

Verifiable Ideal Guard Point Standard For Air Pollution: A Statistical Analysis

Praveen Kumar Bhatt¹, Dr. Sudesh Kumar², Dr. Ankur Nehra^{3*}

ABSTRACT

The Ideal Standard can be characterized as a proclamation about the number of inhabitants in toxin which is set with no strategy by which consistency is to be tried or screened. For instance, with the Best Standard, we can't register the likelihood that a specific checking site will be in control in the approaching year. Barnett and O'Hogan (1997) presented the idea of the Factual Obvious Ideal Norm (SVIS). The thought is to join the Optimal Norm with a genuinely based rule of execution. To this end, conventional factual apparatus might be utilized. We might utilize Neyman Pearson's Approach of Speculation testing to build SVIS. In this paper, we develop SVIS because of the Neyman Pearson Speculation testing system and research the Air Nature through SVIS.

KEYWORDS:

SPRT, Operating Characteristic, Ideal Standard, Air pollution, Standard, Realizable standard, Statistically Verifiable Ideal Standard (SVIS), Exceedences,

MSC: 62P12, 62B05

1. INTRODUCTION

The construction of SVIS using either hypothesis testing or a confidence interval approach. These SVIS were constructed by imposing the constraints on $P(X)$ (or φ_{1-t} quantile of the distribution of X). These SVIS can be described as single-level SVIS as these provide benefits of doubt either to the compiler or regulator. If we set the hypothesis $I_o : \omega \leq \omega_0$, the regulator has to give strong evidence to reject the hypothesis. If we set $I_o : \omega \geq \omega_0$ then the compiler will need strong evidence to reject the hypothesis. That is compiler will need to maintain the pollution level below to ω_0 . In this manner, from the above conversation, we see that there is an irreconcilable situation as the single-level SVIS allows being vindicated either to the controller or polluter. Thus, we want SVIS which can deal with both the polluter and controller risk. To this end, we want a twofold level SVIS. Barnett (1979) examined the idea of genuinely certain ideal gatekeeper point guidelines (SVIGPS) which are twofold level SVIS. The thought behind the idea of SVIGPS can be portrayed underneath. To cover the two dangers, we accept that the polluter and controller both consent to think twice about it. So instead of setting the single-level standard ω_0 both the regulator and polluter are ready to set upper and lower guard points respectively around the old single standard level ω_0 . The regulator sets an upper guard point above ω_0 at ω_2 with the assurance that at this level, pollution will be detected with probability $1 - \tau$ (a is small) Polluter sets a guard point below ω_0 at ω_1 with the assurance that for compliance the probability is $1 - \nu$ where ν is small. A defined standard is fair to both regulator and polluter for some appropriate value of ω_1 , ω_2 , τ and ν . When $\omega > \omega_2$ the probability of failing the standard is not less than $1 - \tau$ and when $\omega < \omega_1$, the probability of compliance of the standard is not less than $1 - \nu$. When ω is between ω_1 and ω_2 then there is uncertainty and we need more information in terms of observations. Hence for specified τ and ν , the concept of SVIGPS can be explained statistically as below:

If the mean pollution level ω of the pollutant exceeds ω_2 then the probability of failing the standard is at least $1 - \tau$ and the population is declared as out of compliance. If the mean pollution level ω is below ω_1 then probability is at least $1 - \nu$ for compliance of the population. To develop the above-explained SVIGPS, we will use the hypothesis testing framework based on the sequential probability ratio test (SPRT) developed by A. Wald (1947). We know that in the Neyman Person hypothesis testing approach, it is impossible to control both types of error (risk) τ and ν i.e. τ and ν cannot be made arbitrarily small for the fixed value of sample size n (say). In the sequential testing approach, the sample size is not fixed and both types of error (risk) τ and ν can be controlled. Thus, in this paper, we shall construct SVIGPS based on a hypothesis testing framework using a sequential probability ratio test.

