International Journal of Psychosocial Rehabilitation, Vol. 24, Issue 02, 2020
ISSN: 1475-7192

Modelling and Forecasting the Mortality Risks to Investigate its Impact on Insurance Companies in Malaysia: Poisson Lee-Carter Model and Cairns-Blake-Dowd Model

[1]Ong Ying Ming, [2]Raja Rajeswari, [3]Hazlina Darman

Abstract--*The objective of this paper is to model and forecast the future mortality rates based on the data of insurance companies in Malaysia. This paper compares and applies three stochastic mortality models which are Lee-Carter, Poisson Lee-Carter and Cairns-Blake-Dowd models to forecast mortality rates for population of Malaysia. All genders and all single age data are fitted to all the models from year 2000 to year 2015. Based on the results, it shows that the fitted and forecasted values are having the different pattern of time index for Lee-Carter of family model and Cairns-Blake-Dowd model. To estimate the appropriate model among these models used in this study, the maximum log likelihood, AIC and BIC are used. According to the results of goodness of fit, Poisson Lee-Carter model has the best and the lowest values compared to other two models. The results from MAE and MAPE indicate that the forecast values are closer to the actual value. Therefore, Poisson Lee-Carter model is a good fit model to forecast Malaysia mortality rate.*

Keywords--*Forecasting,Mortality rate, Lee-Carter model, Poisson Lee-Carter model, Cairns-Blake-Dowd model*

I. INTRODUCTION

Life expectancy has improved significantly around the world. However, the rate of the mortality improvement appears to have slowed in some countries (Crawford, et al., 2008). Mortality risk is defined as the risk of people dying whereas longevity risk is the risk that the life of people is longer than expected (Cairns, 2013).The mortality improvement is one of the mortality assumption that must be considered in the pricing policy. Hence, the study that involved the mortality risks becomes vital for all insurance industries to alleviate or avoid the related risk contrary significances of financial (Hu & Cox, 2005).

Malaysia is one of the developing countries that is considered as a high longevity risk country in Asia where the mortality risk in Malaysia is considered low. Based on the data from Department of Statistics Malaysia,

[1]*School of Actuarial Science, Mathematics, and Qualitative Study (SOMAQS), Asia Pacific University of Technology and Innovation, Malaysia tp038242@mail.apu.edu.my*
[2]*School of Actuarial Science, Mathematics, and Qualitative Study (SOMAQS), Asia Pacific University of Technology and Innovation, Malaysia raja.rajeswari@staffemail.apu.edu.my*
[3]*School of Actuarial Science, Mathematics, and Qualitative Study (SOMAQS), Asia Pacific University of Technology and Innovation, Malaysia hazlina.darman@staffemail.apu.edu.my*

DOI: 10.37200/IJPR/V24I2/PR200395
Received: 18 Dec 2019 | Revised: 03 Jan 2020 | Accepted: 15 Jan 2020

there has an outstandingly rise in expectancy of life at birth of males and females from 1970 to 2005. It represents an improvement of 7.88 years and 7.89 years for males and females, accordingly (Ibrahim & Siri, 2015).

Lee-Carter (LC) Model is significant for modelling and forecasting the mortality risk and longevity risk. In 1992, the LC Model was introduced by Ronald D. Lee and Lawrence Carter and it has been used for forecasting mortality in the world (Gogola, 2014). Being one of the best suitable models among others in terms of the prediction error and easy implementation have made Lee-Carter (LC) Model very well-known for the modelling and forecasting the mortality rates over time, according to the studies conducted by Melnikov and Romaniuk (2006) and alsoKoissi, et al. (2006). In Malaysia, there are few studies that used LC Model to model and forecast the mortality rate (Ngataman, et al., 2016); (Kamaruddin & Ismail, 2018). A research in Japan used LC Model to forecast the death rates of Japan citizens and found the modification of the LC Model (Wilmoth, 1996). Besides, Lee and Rofman (1994) also used LC Model on their research to model the mortality rate in Chile. This same goes to Lee and Nault (1993) that have tested Canadian data with LC Model. Despite of this, there is little or even none of the studies that used Poisson Lee-Carter (PLC) Model in Malaysia. According to the research of Brouhns, et al. (2002), Log-Bilinear Poisson version of LC Model was able to obtain better accuracy despite the volatility of the death rates. Brouhns, et al. (2005) also conducted a research to apply simulation procedures to the extended of LC Model with the objective to improve the survival rate and longevity risk by measuring ambiguity. Most of the studies are comparing different methods of forecasting, such as the LC Model and PLC Model (Stoeldraijer, et al., 2013). However, most of the studies do not manage to conclude the best model that gives the best forecasting value. One of the reason is the death index of the world will not be linear all the time but it will only be highly linear for most of developed countries.

Meanwhile, the Cairns-Blake-Dowd (CBD) Model is one of the stochastic mortality models that is used to generate death indices from the time-varying parameters (Li, et al., 2012). In the CBD Model, the two components of risks that will be considered are:(i) systematic risks and (ii) unsystematic risks (Ibrahim & Siri, 2015). The model involves the risks in the model and forecast the mortality rates and also improvement of mortality. The mortality risk of systematic is the risk that can be undiversified through increasing the insurance portfolio (Gatzert & Wesker, 2014). In addition, it is the unexpected risk deviations from the expected death rates. The systematic mortality risk is demonstrated and accounted in dissimilar methods. In order to estimating the mortality risk, the mortality improvement is one of the ways that is used for the modelling of mortality.

1.1Statement of problem

Many developing countries are having incomplete statistics systems to calculate or monitor the mortality risk. One of the problems in mortality risks is life expectancy increases for most countries as shown by many researchers (Biehl, 2013). The mortality risks tend to be reduced over time for both male and female (Ibrahim & Siri, 2015). Due to decreasing of mortality risks, the life expectancy at birth inclines over time for both genders. Government is one of the main bodies that will look into the mortality risks which will be used to estimate the death rates (Biehl, 2013). Another body that concerned with this is the insurance companies. In order to price the insurance contracts or policies, they will look into the mortality rates and mortality risk (Bett, 2017).

