthogonal transformation

$$\mathbf{B}_l = (b_{ij})_{i,j=1}^n, \quad (16)$$

is introduced, the elements of which coincide with the identity matrix, with the exception of four elements defined as follows: $b_{\tau\tau} = -b_{ss} = b_{\tau s} = b_{s\tau} = 2^{-1/2}$. The matrix

$$\hat{\mathbf{P}}_{k+1}^{(r)} = \mathbf{B}_l \hat{\mathbf{P}}_{k+1}^{(r)} \mathbf{B}_l,$$

is calculated, then the τ -th column and the s -th row are rearranged from the $(l+1)$ -th column and $(l+2)$ -th row so that the resulting $(l+1)$ -th diagonal element is the largest. Next, a recount is made in accordance with (14), (15) of the elements of cell $\hat{\mathbf{P}}_{k+1,3}^{(r)}$ and the transition to the next factorization step is carried out, while $\mathbf{P}_{k+1}^{(r+1)}$ is taken as received $\hat{\mathbf{P}}_{k+1}^{(r+1)}$.

As soon as $|\rho_{\tau\tau}^{(r)}| \leq \varepsilon$ and $|a_{\zeta\tau}^{(r)}| \leq \varepsilon$, the factorization process stops and the non-orthogonal factorization of the matrix F is determined in the form

$$\mathbf{P}_{k+1,\varepsilon} = \hat{\mathbf{U}}_\varepsilon^T \hat{\mathbf{I}} \hat{\mathbf{U}}_\varepsilon, \quad \hat{\mathbf{U}}_\varepsilon = \mathbf{U}_l \mathbf{B}_{(l)},$$

where the upper trapezoidal matrix $\mathbf{U}_l = (\mathbf{P}_{k+1,1}^{(r)}; \mathbf{P}_{k+1,2}^{(r)})$ is composed of cells $\mathbf{P}_{k+1,1}^{(r)}$ and $\mathbf{P}_{k+1,2}^{(r)}$ of the resulting matrix (13); $\mathbf{B}_{(l)} = \mathbf{B}_l \dots \mathbf{B}_1$, where $\mathbf{B}_i = \mathbf{I}$ if conversion (16) was not performed; $\hat{\mathbf{I}}$ – diagonal matrix with a l -th diagonal element defined as $\hat{i}_l = \text{sign} f_{ll}^{(r-1)}$.

If the symmetric matrix \mathbf{P}_{k+1} of order n has a rank of $r \leq n$ and the regularization parameter is taken $\varepsilon = 0$ [17-21], then in the regularized Cholesky method exactly r factorization steps and

$$\mathbf{P}_{k+1,\varepsilon} = \mathbf{P}_{k+1}, \quad \mathbf{P}_{k+1}^+ = \mathbf{B}_{(r)} \mathbf{U}_r^+ \hat{\mathbf{I}} (\mathbf{U}_r^+)^T \mathbf{B}_{(r)}^T$$

will be taken.

If, in addition, \mathbf{P}_{k+1} is a non-negative definite matrix, then the leading element is the diagonal element, $\hat{\mathbf{I}}$ is the identity matrix, and thus

$$\mathbf{P}_{k+1,\varepsilon} = \mathbf{U}_r^T \mathbf{U}_r = \mathbf{P}_{k+1}, \quad \mathbf{P}_{k+1,\varepsilon}^+ = \mathbf{U}_r^+ (\mathbf{U}_r^+)^T.$$

By virtue of [11,22], the optimal current estimate of the state vector is determined using the relation:

$$\begin{aligned}\hat{\theta}_{k+1|k+1} &= M[\theta_{k+1} | y_1^*, \dots, y_k^*, y_{k+1}^*], \\ M[\theta_{k+1} | y_1^*, \dots, y_k^*, y_{k+1}^*] &= M[\theta_{k+1} | y_1^*, \dots, y_k^*] + M[\theta_{k+1} | \tilde{y}_{k+1|k}^*], \\ \tilde{y}_{k+1|k}^* &= y_{k+1}^* - M[y_{k+1}^* | y_1^*, \dots, y_k^*], \\ \hat{\theta}_{k+1|k+1} &= \hat{\theta}_{k+1|k} + M[\theta_{k+1} | \tilde{y}_{k+1|k}^*].\end{aligned}\quad (17)$$