Corresponding Author: Dr. Ankur Nehra

1. Research Scholar, Department of Statistics, Sunrise University, Alwar, Rajasthan

Email Id: bhatt90praveen@gmail.com

2. Department of Statistics, Sunrise University, Alwar, Rajasthan

3. Department of Mathematics, Dhanauri P.G. College, Dhanauri, Haridwar, Uttarakhand, 249404, India

Email id: nehradpgc123@gmail.com

2. SEQUENTIAL PROBABILITY RATIO TEST (SPRT)

In this part, we will examine SPRT for testing basic invalid speculation against a straightforward elective theory with the assistance of the probability capability.

Let X be an irregular variable having a likelihood dissemination capability

$$f(x, \varphi); \varphi \in \mathcal{D}; x \in R; \mathcal{D} = \{\varphi_0, \varphi_1\}$$

Suppose we wish to test the hypothesis

$$I_0 : \varphi = \varphi_0 \text{ versus}$$

$$I_1 : \varphi = \varphi_1$$

$$\text{and } \{X_i, i \geq 1\}$$

is a sequence of random variables generated through simple random sampling from the probability density function $f(x, \varphi)$. The SPRT $\phi(x)$ for testing I_0 versus I_1 can be described below:

At the n th stage of the experiment if

1. $\gamma_n(x) \geq A$

stop sampling with the rejection of I_0

2. $\gamma_n(x) \leq B$

stop sampling with acceptance of I_0

3. $B < \gamma_n(x) < A$

continue sampling by taking one more observation

where

$$\gamma_n(x) = \frac{L(x, \varphi_1, n)}{L(x, \varphi_0, n)},$$

$$L(x, \varphi_j, n) = \prod_{i=1}^n f(x_i, \varphi_j), \quad j = 0, 1$$

and A and B are some constants that are obtained in such a way that SPRT has the specified strength (α, β)

where

$$\eta = P[\text{Type I error}] = P[\text{Reject } I_0 \mid I_0 \text{ is true}] \text{ and } \nu = P[\text{Type II error}] = P[\text{Reject } I_0 \mid I_1 \text{ is true}]$$

SPRT can also be understood in another way in terms of S_n as below:

At the n^{th} stage of sampling if

1. $S_n \geq a$

Stop sampling with rejection I_0 . Where $a = \log A$

$$2. S_n \leq b$$

Stop sampling with acceptance of I_0 , Where $b = \log B$

$$3. b < S_n < a$$

The result is uncertain and another observation is taken.

Where

$$S_n = \sum V_i = \log [\gamma_n(x)] \text{ and}$$

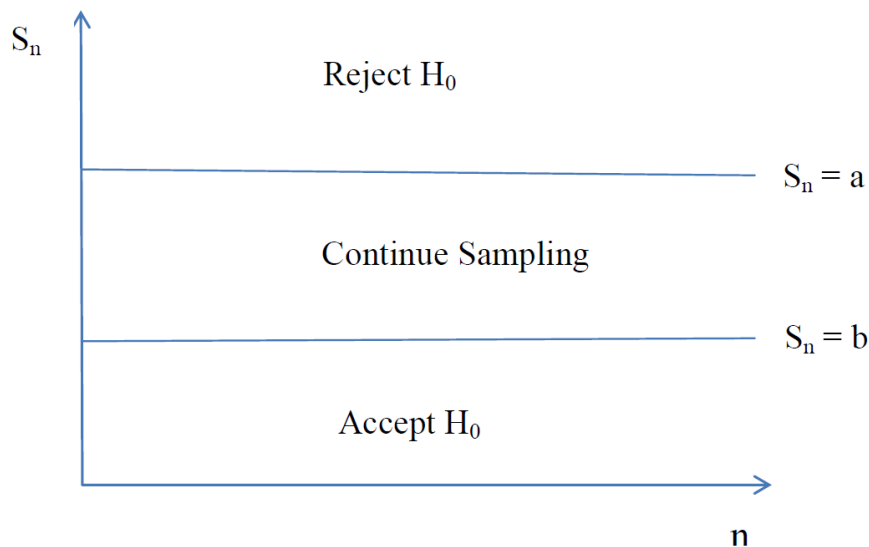
$$V_i = \log \frac{f(x, \varphi_i)}{f(x, \varphi_0)}$$