DOI: 10.37200/IJPR/V24I2/PR200395
Received: 18 Dec 2019 | Revised: 03 Jan 2020 | Accepted: 15 Jan 2020

Therefore, government, insurance industries and corporate bodies are acquiring more knowledge of future mortality rates. The future forecasting of mortality will be important in the future where all related bodies will consider it as part of the assets or liabilities of their industries depending on whether it is increasing or decreasing.

1.2Research Objective

General Objective

Modelling and forecasting the future mortality rates based on the data in Malaysia.

Specific Objective

1. To find out the appropriate model among Stochastic Mortality Modellings.
2. To forecast the future mortality risks by using appropriate model.
3. To compare the insurance related products pricing based on the future and actual mortality risks.
4. To measure the forecast accuracy of the insurance related products pricing.

II. METHODOLOGY

Poisson Lee-Carter (PLC)Model will be conducted in this research where it involves different factors and the number of deaths that follows Poisson distribution. Although LC Model gives good fit for all the mortality data, it makes no assumption about the smoothness degree in death rates across ages and years (Blake, et al., 2018). In order to fully explain the mortality data, the original Cairns-Blake-Dowd model is available which includes assumption of smoothness in death rates across ages in the same year but not between years (Blake, et al., 2018). Therefore, another model needed for modelling and forecasting the mortality in this study is Cairns-Blake-Dowd (CBD) Model. In fact, not all stochastic mortality models are suitable for modelling and forecasting the mortality and longevity risks. A research shows LC Model has a good fitness to the Malaysian's mortality risk. However, it is not enough to predict the data. Thus, modelling and forecasting the mortality and longevity risks in Malaysia can be done by using an extended model from LC Model which is Poisson Lee-Carter Model and Cairns-Blake-Dowd Model (Kamaruddin & Ismail, 2018).

The first model used to measure the mortality risk is Lee-Carter (LC) Model, but instead of using homoscedastic error terms of the original Lee-Carter Model, it is better to use a Poisson variable (Brouhns, et al., 2002)because the death rates at older ages are more unstable compared to those at younger ages (Li, et al., 2012). The assumptions of numbers of death are given as below:

$$D_{x,t} \sim Poisson\left(E^{c}_{x,t}\mu_{x,t}\right)$$

Or

$$D_{x,t} \sim Binomial\left(E^{0}_{x,t}, q_{x,t}\right)$$

where

DOI: 10.37200/IJPR/V24I2/PR200395
Received: 18 Dec 2019 | Revised: 03 Jan 2020 | Accepted: 15 Jan 2020

$\mathbb{E}\left(\frac{D_{x,t}}{E^c_{x,t}}\right) = \mu_{x,t}and\mathbb{E}\left(\frac{D_{x,t}}{E^0_{x,t}}\right) = q_{x,t}$ given that:

$D_{x,t}$ = The number of deaths

$E^i_{x,t}$= The known exposure to the risk of death, $i = c, 0$.

General model of Poisson Lee-Carter and Lee-Carter Models are given by the following formula from *R* package "StMoMo":

$\eta_{x,t} = a_x + \beta^{(1)}_x k^{(1)}_t$

where

$\eta_{x,t}$: The link function of mortality, $\eta_{x,t} = logit(q_{x,t}) = \log\left(\frac{q_{x,t}}{1-q_{x,t}}\right)$

a_x : The static age function

$\beta^{(1)}_x$: The pattern of mortality changes across ages, age modulating term

$k^{(1)}_t$: The general trends of mortality through time, period index

One of the best way to project mortality and forecast the period index $k^{(1)}_t$ is using ARIMA processes (Villegas, et al., 2018). The response variable of link function $\eta_{x,t}$ depends on the mortality data which transforms a force of mortality measure into a suitable modelling form (Bozikas & Pitselis, 2018). If the numbers of deaths at age *x* and year follow a Binomial distribution, then the initial exposures to risk of deaths should be used in Lee-Carter Model where the standard Lee-Carter Model use the logit expression for the probability of death.

For Poisson Lee-Carter Model, the central exposures to risk of deaths should be used when the variable of the numbers of deaths at age *x* and year is under Poisson distribution assumption while the link function under Poisson Lee-Carter model takes the form as log expression (Bozikas & Pitselis, 2018). Lee and Carter (1992) advised the following set of parameter constraints to certify identifiability of the model which are the time index and the age modulating term, $k^{(1)}_t$ and $\beta^{(1)}_x$ where $\sum_t k^{(1)}_t = 0$ and $\sum_x \beta^{(1)}_x = 1$.

The second model used in this research is Cairns-Blake-Dowd (CBD) Model. It has played a vital role in forecasting mortality at higher ages, for example, ages starting at 60 and above. Thus, it becomes better model to model and estimate the mortality risk after a certain age. In this study, the variable of numbers of death is under Poisson distribution assumption.Then the CBD model is defined as below by the R package "StMoMo":

$D_{x,t} \sim Poisson\left(E^c_{x,t}\mu_{x,t}\right)$

$\eta_{x,t} = k^{(1)}_t + k^{(2)}_t(x - \bar{x}) + \varepsilon_{x,t}$

where

$D_{x,t}$: The number of deaths

$E^c_{x,t}$: The central exposure to the risk of death

DOI: 10.37200/IJPR/V24I2/PR200395
Received: 18 Dec 2019 | Revised: 03 Jan 2020 | Accepted: 15 Jan 2020

$\eta_{x,t}$: The link function of mortality, $\eta_{x,t} = \log\left(\frac{q_{x,t}}{1-q_{x,t}}\right)$

$k_t^{(1)}$: Time index in stochastic process and the intercept of the model

$k_t^{(2)}$: Time index of stochastic process and the slope of the model

$\bar{x}$: The mean age of the considered interval of ages

$\varepsilon_{x,t}$: The error term and follows Normal distribution with mean = 0 and variance $=\sigma_\varepsilon^2$.