Now using equations (12) and (17), we can obtain the optimal filter equation:

$$\hat{\theta}_{k+1|k+1} = A_k^0 \hat{\theta}_{k|k} + K_{k+1}[y_{k+1}^* - F_{k+1} A_k^0 \hat{\theta}_{k|k}] + [I - K_{k+1} F_{k+1}] D_k y_k^*, \quad \hat{\theta}_{0|0} = 0, 111,$$

where

$$A_k^0 = I - D_k F_k.$$

Based on (4) and (12), we can write:

$$\varepsilon_{k+1|k} = A_k^0 \varepsilon_{k|k} + \Gamma_k v_k,$$

where

$$\Gamma_k = (I, -D_k); \quad v_k = (w_k^T, v_k^T)^T,$$

with

$$L_k = M[v_k v_k^T] = \begin{pmatrix} Q_k & S_k \\ S_k^T & R_k \end{pmatrix},$$

$$\Gamma_k M[v_k \varepsilon_{k|k}^T] = 0.$$

Then

$$P_{k+1|k} = A_k^0 P_{k|k} A_k^{0T} + \Gamma_k L_k \Gamma_k^T,$$

$$P_{k|k} = [I - K_k F_k] P_{k|k-1}, \quad P_{0|0} = P_0. \quad (18)$$

From equations (9) and (17) we find

$$\hat{\theta}_{k+1|k} = \hat{\theta}_{k|k-1} + K_k^0 [y_k^* - F_k \hat{\theta}_{k|k-1}],$$

where

$$K_k^0 = K_k + W_k,$$

or by virtue of (8) and (10):

$$K_k^0 = [P_{k|k-1} F_k^T + S_k] G_\alpha(P_{k+1}).$$

Then we get

$$P_{k+1|k} = A_k^0 P_{k|k-1} A_k^{0T} - A_k^0 P_{k|k-1} F_k^T G_\alpha(P_{k+1}) F_k P_{k|k-1} A_k^{0T} + \Gamma_k L_k \Gamma_k^T. \quad (19)$$

Equation (19) is a discrete Riccati equation, the research methods of which are given, for example, in [10]. Matrix D_k arises as the natural designation of the transfer matrix in equation (12). Note that the assumption that

$M(\theta_0) = 0$ is irrelevant and can be taken into account by introducing the initial condition of the optimal filter equation:

$$\hat{\theta}_{0|0} = M(\theta_0).$$

The constructed algorithm generates an estimate of

$$\hat{\theta}_{k|k} = M(\theta_k | y_i^*, 0 \leq i \leq k)$$

by processing the current measurements y_k^* in conjunction with previous measurements y_{k-1}^* .

The structure of the filtering algorithm for mutually correlated noises, as well as the structure of a similar algorithm for the case of uncorrelated noises, obviously splits into two structural units “forecast” and “correction” characteristic of the least squares method. Like the filter for the case of uncorrelated noise, the recurrent filter in question represents a negative feedback system. The functionality of such a system depends on its dynamic properties, on the stability and quality of the corresponding algorithm.

The stability of the filtering algorithm under mutually correlated noises as a property of a closed loop of the algorithm, independent of external factors, characterizes, according to

$$\hat{\theta}_{k+1|k+1} = A_k^0 \hat{\theta}_{k|k} + D_k y_k^* + K_{k+1} \{y_{k+1}^* - F_{k+1} [A_k^0 \hat{\theta}_{k|k} + D_k y_k^*]\},$$

the features of solutions of the homogeneous equation

$$\hat{\theta}_{k+1|k+1} = [I - K_k F_k] A_k^0 \hat{\theta}_{k|k}.$$

The quality of the recurrent filtering algorithm for mutually correlated noise is determined by the equation of the second moments

$$P_{k+1|k+1} = A_k^0 P_{k|k} A_k^{0T} - A_k^0 P_{k|k} A_k^{0T} F_{k+1}^T K_{k+1}^T + Q_k^0 - Q_k^0 F_{k+1}^T K_{k+1}^T - K_{k+1} F_{k+1} A_k^0 P_{k|k} A_k^{0T} + \\ + K_{k+1} F_{k+1} A_k^0 P_{k|k} A_k^{0T} F_{k+1}^T K_{k+1}^T - K_{k+1} F_{k+1} Q_k^0 + K_{k+1} F_{k+1} Q_k^0 F_{k+1}^T K_{k+1}^T + K_{k+1} R_{k+1} K_{k+1}^T$$

or, respectively, by the system of equations $P_{k+1|k} = A_k^0 P_{k|k} A_k^{0T} + Q_k^0$ and (18) under condition (10).

The structure of the constructed algorithm clearly reveals in it the continuity with classical filtering algorithms for uncorrelated noises and indicates the possibility of using theoretical results obtained for uncorrelated noises for filtering algorithm analysis with mutually correlated noises.

IV. Conclusion

The above algorithms make it possible to stabilize the matrix inversion procedure when assessing the state of stochastic objects and thereby improve the accuracy of determining the true state vector estimate under perturbation of the object and observer parameters.

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