Along these lines, to distinguish high contamination occasions as they happen, we gather perceptions consecutively, and consistency testing is finished after every perception is gotten

Beneath we give the graphical portrayal of SPRT for testing

$$I_0 : \varphi = \varphi_0 \text{ sensus}$$

$$I_1 : \varphi = \varphi_1 \text{ in terms of } S_n .$$



Graphical Representation of SPRT

According to Wald (1947), some important properties of SPRT are as follows:

$$1. A \leq \frac{1-\nu}{\tau} \text{ and } B \geq \frac{\nu}{1-\tau}$$

$$\text{If } A \leq \frac{1-\nu}{\tau} \text{ and } B \geq \frac{\nu}{1-\nu}$$

then $\tau' + \nu' \leq \tau + \nu$ where (τ', ν') is the actual strength of SPRT for

$A = \frac{1-\nu}{\tau}$ and $B = \frac{\nu}{1-\tau}$ is the required strength.

2. SPRT terminates ultimately with probability one.
3. The OC function $L(\theta)$ of SPRT is given by

$$L(\varphi) \geq \frac{A^{h(\varphi)} - 1}{B^{h(\varphi)} - A^{h(\varphi)}}$$

Where $A = \frac{1-\nu}{\tau}$ and $B = \frac{\nu}{1-\tau}$

The value of $h(\varphi)$ is so obtained such that

$$P[L^{h(\varphi)}] = 1,$$

$$P\left[\frac{f(x, \varphi_2)}{(x, \varphi_1)}\right]^{h(\varphi)} = 1$$

4. The ASN function $E(n | \theta)$ is given by

$$P(n | \varphi) \approx \frac{bL(\varphi) + a[1 - L(\varphi)]}{P_\varphi(P)}$$

5. For all tests of

$$I_0 : \varphi = \varphi_0 \text{ verses}$$

$$I_0 : \varphi = \varphi_1$$

having strength (τ, ν) , the SPRT has the least possible values of $P(n | \varphi)$.

It is essential to take note of that the above property VI of SPRT is significant according to the hypothetical perspective. This property is demonstrated by Wald and Wolfowitz (1948).

3. CONSTRUCTION OF SVIGPS

In this segment, we will examine the development of SVIGPS through Wald's consecutive likelihood proportion test (SPRT) for

$$I_0 : \omega = \omega_1 \text{ against}$$

$$I_1 : \omega = \omega_2 (> \omega_1).$$

If the random variable

$$Y \sim \ln N(\omega, \rho^2), \text{ then}$$

$$X = \log Y \sim N(\omega, \rho^2)$$

The probability density function (pdf) of X is given by:

$$f(x, \omega, \rho) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\omega)^2}{2\sigma^2}\right] \quad \dots (1)$$

For the development of SPRT for testing

$$I_0 : \omega = \omega_1 \text{ against}$$

$$I_1 : \omega = \omega_2,$$

we assume that ρ^2 is known. If ρ^2 is known then without loss of any generality, we take $\rho^2 = 1$. For PRT observations are collected sequentially and at each stage of the experiment we compute ΣX_i . Then SPRT can be described in terms of

$$\Sigma X_i \text{ to test } I_0 : \omega = \omega_1 \text{ against}$$

$$I_1 : \omega = \omega_2 (> \omega_1).$$

Note that

$$V_i = \ln \left[\frac{f(x_i, \omega_2)}{f(x_i, \omega_1)} \right]$$

$$V_i = \ln \left[\exp \left(-\frac{(x_i - \omega_2)^2}{2} + \frac{(x_i - \omega_1)^2}{2} \right) \right]$$

$$V_i = \ln \left[\exp \left(-\frac{1}{2} [\omega_1^2 - \omega_2^2 + 2x_i(\omega_2 - \omega_1)] \right) \right] \quad \dots (2)$$

$$V_i = -\frac{1}{2} [\omega_1^2 - \omega_2^2 + 2x_i(\omega_2 - \omega_1)] \quad \dots (3)$$

$$S_n = \Sigma V_i = \Sigma x_i(\omega_2 - \omega_1) + \frac{n}{2}(\omega_1^2 - \omega_2^2) \quad \dots (4)$$