In addition, the time indexes in stochastic process as $k_t^{(1)}$ and $k_t^{(2)}$ replicate time-related effects to the models that fitted the past data. Furthermore, the CBD Model has three significant principles for the mortality indices where (i) it can reproduce the varying age-trend of the improvement of mortality, but not only for the overall level, (ii) it also has the new-data-invariant property in CBD model and (iii) have clear interpretations (Li, et al., 2012). The time indexes are also modelled and forecasted by using ARIMA processes to fit the CBD Model. In CBD Model, the two age-period terms with pre-specified age modulating parameters and without static age function and cohort effect where $\beta_x^{(1)} = 1$ and $\beta_x^{(2)} = (x - \bar{x})$ CITATION And06 \l 17417 | (Cairns, et al., 2006)}. In this study, the parameter of the CBD model is assumed to be Poisson distribution of deaths with a log link function.

The data that is used in this study are from age 0 to age 95 whether it is either single age or grouped into 5-years age group and genders. In addition, numbers of deaths, mortality rate and population of Malaysia at age *x* are from year 2000 to 2016 from WHO online website. The data technique that is used in this study is *R*programme. For Lee-Carter model, the assumption of deaths follows Binomial distribution. Whereas, for the Cairns-Blake-Dowd and Poisson Lee-Carter Models,theirassumptions of deaths follow Poisson distribution.

To determine the best fitted model among those models, the value of maximum log-likelihood, information criteria will be used. To define the best parameter estimates of the stochastic mortality models, we shall maximize the log-likelihood model. The formula of log-likelihood can be defined in general but the formula will be different when the assumption of deaths is different (Villegas, et al., 2018).

The Poisson distribution of deaths assumption is given below:

$$dev(x,t) = 2\left[d_{xt}\log\left(\frac{d_{xt}}{\hat{d}_{xt}}\right) - \left(d_{xt} - \hat{d}_{xt}\right)\right]$$

The Binomial distribution assumption of deaths is given below:

$$dev(x,t) = 2\left[d_{xt}\log\left(\frac{d_{xt}}{\hat{d}_{xt}}\right) + \left(E_{xt}^0 - d_{xt}\right)\log\left(\frac{E_{xt}^0 - d_{xt}}{E_{xt}^0 - \hat{d}_{xt}}\right)\right]$$

To identify the best models by maximizing the log-likelihood, the value must be small and closer to 0 smaller value indicates the best fit of the parameters of models.

DOI: 10.37200/IJPR/V24I2/PR200395
Received: 18 Dec 2019 | Revised: 03 Jan 2020 | Accepted: 15 Jan 2020

In general, to compare the models, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) can be used (Bozikas & Pitselis, 2018). The correction of the AIC, the $AIC\,(c)$, is quite similar to AIC but it is more suitable when the sample size is small. The lower the values of AIC and $AIC\,(c)$, the more preferable the model is. Some researchers used the stock-recruitment to indicate that the AIC and BIC are both suitable models and concluded that BIC is better than AIC. Therefore, AIC, $AIC\,(c)$ and BIC will be used in this study. The AIC(c) and AIC models is given below:

$$AIC_i(c) = AIC_i + \frac{2k_i(k_i+1)}{n-k_i-1}$$

and

$$AIC_i = 2k_i - 2\log(\hat{L}_i) \; and \; BIC_i = (\log n)\, k_i - 2\log(\hat{L}_i)$$

where

$\hat{L}_i$ = the maximum likelihood estimate

k_i = the numbers of the effective parameters estimated

The period indexes, $k_t^{(i)}$ are assumed to be independent for each mortality model by using independent ARIMA (*p, d, q*) processes respectively which is described as:

$$(1 - \emptyset_1 L - \cdots - \emptyset_p L^p)(1-L)^d k_t = \delta_0 + (1 + \theta_1 L + \cdots + \theta_q L^q) e_t$$

where

L^p = the time lag operator based on d periods back with shifting data

δ_0 = constant drift parameters

$\emptyset_i$ = The Autoregressive coefficients with $i = \{1,2,3, \ldots p\}$

θ_i = The moving average parameters with $i = \{1,2,3, \ldots q\}$

e_t = white noise processes

In this study, the information criterions for parameters of ARIMA process will be used to estimate the best parameters and coefficients for the best forecast mortality model. We will price the insurance related products by using an appropriate mortality modelling method. In this study, the insurance related products that will be applied are: whole life insurance contract and whole life annuity contract which one is for death benefit and another one is for survive benefit. Both products will be affected by mortality rates. In general, the calculations of the discrete whole life assurance isgiven as:

$$A_x = \sum_{k=0}^{\omega-1} V^k \, q_{x+k} k p_x$$

and discrete whole life annuity is given as:

$$\ddot{a}_x = \sum_{k=0}^{\omega-1} V^k \, k p_x$$

where

DOI: 10.37200/IJPR/V24I2/PR200395
Received: 18 Dec 2019 | Revised: 03 Jan 2020 | Accepted: 15 Jan 2020

V^k : Discount factors, $V^k = \left(\frac{1}{1+i}\right)^k, i = interest\ rate$

$_kp_x$: The probability of survive k-year for age x.

q_{x+k} : The probability of die for age (x + k) in one year

ω : The omega age which is the maximum age

Based on the formula above, the insurance related products are calculated by considering the factors from mortality. In this study, we will use the actual mortality to compare the estimated mortality by using the appropriate model. To evaluate the accuracy of the forecasted value in pervious section of insurance related products, the forecast measurements that will be used are Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE). The lower values of MAE and MAPE provide a better insurance related forecasts.The measures of forecast's error for each model are given by:

$$MAE_k = \frac{1}{k-(\omega-1)+1}\sum_{k}^{(\omega-1)}|Y-X| * 100$$

$$MAPE_k = \frac{1}{k-(\omega-1)+1}\sum_{k}^{(\omega-1)}\left|\frac{Y-X}{X}\right|$$

where

k = The age selected, which means the age starts the insurance product

Y = The forecasted value by using forecasted mortality, $\hat{q}_k\ or\ \hat{A}_k\ or\ \hat{\ddot{a}}_k$