Now for testing I_0 against I_1 , the SPRT $\phi(x)$ can be stated as and below:

At the n^{th} stage of sampling if

$$1. S_n \geq a$$

Stop sampling with the rejection of H_0

Where

$$a = \log A = \log \frac{1-\nu}{\tau}$$

And

$$1. S_n = \Sigma V_i$$

$$= \Sigma x_i(\omega_2 - \omega_1) + \frac{n}{2}(\omega_1^2 - \omega_2^2)$$

$$= \Sigma x_i(\omega_2 - \omega_1) + \frac{n}{2}(\omega_1^2 - \omega_2^2) \geq a$$

$$= \sum x_i \geq \frac{a - \frac{n}{2}(\omega_1^2 - \omega_2^2)}{(\omega_2 - \omega_1)} \quad \dots (5)$$

2. $S_n \leq b$

$$\sum x_i \leq \frac{b - \frac{n}{2}(\omega_2^2 - \omega_1^2)}{(\omega_2 - \omega_1)} \quad \dots (6)$$

Stop sampling with acceptance of H_0

Where

$$b = \log B = \log \frac{\nu}{1 - \tau},$$

$$b < S_n < a$$

$$\frac{b - \frac{n}{2}(\omega_1^2 - \omega_2^2)}{(\omega_2 - \omega_1)} < \sum x_i < \frac{a - \frac{n}{2}(\omega_1^2 - \omega_2^2)}{(\omega_2 - \omega_1)} \quad \dots (7)$$

The result is uncertain and another observation is taken.

4. OPERATING CHARACTERISTIC FUNCTION OF SPRT

The OC (Operating Characteristic) function of SPRT is given by

$$L(\omega) \approx \frac{A^{h(\omega)} - 1}{B^{h(\omega)} - A^{h(\omega)}} \quad \dots (8)$$

where

$$A = \frac{1 - \nu}{\tau} \quad \text{and} \quad B = \frac{\nu}{1 - \tau}$$

The value of $h(\mu)$ is so obtained such that

$$P[L^{b(\omega)}] = 1,$$

$$P \left[\frac{f(x, \omega_2)}{f(x, \omega_1)} \right]^{h(\omega)} = 1$$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x-\omega)^2} \left(\frac{e^{-\frac{1}{2}(x-\omega_2)^2}}{e^{\frac{1}{2}(x-\omega_1)^2}} \right) dx = 1 \quad \dots (9)$$

Simplifying equation (9), we get

$$h(\omega) = \frac{(\omega_2 + \omega_1) - 2\omega}{(\omega_2 - \omega_1)} \quad \dots (10)$$

Now taking different values of $h(\omega)$. we shall get the different value of ω and by using equation (8), we get $L(\omega)$.
 OC function.

5. AVERAGE SAMPLE NUMBER FUNCTION OF SPRT

The ASN (Average Sample Number) function is approximately given by

$$P(n | \omega) \approx \frac{bL(\omega) + a[1 - L(\omega)]}{P_\omega(V)} \quad \dots (11)$$

Where

$$a = \log A = \log \frac{1 - \nu}{\tau},$$

$$b = \log B = \log \frac{\nu}{1 - \tau}$$

$$P_\mu(V) = \frac{1}{2}(\omega_2 - \omega_1)(\omega_1 + \omega_2 - 2\omega) \quad \dots (12)$$

Now putting the value from equation (5.12) in equation (5.11), we will get the ASN function. and

6. CONSTRUCTION OF SVIGPS FOR THE POLLUTANT NO₂

In this section, we illustrate the construction of SVIGPS with an example based on the test discussed in the above section. Specifically, we construct SVIGPS for the pollutant NO₂.

According to the NAAQ standard, 24 hourly monitoring values of the pollutant NO₂ should not exceed 80 µg/m³. So instead of setting the single level standards at $\omega_0 = 80$ both regulator and polluter are ready to set upper and lower guard points respectively around the single standard level at ω_0 .

Suppose the regulator set an upper guard point above ω_0 at 85 with the guarantee that at this level pollution will be detected with probability $1 - \tau$ ($\tau = 0.05$ say). And polluter set the guard point below ω_0 at 75 with the guarantee of compliance probability $1 - \nu$ ($\nu = 0.05$) Now we will construct SVIGPS as below:

Our test of the hypothesis will be:

$$I_0 : \omega = 75 \text{ against}$$

$$I_1 : \omega = 85 .$$

The SPRT $\phi(x)$ for testing I_0 against I_1 can be described as below:

i) We will reject I_0 if

$$S_n \geq a$$

Where

$$0.15 \sum x_i - 1.12 \frac{n}{2} \geq a$$

$$\sum x_i \geq \frac{3.01 + 1.09 \frac{n}{2}}{0.10}$$

Where

$$a = 3.01$$

ii) We will accept H_0 if $S_n \leq b$

$$0.15\sum x_i - 1.08 \frac{n}{2} \leq b$$

$$= \sum x_i \leq \frac{-3.01 + 1.08 \frac{n}{2}}{0.10};$$

Where

$$b = 0.199$$

iii) The result is uncertain and another observation is taken if

$$b < S_n < a, b^* < \sum x_i < a^* \text{ where}$$

$$a^* = \frac{3.01 + 1.08 \frac{n}{2}}{0.10} \text{ and}$$

$$b^* = \frac{-3.01 + 1.08 \frac{n}{2}}{0.10}$$

Now for the different stage of the experiment, the above test is described in the table below

| Sampling Stage | NO ₂ | Log NO ₂ | $\sum x_i$ | a^* | b^* | Result | Conclusion |
|----------------|-----------------|---------------------|------------|--------|-------|------------------------|-------------------|
| 1 | 43.01 | 4.011 | 4.011 | 29.008 | -20.5 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 2 | 42.04 | 3.727 | 7.485 | 33.25 | -16.2 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 3 | 41.2 | 3.718 | 11.2 | 37.625 | -11.8 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 4 | 51.09 | 3.928 | 15.13 | 42 | -7.42 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 5 | 48.88 | 3.889 | 19.02 | 46.375 | -3.04 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 6 | 43.08 | 3.756 | 22.78 | 50.75 | 1.333 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 7 | 42.99 | 3.775 | 26.55 | 55.125 | 5.708 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 8 | 38.99 | 3.664 | 30.22 | 59.5 | 10.08 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 9 | 39.98 | 3.689 | 33.9 | 63.875 | 14.46 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 10 | 40.95 | 3.716 | 37.62 | 68.25 | 18.83 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 11 | 39.07 | 3.659 | 41.28 | 72.625 | 23.21 | $b^* < \sum x_i < a^*$ | Continue Sampling |

| | | | | | | | |
|----|-------|-------|-------|---------|-------|------------------------|-------------------|
| 12 | 39.92 | 3.692 | 44.97 | 77 | 27.58 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 13 | 37.2 | 3.617 | 48.59 | 81.375 | 31.96 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 14 | 38.06 | 3.634 | 52.22 | 85.75 | 36.33 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 15 | 37.08 | 3.601 | 55.82 | 90.125 | 40.71 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 16 | 34.9 | 3.562 | 59.38 | 94.5 | 45.08 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 17 | 38.02 | 3.625 | 63.01 | 98.875 | 49.46 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 18 | 30 | 3.397 | 66.41 | 103.25 | 53.83 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 19 | 34.97 | 3.569 | 69.98 | 107.625 | 58.21 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 20 | 34.88 | 3.57 | 73.55 | 112 | 62.58 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 21 | 26.93 | 3.25 | 76.8 | 116.375 | 66.96 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 22 | 33.07 | 3.533 | 80.33 | 120.75 | 71.33 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 23 | 31.99 | 3.469 | 83.8 | 125.125 | 75.71 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 24 | 32.9 | 3.512 | 87.31 | 129.5 | 80.08 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 25 | 32.00 | 3.472 | 90.78 | 133.875 | 84.46 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 26 | 40.01 | 3.695 | 94.48 | 138.25 | 88.83 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 27 | 30.01 | 3.407 | 97.88 | 142.625 | 93.21 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 28 | 29.99 | 3.421 | 101.3 | 147 | 97.58 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 29 | 29.01 | 3.362 | 104.7 | 151.375 | 102 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 30 | 33.98 | 3.529 | 108.2 | 155.75 | 106.3 | $b^* < \sum x_i < a^*$ | Continue Sampling |
| 31 | 26.02 | 3.298 | 114.9 | 164.5 | 115.1 | $\sum x_i < b^*$ | Accept H_0 |