X = The actual value by using actual mortality, $q_k\ or\ A_k\ or\ \ddot{a}_k$

III. Results and Discussions

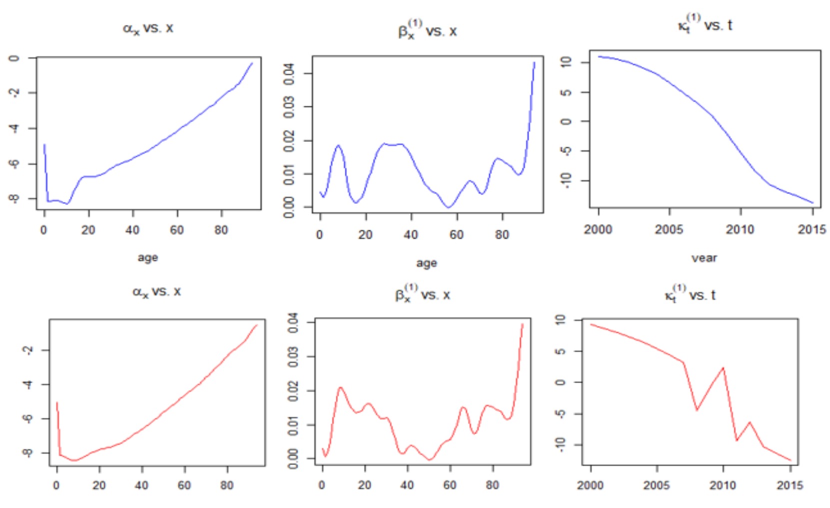

Figure 4.1.1 LC model: a_x, $\beta_x^{(1)}$ and $k_t^{(1)}$ estimated parameters for males (**top** panels) and females (**bottom** panels), aged 0-94, fitted in 2000-2015.

DOI: 10.37200/IJPR/V24I2/PR200395
Received: 18 Dec 2019 | Revised: 03 Jan 2020 | Accepted: 15 Jan 2020

International Journal of Psychosocial Rehabilitation, Vol. 24, Issue 02, 2020
ISSN: 1475-7192

The models used in this study are Lee-Carter (LC), Poisson Lee-Carter (PLC) and Cairns-Blake-Dowd (CBD) models. The parameters of these three models are estimated. Prior to this, the appropriate model must be chosen using

.

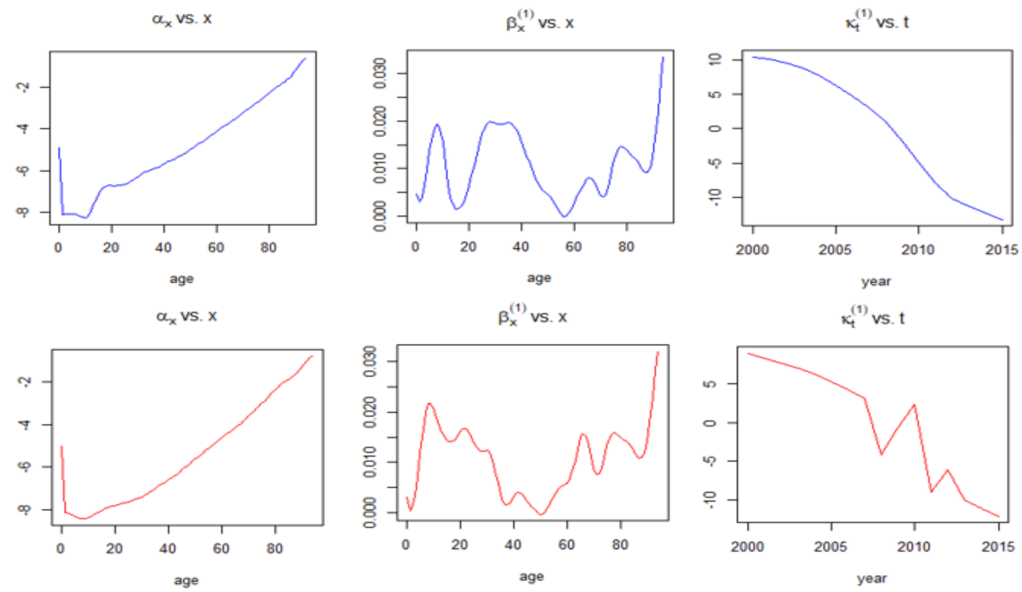

Figure 4.1.2 PLC model: a_x, $\beta_x^{(1)}$ and $k_t^{(1)}$ estimated parameters for males (**top** panels) and females (**bottom** panels), aged 0-94, fitted in 2000-2015.

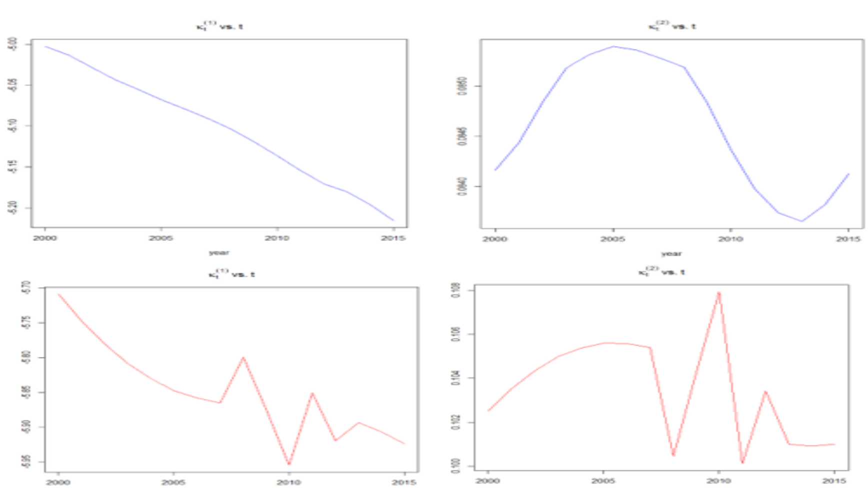

Figure 4.1.3 CBD model:$k_t^{(1)}$and $k_t^{(2)}$ estimated parameters for males (**top** panels) and females (**bottom** panels), aged 0-94, fitted in 2000-2015.