The OC and ASN functions of the above test are described in figure 1 and figure 2 respectively.

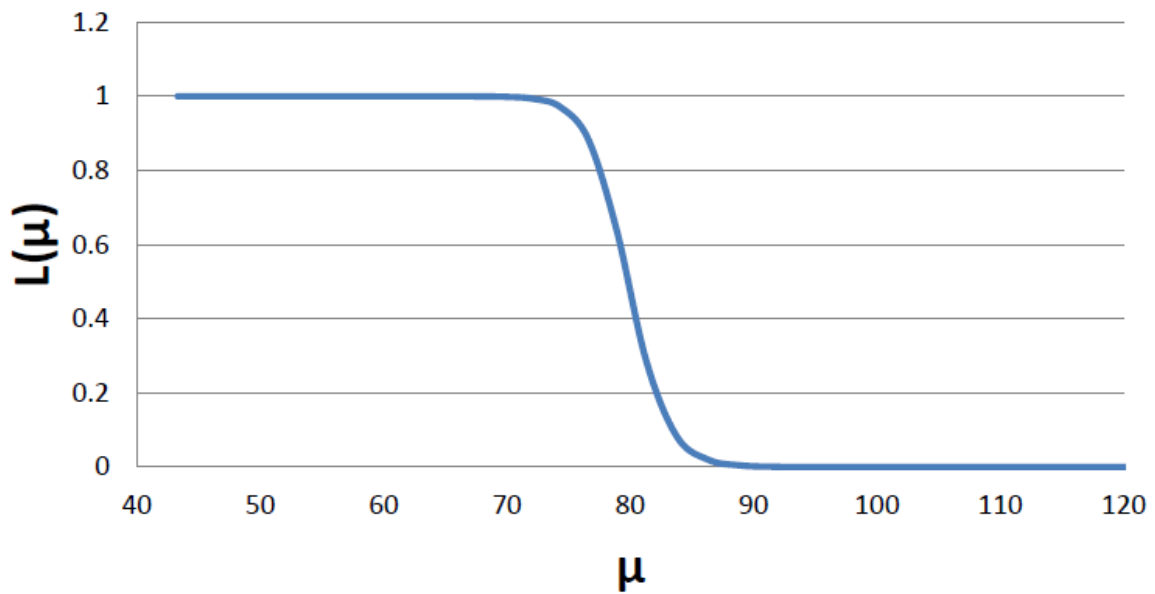


Figure 1: OC Curve for $I_0 : \omega_1 = 75, I_1 : \omega_2 = 85, \rho = 1, \tau = \nu = 0.05$

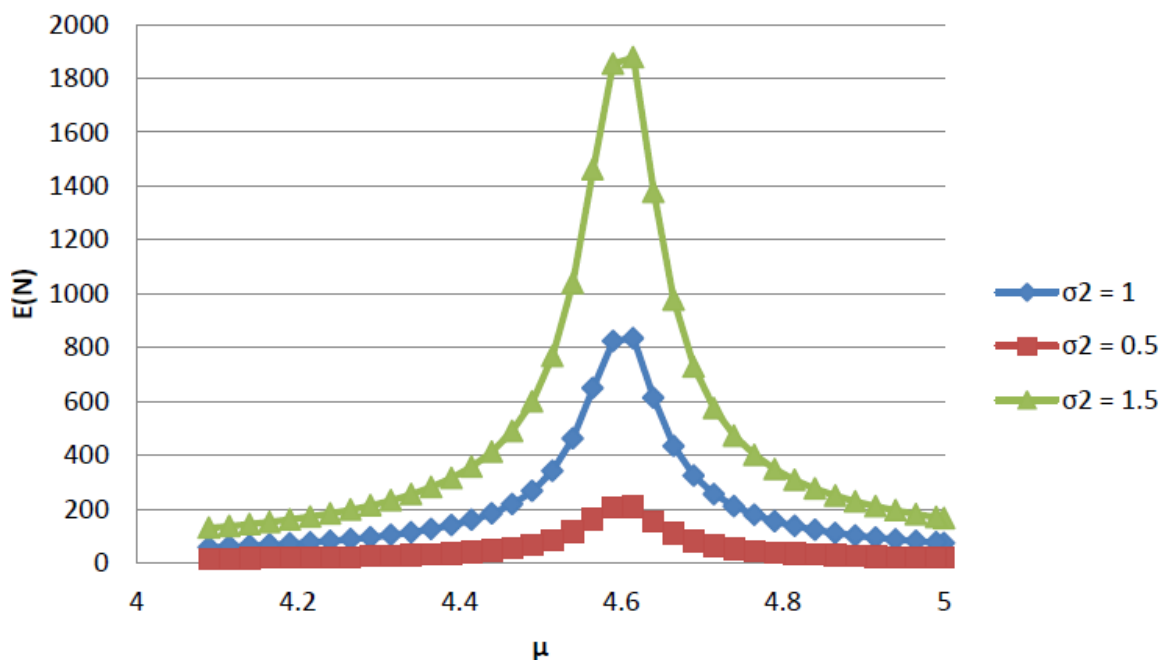


Figure 2: ASN Curve for $H_0 : \omega_1 = 75, I_1 : \omega_2 = 85, \tau = \nu = 0.05$

7. CONCLUSION

The Air idea of a city is checked with the help of the Best Standard set by the Regulatory body. The Ideal Standard can be portrayed as a decree about the quantity of occupants in poison which is set with no technique by which consistency is to be attempted or screened. For example, with the Best Norm, we can't enroll the probability that a particular checking site will be in charge in the upcoming year. Barnett and O'Hogan (1997) introduced the possibility of the Genuine Clear Ideal Norm (SVIS). The idea is to get the Ideal Norm together with a truly based rule of execution. To this end, customary verifiable contraption may be used. We could use Neyman Pearson's Methodology of Theory testing to fabricate SVIS. In this paper, we foster SVIS on account of the Neyman Pearson Hypothesis testing framework and examine the Air Nature through SVIS.