DOI: 10.37200/IJPR/V24I2/PR200395
Received: 18 Dec 2019 | Revised: 03 Jan 2020 | Accepted: 15 Jan 2020

International Journal of Psychosocial Rehabilitation, Vol. 24, Issue 02, 2020
ISSN: 1475-7192

In Figure 4.1.1 – 4.1.3 shows the estimated parameters of the three models. Figure 4.1.1 is for LC model under Binomial assumption and it is similar to the LC model under Poisson assumption which is under Figure 4.1.2. The static age function, a_x, is estimated by using the death rates so the patterns of LC and PLC are shown in Figure 4.1.4.

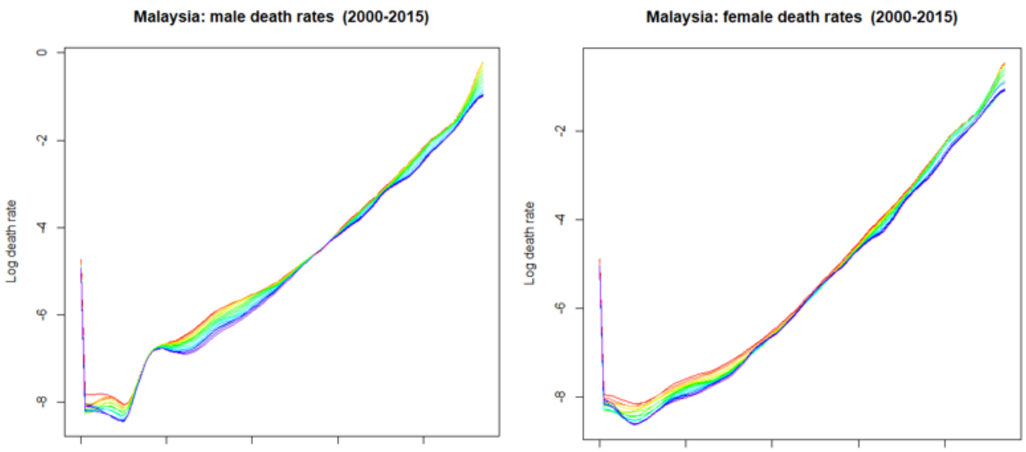

Figure 4.1.4 Log death rates for male (**left**) and female (**right**), aged 0-94, year 2000-2015.

Figure 4.1.4 shows the pattern of death rates is increasing annually by fitting the actual mortality data for Malaysia. From the figure above, LC and PLC models seem to have a better fitting compare to CBD model. Thus, even though the LC and PLC models are under different assumptions of deaths but both provide the similar pattern. It only have a slight difference on the values of coefficients parameters.

The most significant way is by looking at the values of those goodness of fit diagnostics which include the values of log-likelihood, AIC, BIC and etc. According to the formula of BIC, it penalizes the model parameters stronger than AIC and AIC(c). BIC will favor the models with lesser parameters than AIC (Bozikas & Pitselis, 2018). The most appropriate model has the best and smaller value of BIC.

Table 4.1.1The maximum log likelihood and the number of effective parameters along with AIC and BIC values of the mortality models for males and females with ranking order.

Males						
Model	**Maximum Log Likelihood**	**Effective Parameters**	**AIC**	**BIC**	**Deviance**	**Rank**
LC Model	-6242.939	204	12893.88	13980.48	454.33	**2**
PLC Model	-6233.410	204	12874.81	13961.41	404.80	**1**
CBD Model	-64244.790	32	128553.60	128724	116427.58	**3**

DOI: 10.37200/IJPR/V24I2/PR200395
Received: 18 Dec 2019 | Revised: 03 Jan 2020 | Accepted: 15 Jan 2020

Females						
Model	**Maximum Log Likelihood**	**Effective Parameters**	**AIC**	**BIC**	**Deviance**	**Rank**
LC Model	-6507.750	204	13423.50	14510.10	1636.70	**2**
PLC Model	-6506.695	204	13421.39	14507.99	1596.01	**1**
CBD Model	-70637.360	32	141338.70	141509.20	129857.35	**3**

Based on the table 4.1.1, it shows the results for each model that penalize the better model. Consequently, PLC and LC models have the smaller values of log likelihood, AIC and BIC. Unsurprisingly, CBD model holds the worst criteria ranking for both genders. Due to the worst criteria ranking of CBD model, it may be because CBD model is only for retired age which model the mortality risk for age 60 or above. The table also shown that the PLC model is the most appropriate model which has the smallest BIC value, 404.80 compare to LC model. In addition, PLC model is better than LC model where the results of goodness of fit are lower than values of LC Model even though the LC and PLC Models have the same or similar plots that had been discussed earlier. Therefore, PLC model is the most appropriate model for Malaysian's mortality data with all type of gender and age. The ARIMA processes are assumed to be independently for selected model period. We test the ARIMA processes by randomly putting all the parameters value of ARIMA. In order to determine the suitable parameters of ARIMA for the selected model with all genders, goodness of fit is provided.

Table 4.2The Maximum log likelihood, with AIC, AICc and BIC Values of the ARIMA estimates for PLC model for males and females.

Forecast Simulate Arima						
PLC Model for Male						
Arima Method (p,d,q)	**Sigma^2**	**Log Likelihood**	**AIC**	**AICc**	**BIC**	**Rank**
(1,1,0) with drift	0.2382	-10.11	26.22	28.40	28.34	**7**
(0,1,2) with drift	0.1562	-8.07	24.13	28.13	26.96	**5**
(0,2,0)	0.2270	-9.48	20.97	21.30	21.61	**6**
(0,2,1)	0.1152	-5.57	15.15	16.24	16.42	**1**
(0,2,2)	0.1244	-5.57	17.14	19.54	19.05	**2**
(1,1,2) with drift	0.1276	-5.46	20.92	27.59	24.46	**3**
Auto ARIMA (1,2,0) with drift	0.1579	-6.61	17.23	18.32	18.51	**4**
PLC Model for Female						
Arima Method (p,d,q)	**Sigma^2**	**Log Likelihood**	**AIC**	**AICc**	**BIC**	**Rank**
(1,1,0) with drift	11.5900	-38.72	83.44	85.62	85.56	**5**
(0,1,2) with drift	7.4750	-36.19	80.39	84.39	83.22	**2**
(0,2,0)	43.9600	-46.35	94.70	95.03	95.34	**7**

DOI: 10.37200/IJPR/V24I2/PR200395
Received: 18 Dec 2019 | Revised: 03 Jan 2020 | Accepted: 15 Jan 2020

(0,2,1)	15.6900	-39.97	83.94	85.03	85.22	**6**
(0,2,2)	7.8450	-36.06	78.12	80.52	80.04	**1**
(1,1,2) with drift	8.1380	-36.19	82.38	89.05	85.92	**4**
Auto ARIMA (2,1,0) with drift	8.6060	-36.20	80.39	84.39	83.22	**3**

According to Table 4.2, the different estimated parameters of ARIMA provide different values of log likelihood, AIC, AIC(c) and BIC. For males, ARIMA parameter used is ARIMA (0,2,1) where it gives the best results compared to other estimates parameters. ARIMA parameters for PLC model for female is ARIMA (0,2,2) which is different from the parameters for male, but it has the same smaller values as ARIMA (0,2,1) for male. Therefore, for simulating the forecast values of the time index that produce 1000 trajectories to simulate the 1000 paths for the time index of PLC model with ARIMA (0,2,1) and ARIMA (0,2,2), for male and females, respectively.

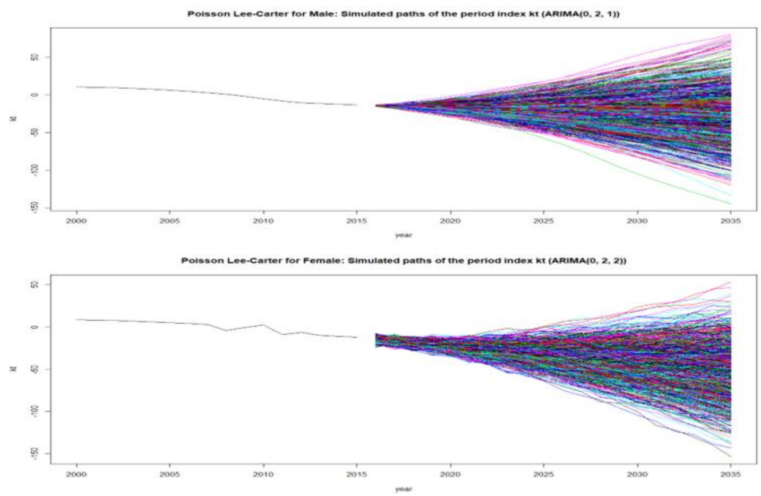

Figure 4.2.1 The simulated paths of the period index with ARIMA (0,2,1) and ARIMA (0,2,2), for males (top panels) and females (bottom panels), year period 2000-2035.

Figure 4.2.1 above shows that the graphs of simulated paths for the PLC model with ARIMA (0,2,1) and ARIMA (0,2,2), for male and female, respectively, indicate that the time indexes for male and female can be fitted by different paths from 2016 to 2035 as shownin Figure 4.2.1. After simulation of period index, the mean of these simulated period index is used for forecasting with 95 percent and 80 percent interval of period index from 2016 to 2035.

DOI: 10.37200/IJPR/V24I2/PR200395
Received: 18 Dec 2019 | Revised: 03 Jan 2020 | Accepted: 15 Jan 2020

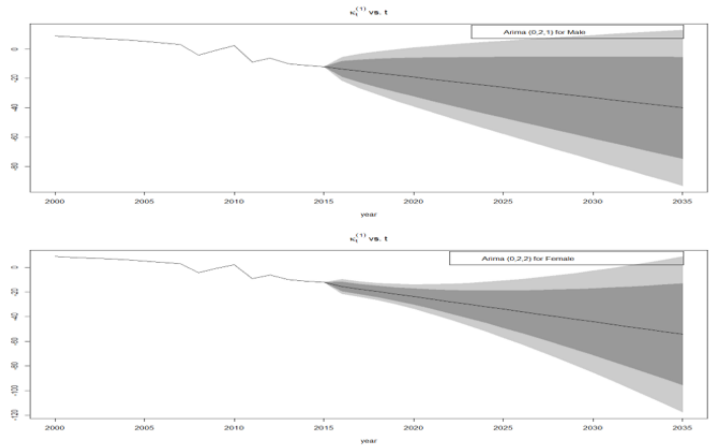

Figure 4.2.2 *The forecasting time index for PLC model with ARIMA (0,2,1) and ARIMA (0,2,2) for male (**top** panel) and female (**bottom** panel)*

Based on Figure 4.2.2, the period index forecast is done by using the mean of 1000 paths simulation of period index. Figure 4.2.2 shows that 95 percent and 80 percent of interval of mean time index. Therefore, the mortality rates of male and female can be forecasted by using the forecasted value of time index from Figure 4.2.2.Thus, Poisson Lee-Carter (PLC) Model is the best model for modelling and forecasting the mortality data of Malaysia for all genders and ages compared to other two models which are Lee-Carter (LC) and Cairns-Blake-Dowd (CBD) models. PLC model is the appropriate model that has the best smaller values of goodness of fit which included the maximum log likelihood, AIC and BIC.