REFERENCES

1. Angus, J. E., 1994: Bootstrap one-sided confidence intervals for the lognormal mean. *The Statistician* 43, No. 3, pp. 395-401.
2. Baker, A. G., 1950: Properties of some tests of sequential analysis. *Biometrika*, 37, 334-346.
3. Barnett, V. and Bown, M. H., 2002: Best linear unbiased quantile estimators for environmental standards. *Environmetrics*, 13 (3), 295-310.
4. Barnett, V. and Bown, M. H., 2002: Setting environmental standards: a statistical approach. *Quantitative Methods in Current Environmental Issues*. Springer-Verlag, London, p99-109.
5. Beer, T., 2000: Setting air quality standards: a case study of the Australian National Environment Protection Measure for Ambient Air Quality. *Environmetrics*, 11, 499-510.
6. Chami, P., Antoine, R. and Sahai, A., 2007: On Efficient Confidence Intervals for the log-normal mean. *Journal of Applied Sciences* 7(13), 1790-1794.
7. Cox, L. H., 2000: Statistical issues in the study of air pollution involving airborne particulate matter. *Environmetrics*, 11,611-626.
8. Dupuis, D. J., 1998: Exceedances over high thresholds: a guide to threshold selection. *Extremes* 1:3; 251-261.
9. Finazzi, F., Scott, E. M. and Fasso, S., 2013: A model-based framework for air quality indices and population risk evaluation, with an application to the analysis of Scottish air quality data. *Appl. Statist.* 62, Part 2, pp. 287-308.
10. Gilbert, R.O. (1987) *Statistical Methods for Environmental Pollution Monitoring*. John Wiley and Sons, New York.
11. Gun A.M., Gupta M.K., and Dasgupta B. 2013: *Fundamental of Statistics*. World Press Private Ltd.
12. Guttorp, P., 2000: Environmental Statistics. *Journal of the American Statistical Association*, vol. 95, No. 449, pp. 289-292.
13. Guttorp, P.,2000. Setting environmental standards: a statistician's perspective. National Research Center for Statistics and the Environment, Technical Report, Number 48.
14. Krishnamoorthy, K. and Mathew, T., 2003: Inference on the means of lognormal distributions using generalized p-values and generalized confidence intervals. *J. of Statistical Planning and Inference* 115, 103-
15. Land, C. E., 1971: Confidence intervals for the linear function of the normal mean and variance. *Ann. Math. Statisti.* 42, 1187-1205.
16. Land, C. E., 1972: An evaluation of approximate confidence interval estimation methods for lognormal means. *Technometrics* 14, 145-158.
17. Land, C. E., 1973: Standard confidence limits for linear functions of the normal mean and variance. *J. Am. Statist. Assoc.* 68, 960-963.
18. Lehmann, E.L., 1986: *Testing of Statistical Hypothesis*. John Wiley and Sons.
19. Martinez-Florez, G., Bolfarine, H. and Gomez, H. W., 2014: The log power normal distribution with application to air pollution. *Environmetrics*, 25, 44–56.
20. Mood, A.M., Graybill F.A. and Boes, D.C. 2006, 5th reprint, Tata McGraw- Hill Publishing Company Limited, New Delhi
21. Niwitpong, S. 2013: Confidence intervals for the mean of lognormal distribution with restricted parameter space. *App. Math. Sci.*, Vol. 7, No. 4, 161 – 166.
22. Olsson, U. 2005: Confidence intervals for the mean of a lognormal distribution. *J. of Statistics Education*, Vol. 13, No. 1.
23. RaoM.N, and Rao HVN. 1989: *Air pollution* Tata McGraw Hill Publishing Company Limited. New Delhi.
24. Rohatgi, V. K. and Saleh, A. K. 2008, 2nd Edition: *An Introduction to Probability and Statistics*. Wiley India Pvt Ltd.
25. Tanushree Shekhawat, Yashbir Singh., *Air Quality of Jaipur City Based on Statistical Verifiable Ideal Standard (SVIS)*, *International Journal of Applied Environmental Sciences* ISSN 0973-6077 Volume 13, Number 2 (2018), pp. 121-133.
26. Taralden, G., 2005: A precise estimator for the log-normal mean. *Statistical Methodology* 2, 111-120.
27. Thompson, M.L., Cox, L.H., Sampson, P.D., and Caccia, D.C.,2000.
28. Varotsos, K. V., Tombrou, M. and Giannakopoulos, C., 2013. Statistical estimations of the number of future ozone exceedances due to climate change in Europe. *Journal of geophysical research atmosphere*, vol. 118, issue: 12, pg: 6080-6099.
29. Wald and Wolfowitz, 1948: Optimum character of SPRT, *Ann. Math.Stat.*, 19, 336-339.
30. Wald, A., 1947: *Sequential Analysis*. Wiley, New York.
31. Weerahandi, S. 1993: Generalized confidence intervals. *J. Am. Statist. Assoc.* 88, 899-905.
32. Zhang, B. L., Guan, Y., Leaderer, B. P. and Holford, T. R., 2013: Estimating daily nitrogen dioxide level: exploring traffic effects. *The Annals of Applied Statistics*, Vol. 7, No. 3, 1763-1777.
33. Zhou XH, Gao S. 1997: Confidence intervals for the log-normal mean. *Statistics in medicine*, vol. 16, 783-790, John Wiley & Sons Ltd.
34. Zhou XH, Gao S., and Hui, S. L., 1997: Methods for comparing the means of two independent log-normal samples. *Biometrics* 53, 1129- 1135.