There are few assumptions in this study that match the insurance related products pricing. Firstassumption is the interest rate is 10 percent per annual for single premium that based on Bank Negara Malaysia. Second assumption is the death benefit and survival benefit is RM1000 and paid at end of the year for whole life assurance and whole life annuity. Thirdly, the single premium is selected for the whole life assurance and whole life annuity without all expenses.It is easy to calculate the premium using the equivalence principle of pricing. Last assumption is the ages that enters the insurance policies are 20year and above, 60 years and above, and 65 years and above. The person whose age is 20 is selected due to the employment act of Malaysia where the person must be age 18 years or above then they are qualified to buy insurance.Age 60 and age 65 are chosen because it is the retired age in Malaysia.

In this section, we also measure theaccuracy of forecast the mortality risks for forecast value to the actual value. To measure the errors between the observed and the predicted values for the same period was evaluated by the forecast power of mortality model (Bozikas & Pitselis, 2018).

DOI: 10.37200/IJPR/V24I2/PR200395
Received: 18 Dec 2019 | Revised: 03 Jan 2020 | Accepted: 15 Jan 2020

Table 5.2Values of mean absolute error (MAE) and mean absolute percentage error (MAPE) measures of the forecasting period 2016 using actual rates for male and female with all ages.

	Male	Female
MAE	0.000615	0.001207
MAPE	3.568%	3.613%

Based on Table 5.2, it shows the MAE and MAPE of mortality risks (q_x) for male and female, which indicate the forecast value of mortality risks is close to the actual mortality risk. For example, the MAE for male is 0.000615, which shows that the forecast value need to plus or minus the MAE value. Based on the result of MAE for male and female, the MAE value for male is closer to zero which is better than female. The MAPE for male also shows the forecast value for male is better than female. It indicatethat PLC model for forecasting mortality is more suitable for mortality data of male in Malaysia compared to female.

Insurance products pricing is calculated by using equivalence principle. It uses the expected present value of the death benefits and the premiums with discount rates and some factors of probability including mortality risks and survival probability. The pricing of the insurance products uses the forecast mortality data and actual mortality data. To prove that the results on the estimated value is fitted well for Malaysian's mortality data, the MAE and MAPE of insurance related products pricing is provided which include whole life assurance and whole life annuity for all genders and age 20 and above, age 60 and above and age 65 and above based on the assumptions in this study.

Table 5.3Values of mean absolute error (MAE) and mean absolute percentage error (MAPE) measures of whole life assurance and whole life annuity for male and female with age 20, age 60 and age 65.

Male						
	Whole life assurance			whole life annuity		
	Age = 20	Age = 60	Age = 65	Age = 20	Age = 60	Age = 65
MAE	0.003301	0.005131	0.005402	0.021391	0.024538	0.023026
MAPE	2.033%	1.159%	1.115%	0.345%	0.528%	0.549%
Female						
	Whole life assurance			whole life annuity		
	Age = 20	Age = 60	Age = 65	Age = 20	Age = 60	Age = 65
MAE	0.00371	0.007819	0.008877	0.015072	0.02916	0.03064
MAPE	1.377%	2.189%	2.407%	0.328%	0.676%	0.752%

Based on the Table 5.3, it shows the pricing of MAE and MAPE, which indicates that all pricing is good when all the forecast errors are small and approaching zero. All the pricings are basically using the 10 percent

DOI: 10.37200/IJPR/V24I2/PR200395
Received: 18 Dec 2019 | Revised: 03 Jan 2020 | Accepted: 15 Jan 2020

interest rate of single premium for assurance and annuity. From the Table 5.3, whole life assurance for male has the best MAE and MAPE compared to female when male's age is elder and enter the policy. According to the mortality data of Malaysia, male has higher risk compared to the female who has same age as male. Thus, males can buy whole life assurance when they are young. Whereas, annuity is the best for the females when they are young.
The single premium for whole life assurance and annuity for male and female with age 20, age 60 and age 65 enter the contract that is calculated and shown as below:

Table 5.3.1The single premium by using forecast and actual values for male and female with the person who age 20, age 60 and age 65 enter the policy.

Single Premium						
Male						
	Actual			Forecast		
	Age = 20	Age = 60	Age = 65	Age = 20	Age = 60	Age = 65
Assurance	20.701	227.82	290.571	21.396	231.193	294.118
Annuity	10771.87	8472.07	7768.441	10764.35	8440.367	7737.557
Female						
	Actual			Forecast		
	Age = 20	Age = 60	Age = 65	Age = 20	Age = 60	Age = 65
Assurance	11.212	182.246	243.861	11.123	180.84	241.871
Annuity	10875.79	8963.349	8266.134	10876.99	8981.176	8288.648

The single premium is calculated based on the sum assured of RM 1000 with 10 percent interest rate. From the Table 5.3.1 above, it shows the forecast single premium close to the actual premium, which indicate that the PLC model is an appropriate model for forecasting the future mortality risks for Malaysia and using the future value to price the insurance related products.

IV. CONCLUSION

To conclude this study, mortality risks are not commonly done in studies of Malaysia for forecasting and modelling. This can be one of the limitations due to the lack of Malaysia's studies in mortality. This paper compared and applied three stochastic mortality models which are Lee-Carter, Poisson Lee-Carter and Cairns-Blake-Dowd Models to mortality rates for population of Malaysia. In addition, all genders and all single age data are fitted to all the models from year 2000 to year 2015. Based on the results, it shows that the fitted and forecasted values are having different pattern of time index for Lee-Carter model and Cairns-Blake-Dowd model.

To estimate the most appropriate model, the maximum log likelihood, AIC, and BIC are used. According to the results of goodness of fit, Poisson Lee-Carter model has the best and the lowest values compared to other two models. Thus, the Poisson Lee-Carter model becomes the most appropriate model for Malaysian population and is

DOI: 10.37200/IJPR/V24I2/PR200395
Received: 18 Dec 2019 | Revised: 03 Jan 2020 | Accepted: 15 Jan 2020

used for forecasting the future mortality rates. The estimation of ARIMA processes is conducted for the Poisson Lee-Carter model to simulate the best parameters of ARIMA for future years' time. Therefore, the log likelihood, AIC and BIC are essential for comparing the parameters and the models. The best parameters of ARIMA for male and female are ARIMA (0,2,1) and ARIMA (0,2,2), respectively and these parameters are selected to simulate the mortality index for 1000 paths. Besides that, the mortality rates are forecasted by using the simulated time index which is more accurate.

Last but not least, to determine the forecast accuracy by using the mean absolute error (MAE) and mean absolute percentage error (MAPE) that conducted in this study. All the MAE and MAPE are approaching to zero which indicating that the forecast values are closer to the actual value. We have concluded that Poisson Lee-Carter model is a good fit model to Malaysia mortality data for all genders and ages.

The limitation faced by the researcher in this study in the availability of researches conducted is insufficient on this topic in Malaysia to be used as references. The data are difficult to obtained as it can only be requested from the government or official organisations because it is protected data. In addition, the data do not have single age of mortality data in Malaysia, so some techniques may be needed to separate the data from group to single data as it will take long time to process. Time constraint was also a great challenge faced by the researcher in this research. Due to time constraint and difficulty in accessing the data, this depth of this research may not be as precise and specifically analysed as the data collected and tested is unabridged. Lastly, these obstacles affect the volume of data needed to do modelling and forecasting, which is the main element in this research (Li, et al., 2012). The insurance companies and government bodies can apply this model for modelling and forecasting the mortality rate for all genders and ages.

REFERENCES

1. Biehl, J., 2013. The Impact of life: How Mortality Improvement Affects the Life Insurance Industry Today.. Product Matters!, Issue 85, pp. 23-25.
2. Blake, D., Andrew Cairns, K. D. & Kessler, A., 2018. Still Living With Mortality: The Longevity Risk Transfer Market After One Decade, Edinburgh: The Institute and Faculty of Actuaries.
3. Bozikas, A. & Pitselis, G., 2018. An Empirical Study on Stochastic Mortality Modelling under the Age-Period-Cohort Framework: The Case of Greece with Applications to Insurance Pricing. Risks, 6(44), p. 34.
4. Brouhns, N., Denuit, M. & Vermunt, J. K., 2002. A Poisson log-bilinear regression approach to the construction of projected lifetables. Mathematics and Economics , Volume 31, pp. 373-393.
5. Brouhns, N., Van Keilegom, I. & Denuit, M., 2005. Boostrapping the Poisson Log-Bilinear Model for Mortality Forecasting. Scandanavian Actuarial Journal, Volume 3, pp. 212-224.
6. Cairns, A. J. G., Blake, D. & Dowd, K., 2006. A TWO-FACTOR MODEL FOR STOCHASTIC MORTALITY WITH PARAMETER UNCERTAINTY: THEORY AND CALIBRATION. The Journal of Risk and Insurance,, 73(4), pp. 687-718.
7. Crawford, T., Haan, R. d. & Runchey, C., 2008. Longevity Risk Quantification and Management: A Review of Relevant Literature, United State: Society of Actuaries.
8. Gatzert, N. & Wesker, H., 2014. Mortality Risk and its Effect on Shortfall and Risk Management in Life Insurance. The Journal of Risk and Insurance, 81(1), pp. 57-90.

DOI: 10.37200/IJPR/V24I2/PR200395
Received: 18 Dec 2019 | Revised: 03 Jan 2020 | Accepted: 15 Jan 2020

9. Gogola, J., 2014. A Comparison of Lee-Carter and Cairns-Blake-Dowd Stochastic Mortality Model. Mathematical Methods in Science and Mechanics, pp. 130-134.
10. Hu, Y. & Cox, S. H., 2005. Modelling Mortality Risk from Exposure to A Potential Future Exreme Event and its Impact on Life Insurance. Department of Risk Management and Insurance.
11. Ibrahim, R. I. & Siri, Z., 2015. A Study on Longevity Factor: The Case of Government Pensioner in Malaysia. International Journal of BUsiness and Society, 16(1), pp. 147-161.
12. Koissi, M.-C., Shapiro, A. F. & Högnäs, G., 2006. Evaluating and extending the Lee–Carter model for mortality forecasting: Bootstrap confidence interval. Insurance: Mathematics and Economics, 38(1), pp. 1-20.
13. Lee, R. & Nault, F., 1993. Modeling and forecasting provincial mortality in Canada, Montre'al: World Congress of International.
14. Lee, R. & Rofman, R., 1994. Modelacion y proyeccion de la mortalidad in Chile (Modeling and projecting mortality in Chile). Natas, 22(59), pp. 182-313.
15. Li, J. et al., 2012. Mortality Experience in Asia-Pacific and Modelling and Management of Longevity Risk. Insurance Risk and Finance Research Centre .
16. Maccheroni, C. & Nocito, S., 2017. Backtesting the Lee-Carter and the Cairns-Blake-Dowd Stochastic Mortality Models on Italian Death Rates. Risk 2017, Issue 34, p. 5.
17. Melnikov, A. & Romaniuk, Y., 2006. Evaluating the performance of Gompertz, Makeham and Lee–Carter mortality models for risk management with unit-linked contracts. Insurance: Mathematics and Economics, 39(3), pp. 310-329.
18. Ngataman, N., Ibrahim, R. I. & Yusuf, M. M., 2016. Forecasting the mortality rates of Malaysian population using Lee-Carter method, Malaysia: American Institute of Physics.
19. Stoeldraijer, L., Duin, C. V., Wissen, L. V. & Janssen, F., 2013. Impact of Different Mortality Forecasting Methods and Explicit Assumptions on Projected Future Life Expectancy: The Case of the Netherlands. Journal of Population Sciences., 29(13), pp. 323-354.
20. Villegas, A. M., Millossovich, P. & Kaishev, V. K., 2018. StMoMo: An R Package for Stochastic Mortality. R Package.
21. Wilmoth, J., 1996. Mortality projections for Japan: a comparison of four methods, New York: Oxford University Press.